SOME ASPECTS OF FLUID FLOW

BEING THE PAPERS PRESENTED AT A CONFERENCE ORGANIZED BY

THE INSTITUTE OF PHYSICS

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AND THE

REPORTS OF THE CONFERENCE DISCUSSION GROUPS

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FOREWORD

Any chemist, who has had the experience of going from laboratory to the full scale trials, gains a healthy appreciation of the importance of this subject. One can have the most beautiful chemical reaction in the laboratory but it will be useless unless one can move the reactants, can mix them so that the reacting molecules can actually collide and can separate and move the products !

The science of fluid flow has risen to the greatest heights of technical achievement in the fields related to aircraft, ships and certain new power units, rather than those related to immobile equipment. In the latter fields reasonably effective and economic operation can be obtained by simple "trial and error" methods. On the other hand aircraft would not leave the ground (or certainly would not recover the ground with safety) if it were not for the highly developed science of aerodynamics. Similarly, without the knowledge provided by this science the gas-turbine would not drive its own compressor let alone give an efficient power output.

Possibly due to these advances, fresh attacks are now being made on a wide range of fields and problems related to the processes that form the bases of many industries. It is to these fields that this conference was particularly directed, as shown by the examination of the present position set out in the first four papers. There was no intention to be comprehensive and many important fields are barely mentioned, such as river and tidal problems, ventilation and heat exchangers.

I would like to add one rather homely example, that has no doubt been the experience of many of us in recent years, the mixing of a powder such as dried milk or custard with a liquid such as water or milk. One might be inclined to disregard the instructions on the packet or the advice of the experienced and add the powder to the liquid. Thereafter will inevitably follow a large dissipation of energy with little result. The reverse process of adding first a small amount of liquid to the powder which is then worked to a smooth paste, followed by a second small addition of liquid leading to a thick cream before the final complete addition gives a much better result with less effort. The reason for this is that, only through the higher viscosity of the paste or thick cream can the agitation lead to shear stresses sufficient to break up the agglomerates of small particles of powder and hence to proper wetting and homogenization.

The enterprise of the Institute of Physics in organizing this Conference proved to be fully justified by the large attendance and the wide range of interests represented. This volume of its proceedings should prove of great value to the many scientists and engineers all over the world who are concerned with fluid flow problems.

CHARLES GOODEVE

November, 1950.

PREFACE

Advance proofs of the fifteen papers in this volume were circulated to some one hundred and fifty persons who attended the conference; the papers were therefore only presented in summarized form. The conference divided into parallel discussion groups: groups 1 and 2 on the first afternoon and groups 3, 4 and 5 on the second afternoon. The Chairmen of these groups, assisted by others, prepared summaries of the group discussions for presentation at the final session of the conference. These are printed in this volume, together with a concluding statement and summary.

The Board of the Institute of Physics wishes to place on record its appreciation of the assistance and ready co-operation of individuals and organizations. In particular, its thanks are due to:

(i) Sir Charles Goodeve, the Director of the British Iron and Steel Research Association, for his support and for opening the conference.

(ii) The conference organizing committee which was as follows:

Dr. B. P. Dudding (Chairman of the Committee) General Electric Co. Ltd.)

Mr. R. L. Brown (The British Coal Utilization Research Association)

Mr. F. Gill (Anglo-Iranian Oil Co. Ltd.)

Mr. M. P. Newby (British Iron and Steel Research Association)

Mr. C. I. Rutherford (Imperial Chemical Industries Ltd.)

Dr. E. Seddon (The United Glass Bottle Manufacturers Ltd.)

Dr. R. S. Silver (Federated Foundries Ltd.)

Mr. W. A. Simmonds (Gas Research Board)

Mr. W. F. Simonsen (The Water Tube Boiler Makers' Association)

Mr. M. W. Thring (British Iron and Steel Research Association)

Mr. R. C. Worster (The British Hydromechanics Research Association);

(iii) the authors of the papers;

(iv) the Chairmen and recorders of the discussion groups;

(v) Mr. M. W. Thring who prepared and presented the concluding statement;

(vi) The British Iron and Steel Federation for its kindness in allowing the conference to take place in its house "Ashorne Hill", near Leamington Spa, Warwickshire,

(vii) Messrs. Edward Arnold and Co., for undertaking the publication of the volume and for their willing co-operation in its preparation,

(viii) the printers, Messrs. Metcalfe and Cooper, Ltd., for producing advance proofs during a period of difficulty in the printing industry, and their cordial co-operation.

H. R. Lang,
Secretary and Editor.

The Institute of Physics.

November, 1950.

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1

Survey of Industrial Problems involving Turbulent Mixing of Fluids

By M. P. NEWBY, B.Sc., A.Inst.P. and M. W. THRING, M.A., F.Inst.P.

The British Iron and Steel Research Association.

ABSTRACT. The mixing of fluids is a process of great importance in many industries, for example in mining and in the steel and glass making industries. Several of these industrial applications of turbulence are considered and particular attention is given to the turbulent jet which is frequently used. A brief summary of the theories of turbulent mixing is given and the great difficulty of applying them to practical problems is pointed out. On the other hand, very valuable results can be obtained by the use of models.

1. INTRODUCTION

In chemical plant, furnaces, coal mines and many other industrial systems, the mixing of fluids is one of the key processes. The aim of mixing the fluids may be to bring about a uniform composition of materials or a uniform temperature, or by means of one rapidly moving fluid to set the other in motion; or it may be to allow a chemical reaction to take place between two fluids. It has long been known that turbulent diffusion gives a mixing hundreds of times faster than molecular diffusion and in industrial processes where one of the aims is to obtain a high degree of mixing in as small a volume as possible, turbulent diffusion is nearly always employed. The designer wants advice from the research worker on the best shape for the mixing system to give mixing in a reasonably small volume of apparatus and to have the minimum loss of energy due to friction and to be able to calculate the overall pressure losses, heat loss through the walls and other factors leading to inefficiency. Section II of the present paper attempts to give a more detailed survey of the problems of various industries coming under the general head of turbulent mixing, while in Section III the extent to which fundamental knowledge and various research methods have been applied and can be used is briefly surveyed. Finally, in Section IV the gaps in our present knowledge are indicated and an attempt is made to recommend the methods which must be used by designers under the present state of knowledge.

2. INDUSTRIAL PROBLEMS

The industrial problems which have been put forward by various industries may be broadly surveyed under the three headings of the general behaviour of jets, the mixing of fluids by jets, and other mixing methods.

(*a*) *General behaviour of jets.* One common use of jets is to obtain impact of a gas stream on to a solid surface or on to a bed of broken solids. Some examples of this are the cooling of the outside of glass tank blocks with high velocity air jets, certain forms of combustion system for gases in which the burning gases impinge on a surface and the flow of the jet of air into a fuel bed in, for example, a blast furnace, a nozzle type gas producer, Fig. 1, or the B.C.U.R.A. Downjet furnace Fig. 2.

In the downjet furnace the type of flow occurring is of great interest and not fully studied. The ingoing air enters the chamber as a cold jet and strikes the fuel bed. After combustion has taken place the hot gases

return having a larger volume but much lower velocity. Information is lacking as to the interaction of the ingoing and outgoing streams and the degree of entrainment which is taking place. It is not known how far back the jet can be taken from the bed and still give adequate aeration to the bed.

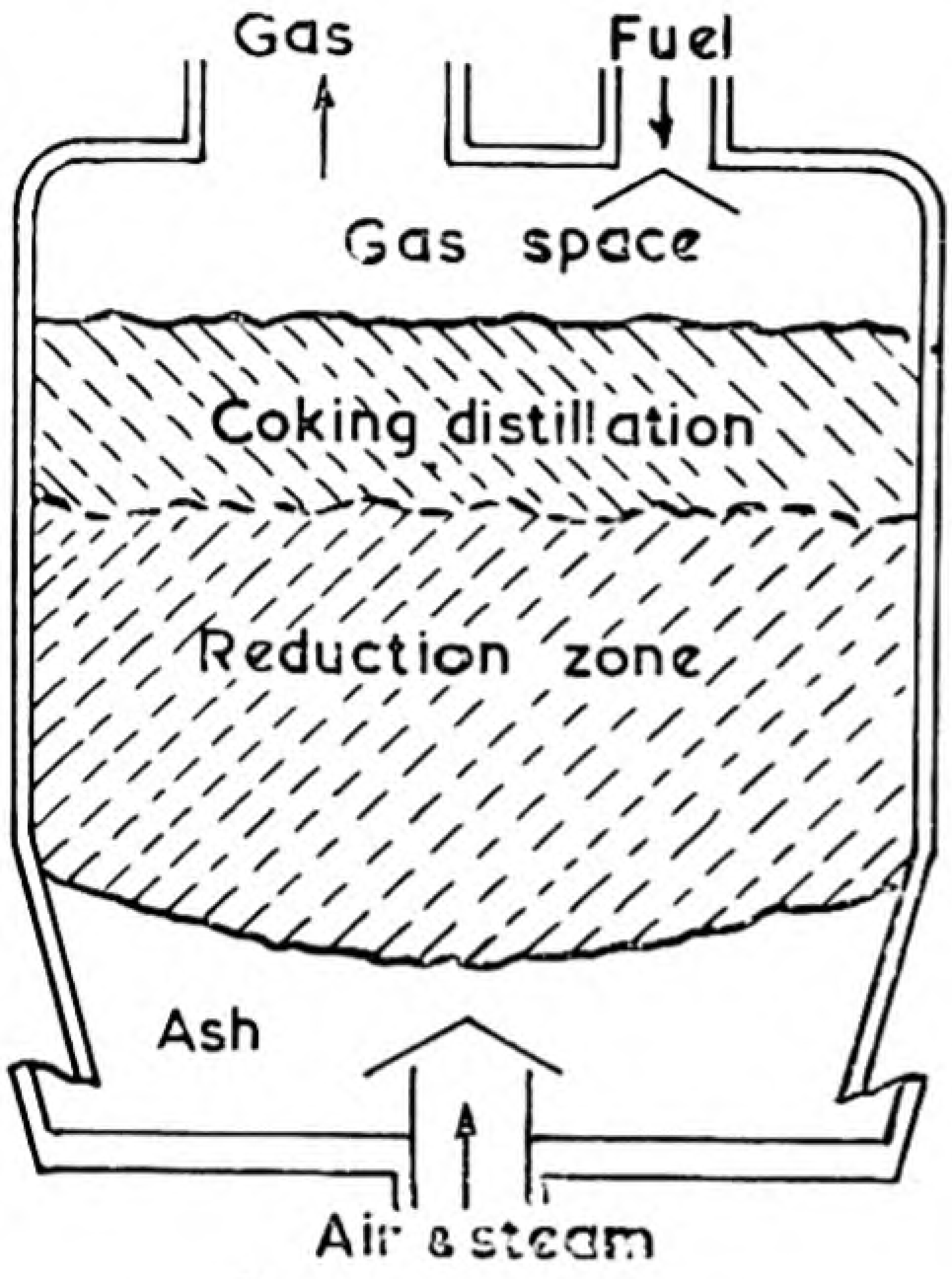

Fig. 1.—Gas producer.

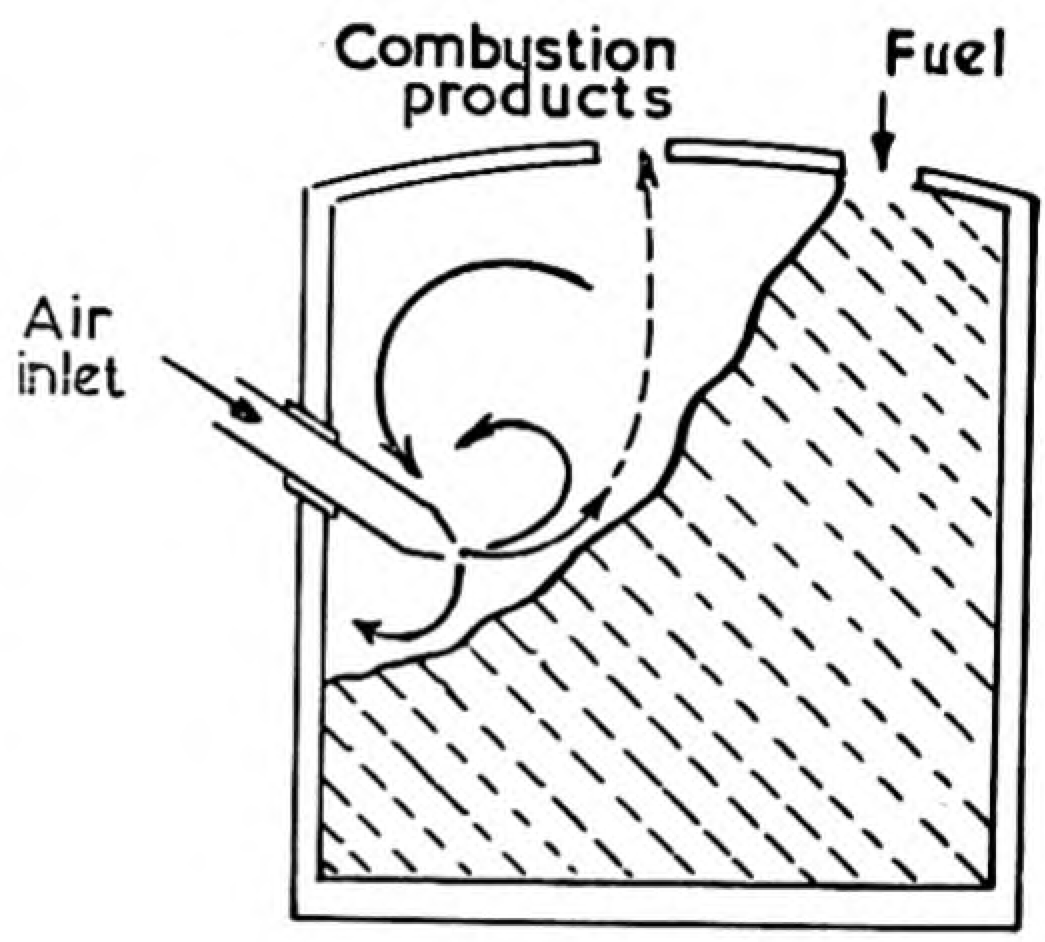

Fig. 2.—Downjet furnace.

As an example of the use of jets in boilers, steam jets are sometimes used to give circulation of the gases above the fuel and so facilitate ignition by entrainment of hot gases. Secondary air for the completion of combustion may be introduced using steam to entrain the secondary air and to import sufficient momentum to mix effectively with the furnace gases. The efficiency of these systems has never been estimated. In these latter cases the aerodynamic problems of the jet itself are further complicated by the actual consumption of the solid particles as they react with the gases and the corresponding formation of a cavity in the fuel bed which results in quite a different flow pattern when the system is reacting from that obtained when cold air emerges from a jet and impinges on cold solids. The subjects on which the designer of these systems would like more information are methods of visualising or predicting the flow pattern of the jet in an enclosed space and when it impinges on solid surfaces.

It is known that the jets which are used for mixing and combustion in many industrial furnaces cause recirculation of some of the combustion gases because the volume of gases entrained by the turbulent jet is greater than the volume of combustion air supplied to the furnace chamber. The recirculation and re-entrainment of waste gases by a flame causes a dilution of the active constituents. This is a disadvantage in so far as it reduces the temperature of the burning gases and as a consequence, reduces the radiant heat transfer. Also, if waste gases are entrained into the gas jet at the expense of air then complete combustion may be delayed and some of the

fuel gases may leave the furnace unburnt. The dilution can, however, be an advantage if the combustion would otherwise be too rapid and cause damage to the furnace lining owing to high local temperatures.

Another instance in which the flow pattern of a jet is of industrial importance arises in air conditioning where the interest lies in the distance from the nozzle at which the jet still preserves a certain fraction of its initial axial velocity[1]. A certain amount of work has been carried out on jets for special purposes. Thus Schmidt[2] has worked out the theory of a jet of warm air rising under its own buoyancy basing it on the Prandtl mixing length theory. He compared his calculations with experiment, using a heating coil as a jet source, but did not obtain very good agreement with theory for velocity and temperature profiles.

(*b*) *The mixing of fluids by jets.* Perhaps the most important industrial application of the mixing of fluids by jets is in the turbulent mixing of fuel, whether in the form of airborne pulverised coal, stream or air atomised oil, or gaseous fuel with a surrounding air flow. As discussed above, the thermodynamically ideal flame would be one in which mixing was as rapid as possible, but in practice the aim is more often to obtain mixing spread out fairly uniformly over a certain definite length in order to avoid excessive local heating. This is more true of high temperature furnaces such as glass tanks and open-hearth steel furnaces than it is of boilers where high rates of mixing and combustion can be more comfortably handled. In the flames of glass and steelmaking furnaces (a model of one is shown in Fig. 3), the air is always highly preheated and when producer gas is the fuel this also

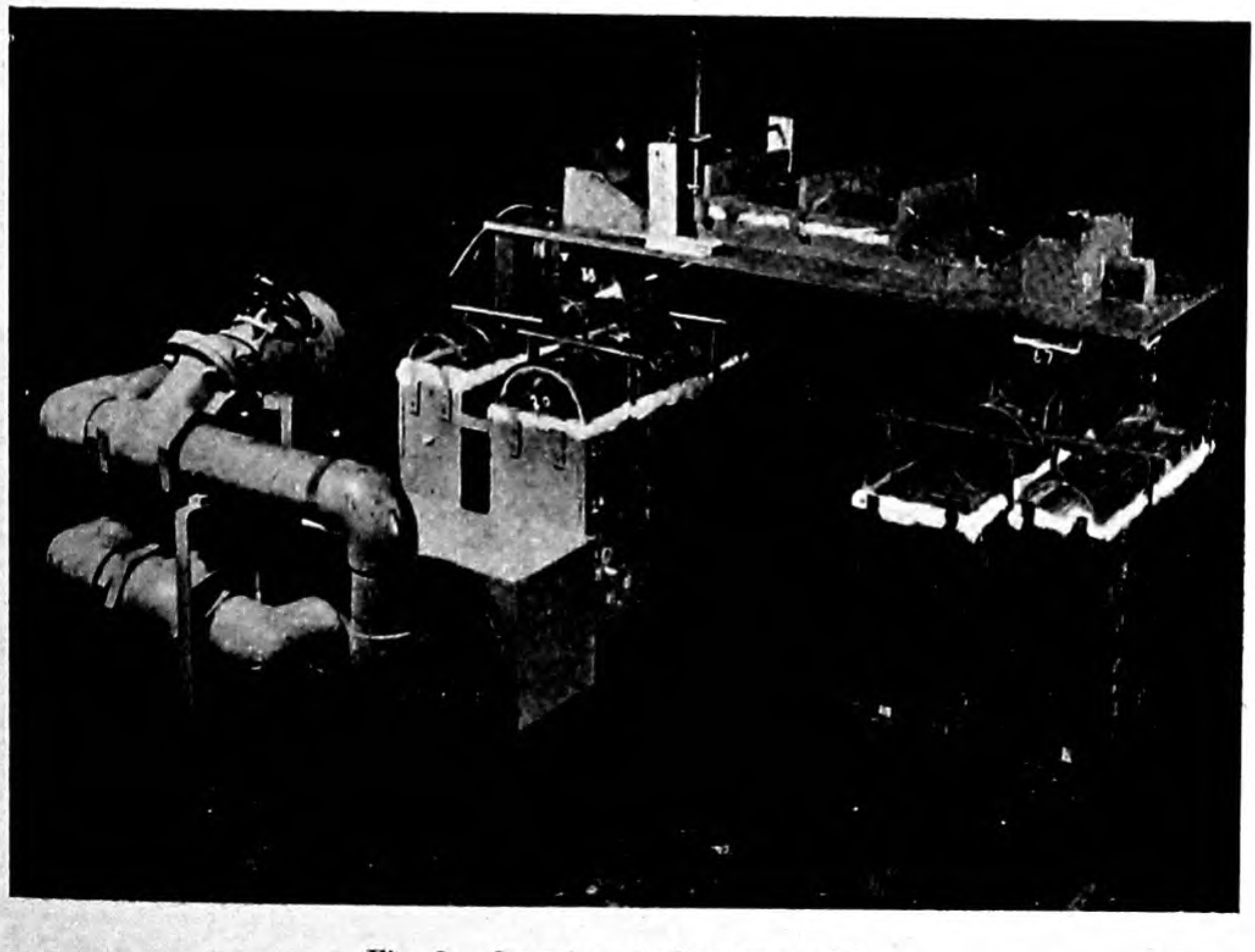

Fig. 3.—Open-hearth furnace model.

is highly preheated. Under these conditions the rate of chemical reaction is very high and it has been shown that combustion of fuel gas and air may be considered as falling instantaneously upon mixing. The development of the flame is thus almost entirely determined by the rate of turbulent mixing so that it is vitally important to know this rate and possibly also to be able to control various stages in the mixing. For example, it may be desirable in order to produce a luminous flame to have relatively slow mixing for a short distance, followed by very rapid mixing. The behaviour of fuel jets is often complicated by the fact that they occur in regions where the walls have a significent effect on the jet(3),(4), and in some cases, such as boilers, the buoyancy arising from the fact that the flame jet is lighter than the surrounding gases also introduces a bending effect which is difficult to predict. A further complicating factor arises when the flames impinge on walls or on the surface of the molten materials as they are usually designed to do in steelmaking furnaces. The expansion of the gases due to the liberation of heat by combustion also causes departures from the behaviour of simple jets although this effect is less when the air and fuel gas are highly preheated before combustion so that their actual further expansion on combustion is only about 50% by volume.

In steelmaking furnaces liquid fuels are nearly always atomised with steam (Fig. 4), although air has been tried in some cases. It is claimed

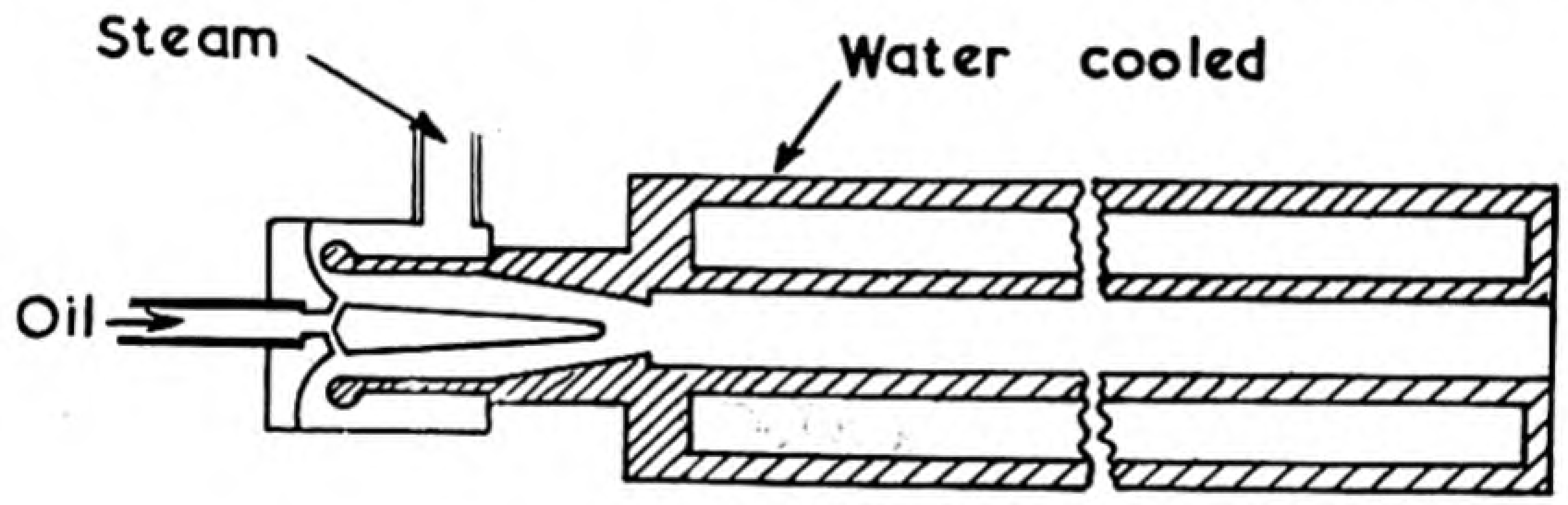

Fig. 4.—Steam atomiser.

that pressure atomisation gives too short and wide a flame which tends to impinge on the side walls and roof and cause excessive local heating. In the glass tank furnace pressure atomisation has been successfully used but in all cases the atomisation processes of large burners are of great importance as there is a great danger of too large droplets emerging from the burner. Pressure atomisation has been more fully studied than steam and air atomisation, mainly because it can be studied on a smaller scale. It is not known how far these latter atomisers could be improved to give the same degree of atomisation with smaller pressure requirements. It is likely that the droplet size has a rather wide size distribution. There is need for a fundamental study, both experimental and theoretical of the breakup of liquids by an impinging gas.

Another very important application of the turbulent jet to mixing is the case where it is used to impart momentum to a surrounding stream in an injector or ejector(5) (Fig. 5). One common example of the use of injectors is to provide the flow to a gas producer. In order to make producer

gas with a fairly high gas efficiency, it is desirable to introduce steam into the air flow to the coal bed. This fact is made use of by using the steam as the actuating fluid in an injector to introduce the air. The single jet injector

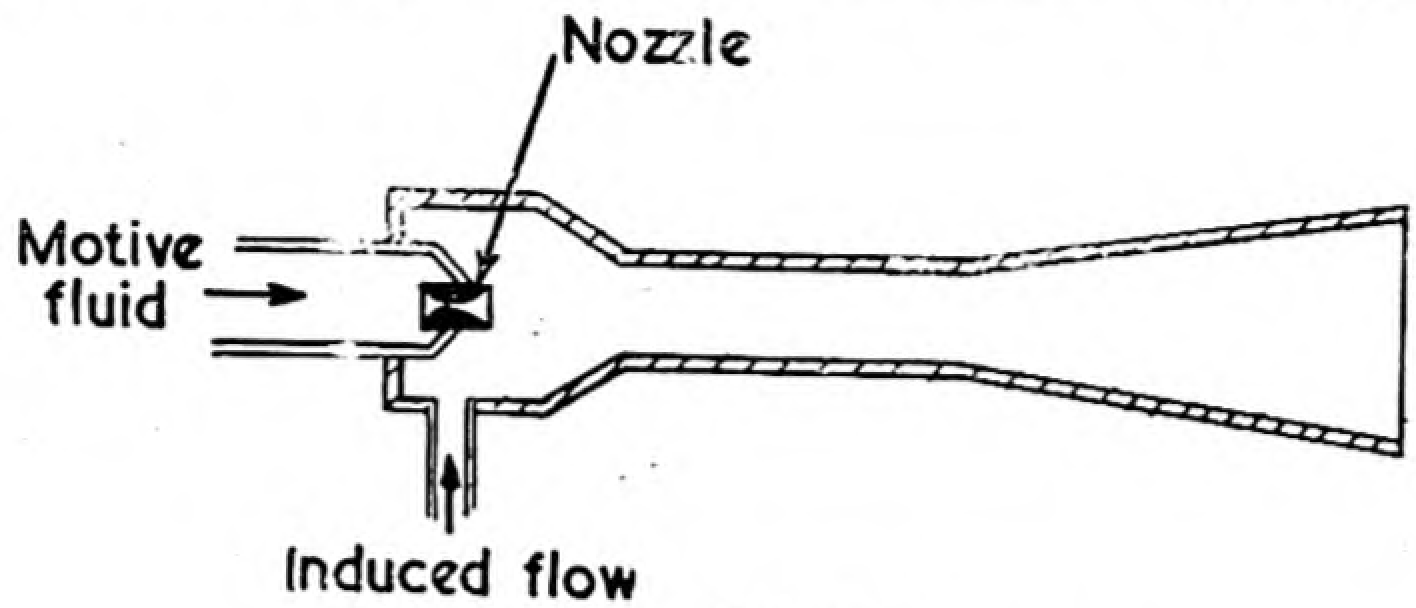

Fig. 5.—Injector.

has in many cases been replaced by a multiple jet system because it takes up less space. For high gas pressures injectors cannot be used and fans are used instead, and a fan takes up even less space than a multiple jet injector.

Design difficulties occur owing to the lack of available space below many producers. With other types, however, the mixing is brought about on the stage and the mixture is subsequently led to the producer which allows greater freedom in design.

Large ejectors are sometimes used for removing hot gases from furnaces and obtaining draught. The efficiencies of these installations only rise to about 15% for single stages. So far the formulæ used for calculating the performance of these large ejectors are essentially empirical(6).

(c) *Mixing of fluids without the use of jets.* Convection currents can be used to obtain mixing in industrial systems, both in the obvious case of mixing parts of the fluid which have a different temperature but also to mix parts with a different combustion ; for example, in glass tank furnaces convection mixing of the batch is of considerable importance although the conditions are not ideal for this method of mixing since the heat is supplied from the top. These currents have been demonstrated by Peychès(7) employing models, Fig. 6. The viscosity of molten glass is such that turbu-

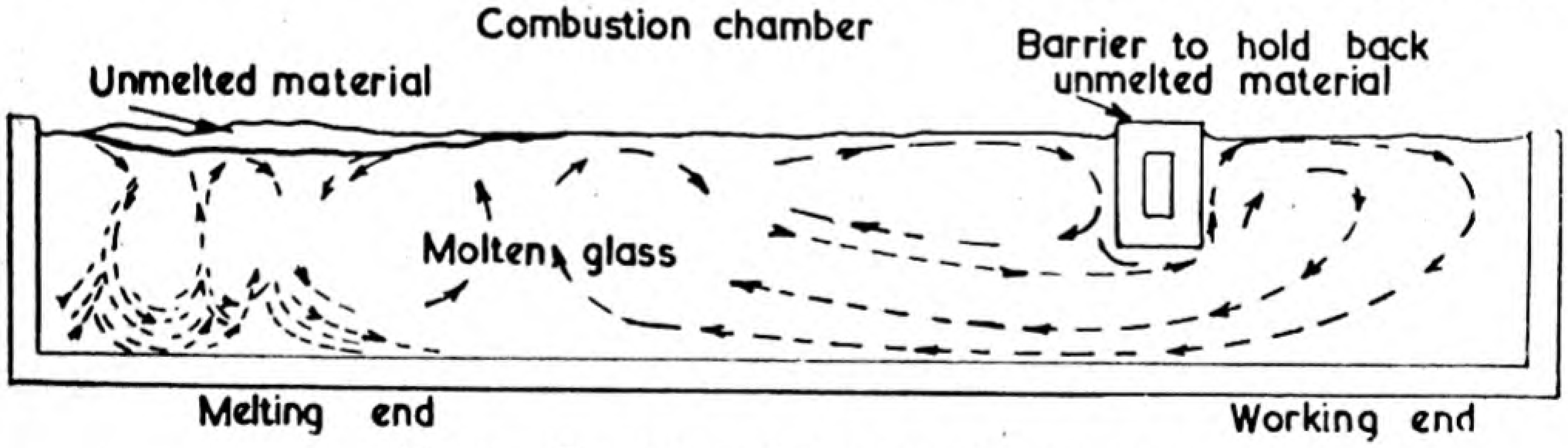

Fig. 6.—Circulation of glass in a glass furnace.

lent flow will not develop generally within the furnace but increased rate of circulation in the bath does produce more homogeneous glass. This is presumably largely brought about by allowing ordinary processes of diffusion to operate between the adjacent masses brought into close proximity by the circulation.

The rate of circulation of glass in the tank may be increased by lengthening the furnace and increasing the heat input to the glass. The heat efficiency of the furnace, however, drops on account of this.

The use of baffles and screens to obtain mixing is of importance in a number of industries, particularly in the mining industry where gases coming from the face may contain 30% and even up to 80% methane and these must be rapidly mixed with ventilating air to give a mixture with less than 6% by volume. Since methane is lighter than air there is also a danger of the formation of a stratified layer of methane near the roof. Mixing is usually obtained at present by hanging a cloth across the returning main air channel a short distance downstream from the source so that there is a gap between the cloth and the roof. This increases the turbulence in the wake of the cloth and brings about the mixing. More information is needed on the efficiency of this mixing and effect of wall roughness on the speed at which mixing is brought about. If mixing occurs appreciably faster above rough roadways, it will be possible to collect samples of gas fully diluted with air nearer to the source. This would be very valuable in allowing samples to be taken before further contamination from other sources had taken place. The use of local jets to accelerate mixing in these cases is also an important possibility provided good mixing can be obtained without excessive power absorption.

A second mining problem concerns the mixing of air which takes place when air from a side gallery joins that in the main air channel. It is not known under what conditions it is necessary to supply the side gallery with its own air and when the mixing which occurs at its entrance can be sufficient to ensure adequate ventilation. No work seems to have been done on the mixing problems which occur in mining.

Problems of dilution are also of great interest in fire fighting. Dilution is already used to a small extent by firemen to improve the visibility. The optimum conditions of jet size and air velocity for this purpose are not known.

(*d*) *Summary of industrial problems.* In all the cases considered above, the designer would like to know the general overall flow pattern, i.e., the average velocity at a point and, in particular, the trajectory of the axis of a jet or of the centre of each stream in a mixing system, and the angle of spread of this stream, or the pattern of lines of equal mixing. Secondly, he would like to know the degree of mixing at various points and the extent to which the higher velocity stream has imparted momentum to the lower velocity one. Thirdly, he would like to know the overall pressure loss, and lastly, he wants to know the effect of changes in the shape of jet, of the surrounding walls, and of any solid objects inserted into the stream.

3. FUNDAMENTAL KNOWLEDGE

(*a*) *General theory of turbulence.* Most of the work on mixing and turbulence has started with a rather different aim than those of the designer of an industrial mixing system, outlined above, and the approach has not proved very fruitful for the present purpose. The method of approach

which has been most extensively followed is a fundamental one so that in the long run it will be of the greatest value, but in the meantime designers have found it necessary to use less fundamental methods such as the use of models. Broadly speaking the position in more fundamental investigations is as follows :—

The earliest treatments were those of Boussinesq[8] and Reynolds[9]. Boussinesq assumed that the turbulence transfers momentum and material by means of packets or eddies so that there is an eddy viscosity which is very much higher than the true physical viscosity and an eddy diffusion which is very much more rapid than molecular diffusion. He does not, however, produce a method by which the eddy viscosity can be calculated, so that it is in fact a parameter which has to be inserted into the results. Reynolds' treatment was somewhat similar but he expressed the stresses in terms of the deviations of the instantaneous velocity components from the time mean velocity components at a given point. Here again no theoretical value can be predicted for the coefficients. The difficulties have become greater as each successive worker has introduced assumptions which attempt to give a more accurate picture of turbulent motion but lead to mathematics which is more and more difficult to apply to industrial problems. Thus Prandtl[10] assumes that the eddy packet maintains its momentum constant for a certain distance, which is called the mixing length, and then merges it with the surrounding fluid. This mixing length is to some extent analogous to the mean free path of the molecule in the behaviour of a streamline gas but is of course very much greater in magnitude. This assumption is not, however, physically justified since the eddy does not remain an entity in the way that a gas molecule retains its motion and then suddenly changes it. Taylor[11] used the assumption that any packet of air has a constant vorticity during turbulent diffusion. Von Kármán[12] introduces the idea of turbulent similarity according to which the mixing pattern is independent of scale. The theory which has been most extensively applied in other fields is Taylor's[11] statistical theory of turbulence in which the correlations between the velocities of particles at different places are considered or of a particle at successive instants.

Quite apart from the fact that none of these theories enable the parameter on which they depend to be predicted from the known physical properties of the gas they can very rarely be applied to industrial problems because the geometry of the industrial system is too complicated. Among the difficulties met in practice are the facts that the turbulence must often be anisotropic and its scale is often comparable with that of the chamber in which it is found, both of which conflict with the assumption of these theories. Some of the theories can be applied to the simple case of an axially symmetrical jet infinitely far from the surrounding walls.

In making an estimate of the degree of mixing occurring in a simple system Boussinesq's early concepts of an eddy viscosity and an eddy diffusion are simple and may be most useful. Although there is no calculation for the distribution of these quantities if they can be estimated by comparison with experimental results from similar systems, results can be

obtained comparatively simply. In special cases the distribution is found to be surprisingly simple. For example, in the case of an axially symmetrical jet the eddy viscosity is constant over the conical core and is proportional to the original diameter and velocity of the jet. The eddy diffusion coefficient is not so nearly constant but for practical purposes may be assumed constant over the centre of the core[13]. Again, in the diffusion across flow between two parallel plane boundaries, the diffusion coefficient is constant across the section except near the walls.

(*b*) *Theory and measurement of turbulent jets.* When a fluid stream issues from a small aperture at a high velocity into a large chamber, the turbulent mixing process at the edge of the high velocity stream soon causes it to entrain gas from the surroundings under conditions in which the forward momentum is conserved. The velocity profile thus spreads out with distance from the aperture of the jet the angle of spread being about 24° for a free jet (Fig. 7). The velocity on the axis remains constant for

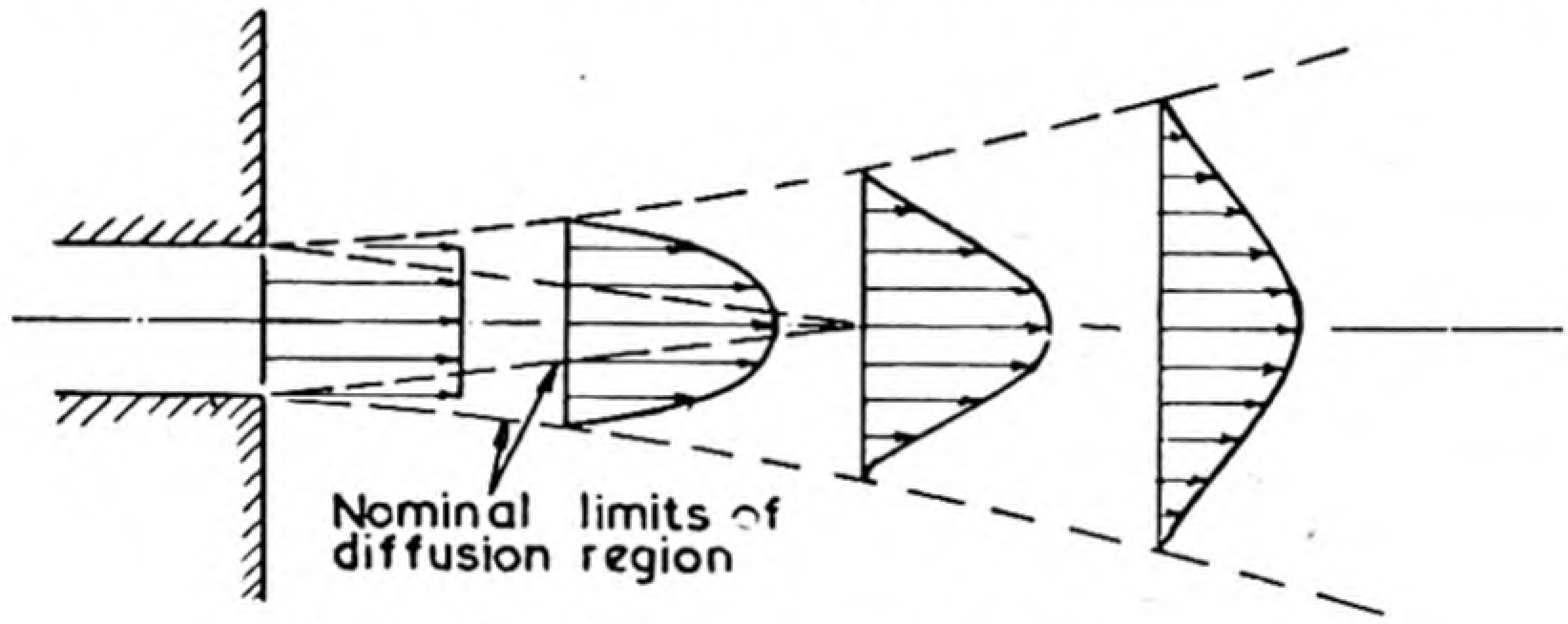

Fig. 7.—Jet diffusion showing velocity profiles.

about six aperture diameters and then begins to fall as the turbulence reaches the centre of the jet. From this point onwards the velocity profile retains its shape while spreading continually. In the case of a jet discharging into an infinite space it spreads out and entrains surrounding gases, and in the case where there are walls within a finite distance of the axis of the jet and an insufficient supply of fluid around the jet at the inlet end to make up that drawn in by the mixing process, there is a back flowing stream which consists of recirculated material from the jet and is re-entrained by the jet.

In the applications of the theories of turbulence to axially symmetrical jets issuing from point sources a number of assumptions are made. These are (*a*) the static pressure gradient is assumed uniform or zero; (*b*) variation of jet behaviour with Reynolds' number cannot be taken into account and so the latter is taken to be great; (*c*) the turbulence is isotropic; (*d*) the density remains constant throughout; (*e*) molecular diffusion in the jet is small compared with the turbulent diffusion. During the calculation an empirical constant must be introduced which is chosen to give the best agreement with experiment. Thus the theories only provide the shape of the mean flow patterns, velocity distribution and diffusion but the magnitude is not predicted.

The two classical treatments of jets are those based on the momentum transport equation of Prandtl[12] and the vorticity transport theory of

Taylor[11]. The Prandtl equation was applied to an axially symmetrical jet originating from a point of Tollmien[14] and the Taylor equation was applied by Howarth[15]. These theories establish the shape of the velocity profile of the jet at any section and show that (*a*) axial velocity on the jet is inversely proportional to the distance from the origin and (*b*) the velocity profiles are similar at any section. Kuethe[16] extended Tollmien's work to the case of a jet with a finite source and established the character of the flow near the source. Experiments by Kuethe and Ruden[17] showed that the solutions were dimensionally correct and that Tollmien's solution was valid for distances greater than about eight source diameters from the origin. Corrsin[18] has also measured the velocity distribution across a jet. The theoretical solution appears to give velocity distribution curves which have peaks which are too sharp.

Work on the temperature distribution in jets has also been carried out and Howarth has applied both the Prandtl and Taylor theories to the temperature distribution in a jet in which he assumed that the coefficients of momentum and heat transfer were the same. It is found in fact, that the heat jet spreads more rapidly than the velocity jet. Recently Hinze and Van der Hegge Zijnen[13] have reverted to the original concepts of Boussinesq.

The inability of the mixing length theory to explain the greater width of the heat jet and the increasing amount of experimental evidence from Hinze and Van der Hegge Zijnen[13], Albertson, Dai, Jensen and Rouse[19], Cleeves and Boelter[20], [21], and Squire[22] as to the exact shape of the velocity profile points to the inadequacy of the early theories and the picture emerges of the axially symmetrical jet as a structure consisting of a uniformly turbulent central region and an outer region of irregular protuberances. The need for further fundamental work on the structure of jets is evident. As far as the designers of furnaces and other systems involving jets are concerned, Tollmien's work can be applied with sufficient accuracy for most purposes when the jet can be treated as though it were flowing into an infinite space. For the calculation of injectors and ejectors, formulæ based on the assumption of a certain percentage loss of momentum in the mixing zone can be used with sufficient accuracy for most purposes, but in almost all other cases it is necessary to work experimentally.

In the case of high velocity jets entering a combustion chamber the concept of a free jet entering a stationary mass of gas is often sufficient to account broadly for their observed behaviour. This is true for example in the case of large flames produced by the burning of jets of atomised oil suspended in steam where the velocity of the jet, as it issues from the nozzle, is high and its momentum is high in comparison with that of the combustion air which surrounds it. In the case of producer gas fired furnaces, on the other hand, the simple concept of a jet issuing from a point source is no longer appropriate on account firstly of the much larger size of the port in comparison with the furnace chamber (the diameter may be 30% of the height of the combustion chamber) and secondly to the fact that the momentum of the combustion air as it enters the furnace chamber is of the same order as that of the fuel gas.

Davis[23] has calculated the change of direction in a jet due to buoyancy on very simple assumptions and obtained reasonable agreement with experiment for measurement in the flame of a combustion chamber of a water tube boiler. Collins[24] has applied the simple jet solution to the oil flames in open-hearth furnaces and Wohlenberg[25] has attempted to calculate combustion rates in turbulent flames by considering the turbulence as greatly increasing the area across which mixing by molecular diffusion can take place.

(c) *Models.* In view of the great difficulty of applying fundamental theory especially when the geometry of the system is at all complicated, it has often been found profitable to study industrial mixing processes by means of aerodynamic models in which the flow is made dynamically similar. Where momentum and viscosity are the most significant physical forces the criterion is that of Reynolds and working on laboratory sized models with cold gases it is possible to employ gas velocities of the same order as those used in furnaces having temperatures of about 1500°C. If water is used as the fluid in the model velocities about 13 times lower may be used for the same sized model. Water models are thus, broadly, valuable for providing a visual indication of the flow conditions and there is no doubt that their use will greatly increase as designers come to appreciate their value. Air models with Reynolds similarity have the advantage over water models that the shape can be more readily changed and velocity measurements can readily be made. Recently also, a new technique for the study of mixing has been developed and applied to open hearth furnace models[26]. On the other hand, air models do not lend themselves very readily to the visual study of flow both because of the higher velocities concerned and because it is not so easy to make the flow visible by the use of indicators. Where other processes, such as gravity in natural convection or surface tension in the formation of droplets are rate determining as well as turbulence, it is not generally possible to obtain similarity between the model and the original merely by adjusting the velocities in the model. Very often, however, it is possible to obtain similarity as far as these other processes are concerned and sacrifice Reynolds' number similarity, provided it works out that the flow is fully turbulent in the model There is no doubt that models are most valuable tools in the hands of designers of industrial equipment.

4. CONCLUSIONS

The discussion given in this paper shows that the fundamental theory available has not yet been able to provide formulæ for industrial calculation and that the simple ideas of the early workers are still the most useful. There is no doubt that in the end the fundamental line of approach will give powerful methods although the calculations for 3-dimensional systems with complex shapes will always remain very inconvenient. It is probable therefore that even when fundamental formulæ are available it will be necessary to make far-reaching simplifying assumptions and hence to rely to some extent on experiment to be sure that they are applicable. In the

meantime all that is available for designers are semi-empirical formulæ such as those recommended from Tollmien's theory for the entrainment of the surrounding atmosphere of a jet in a large space and that found suitable for calculation of injectors. In other cases it is necessary to make use of models.

5. ACKNOWLEDGMENTS

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2

Survey of Industrial Problems involving the Combined Flow of Fluids and Solids

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ABSTRACT. A survey is given of industrial plant in which there is combined flow of fluids and solids. Such plant is found in nearly all industries notably fuel, steel and chemical and includes gas cleaners, fluidised reactors, pneumatic and hydraulic conveyors, coal and ore washers and combustion and gasification appliances. The physical phenomena range from fluids carrying solids in dilute suspension to the other extreme of packed beds of solids through which a fluid is forced. A wide range of particle sizes is found and both gas and liquids are used. Interest is not confined to the properties of the fluid-solids system as a whole, knowledge being needed also about the relative motion of solids and fluid.

INTRODUCTION

This session of the Conference is concerned with industrial plant in which there is combined flow of fluids and solids. Such plant is found in nearly all industries, notably the fuel, steel and chemical industries. The main types of plant are gas cleaners, fluidized reactors, pneumatic conveyors, coal washers and combustion and gasification appliances. The physical phenomena range from fluids carrying solids in dilute suspension to the other extreme of packed beds of solids through which a fluid is forced.

Ten years ago the physical treatment of the combined flow of fluids and solids was in its infancy. In 1937 Rosin(1) had pointed out the importance of the particle size of the solids : shortly after this, in 1943, Dallavalle collected together much of the scattered literature (several hundred references were given) in his book " Micromeritics "(2). In contradistinction to these attempts to give a general treatment of the subject, the engineering texts (3, 4) showed that the design of plant was then—and to a great extent, still is—largely empirical. Little more was attempted than to use, on the one hand, simple and limited modifications of Stokes Law(5) for the fall of spheres in stationary fluids having infinite extent, and on the other, of Einstein's relation (6) for the viscosity of dilute suspensions.

In recent years, the demand for more efficient and more economic plant, with the added stimulus of the newest comer to the field—the fluid-solids technique—has led to much further work, which is bringing out the complexity of the problems, the dependence on the size and shape of the solids, on the density of the suspension and the pattern of the fluid flow. If this Conference assists in accelerating the growth of this fundamental knowledge, great developments in industry may be anticipated.

The information collected by the Institute of Physics from industrial organizations and research establishments reveals a broad group of problems which differ in two respects from those encountered in the rheology of pastes and complex mixes. Firstly, the solids are (in general) larger than the micron and sub-micron sizes that are most relevant to rheological enquiries. Secondly, interest is not confined to the properties of the fluid-solid system

as a whole, knowledge being needed also about the relative motion of solids and fluid. Since rheological subjects have been adequately discussed elsewhere [7, 8], they have been excluded from this survey.

CLEANING DUSTY GAS

Economic and efficient cleaning of dusty gas is necessary not only for the adequate operation and maintenance of plant through which the gas passes, but also to minimise the health hazard. A typical example is provided by blast furnace gas, which prior to cleaning may have 20 grains/ft.^{3} of dust, perhaps half of this being above 100μ in size with 10% below 10μ: it is usually required that such a gas be cleaned to better than 0·005 grains/ft.^{3}. Also the handling of coal, particularly when fines are present, introduces a serious dust hazard well known in the mining industry, but also occurring elsewhere. Thus in a horizontal retort house, loading a coal containing fines can disperse 2 lb. coal/ton coal charged [9]. At a conference in Leeds, on "Dust in Industry" [10] in 1948, it appeared that nearly all industries encounter problems in gas cleaning and dust suppression.

The papers at the Leeds Conference were the first to go at all fully into the physical nature of the dusts. In this connexion the difficulty of obtaining a representative sample of dust [10] from a gas flowing in a duct (e.g., at 10 ft./sec. at about 350°F. in a 10 ft. diameter duct for blast furnace gas) must be noted. Experience has shown that the flow pattern in a duct is not uniform and therefore either the pattern must be determined or a mixing device fitted to improve uniformity[11]. It is not known how far it is possible to mix uniformly both gas and dust; also this must be accomplished for a small loss in pressure. Much work remains to be done on methods for determining the dust burden of a gas (preferably also with an indication of the size of the dust), both for the purpose of testing the efficiency of gas cleaning plant as well as to give a routine control of dust emission.

When the dust burden is high enough, the differing response of dust and gas to a constriction in the flow may be used to estimate the dust burden[12, 13]. For dense coal-air mixtures[14] exceeding 5% of coal by volume, the dielectric constant is a simple function of the dust burden. Light obscuration by dusty gas gives, of course, a neat method of measuring dustiness, but extensive calibration experiments are needed to devise an instrument reliable enough for routine control.* In nearly all these methods the response of the instrument depends both on the size as well as the concentration of the dust in the gas.

All of the methods of gas cleaning depend on a knowledge of three factors: the effect of particle size and shape on its resistance to motion in a fluid [16, 17, 18]; the movement of liquid droplets[19]; and the efficiency

*An even more difficult sampling problem is the estimation of suspended particles in flames, where not only have high temperatures to be withstood by the sampling probes, but also the combustion of the particle needs to be extinguished at the moment of collection. Solution of this problem depends in part on the extension of knowledge of the laws of motion of particles to include burning particles[15], necessary also for the design of pulverised fuel furnace chambers, especially in view of the modern trend towards pressure combustion.

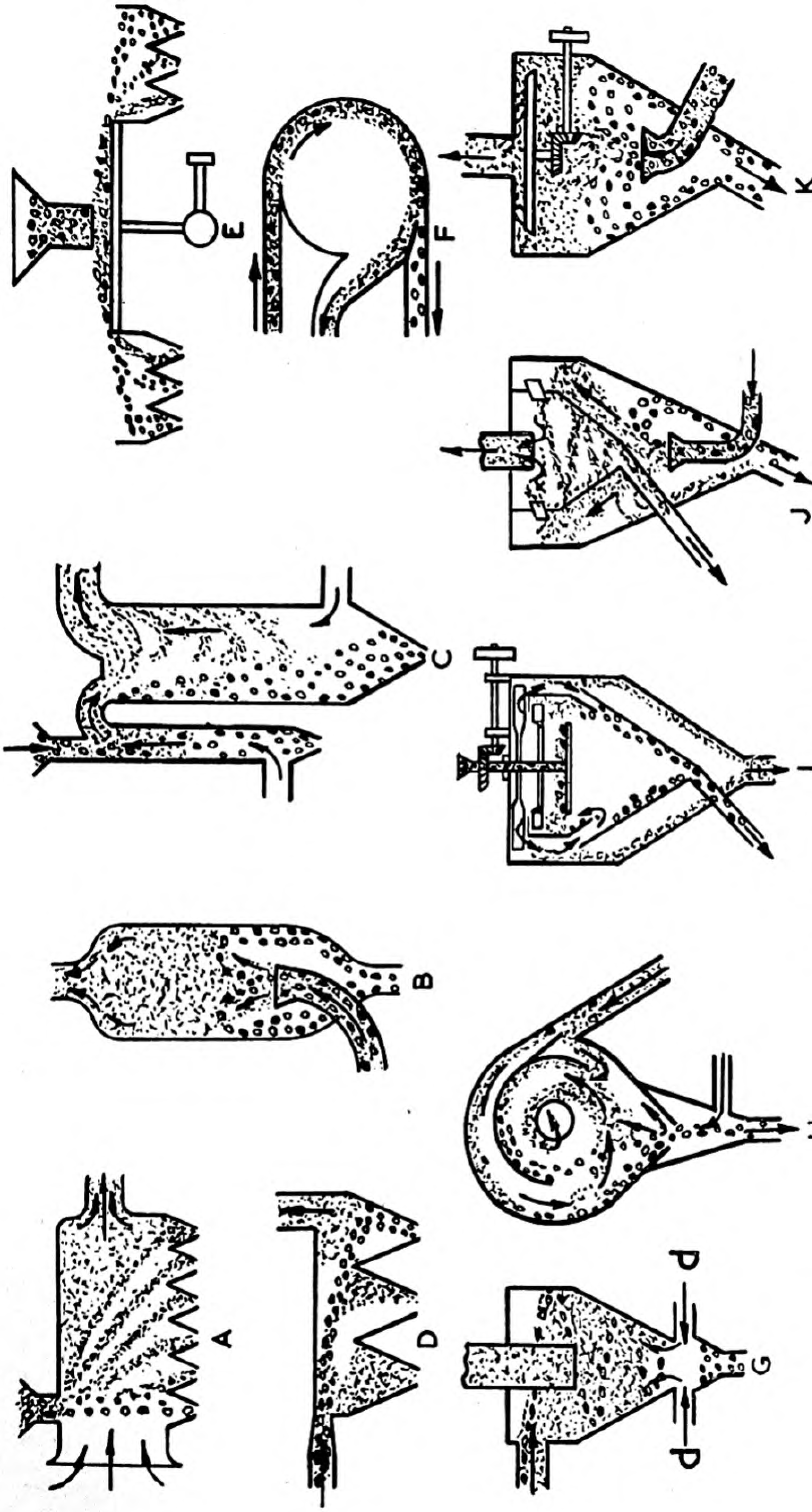

Fig. 1.—Forms of air classifiers.

Typical size of unit *I*: solid throughput—2.5 tons/h; size of solid—95% through 200 B.S. mesh; power required—15 h.p.; diameter—10 ft.

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with which obstructions in the stream collect the particles[20-23]. Motion of the particles is also affected by temperature gradients and electrostatic forces, which last can be, in some cases, of great importance.

One of the commonest designs of gas cleaner is the cyclone (Fig. 1*G*, usually without secondary air inlet *d*, *d*). Attempts have been made[24-27] to measure the inner flow pattern in a cyclone. It seems clear that, as well as the outer swirl, there is also an inner vortex of great importance for efficiency[28]. Knowledge of the flow pattern is needed, not only for the purpose of calculating the size of particle at which a cyclone will cut[29], but also to establish a basis for similarity theory[30], thus enabling model tests to be used to supplement large scale data. The investigations of First[31] go far towards achieving this last objective.

Notwithstanding the lack of exact knowledge of how cyclones work, they are used[32] in all sizes from a few inches diameter to several feet, either singly or in multiple units, to cut out dusts down to 50μ size in the large diameter units with finer cuts still in the smaller cyclones ; there is of course some separation at sizes below the cut size, the efficiency falling off for the finer dust particles in the gas. In the choice of the design to be installed, the pressure drop at the rated capacity is one of the most important factors and the method for calculating pressure drop given by Stairmand[33] appears to be well related to practice. Claims have been made[34] for the use of cyclones with secondary air for separating emery (5μ to 200μ) into 19 sizes at the rate of 10—100 kg./h.

The cyclone is only one of many forms of air classifier (Fig. 1), which include types embodying simple gravitational settling (*A*, *B*, *C*, *D*), those employing centrifugal force to accelerate settling (*F*, *G*, *H*), and those with mechanical means to distribute the particles (whirling disks in *E*, *I*) and to swirl the air (fan in *I*). A major problem in all types is to minimize re-entrainment of coarse particles as they settle out and move towards the outlet. Generally, except for very coarse dusts, some form of centrifugal separation is necessary.

The use of centrifugal force in cyclones and centrifugal separators to separate dust from gas is not, however, the only aerodynamic method of separation. It has been claimed that standing waves induced by gas flowing at an acute angle against slots will enable clean gas to escape through the slots[35] ; the concentrated dusty gas can then be passed to a second separator.

Another group of problems arises when the dusty gas is sprayed with water droplets, usually in the neighbourhood of 100μ in size[36]. This practice is adopted both to improve the efficiency of gas cleaning and to suppress the dispersal of dust from equipment handling a granular material.

Much work is needed on the extent to which model techniques can be used to improve the design of exhaust ventilating systems[37, 38] both with and without water sprays. Particular attention is at present being given to the design of ventilating hoods in coal mines.

In wet methods of suppressing or collecting dust, the water spray contains droplets covering a range of sizes ; these have to be projected into a dust cloud so as to give the maximum number of impacts with the dust ; then the aggregated material must be separated out from the gas. Nonhebel(39) has pointed out the paucity of design data. In spray towers, baffles are often inserted to improve contact between water and dust (Fig. 2) but this is not an efficient method of making a spray. In disintegrators intimate contact is achieved by passing the water and the dusty gas through a fan mechanism. Wet methods bring in their train problems of sludge

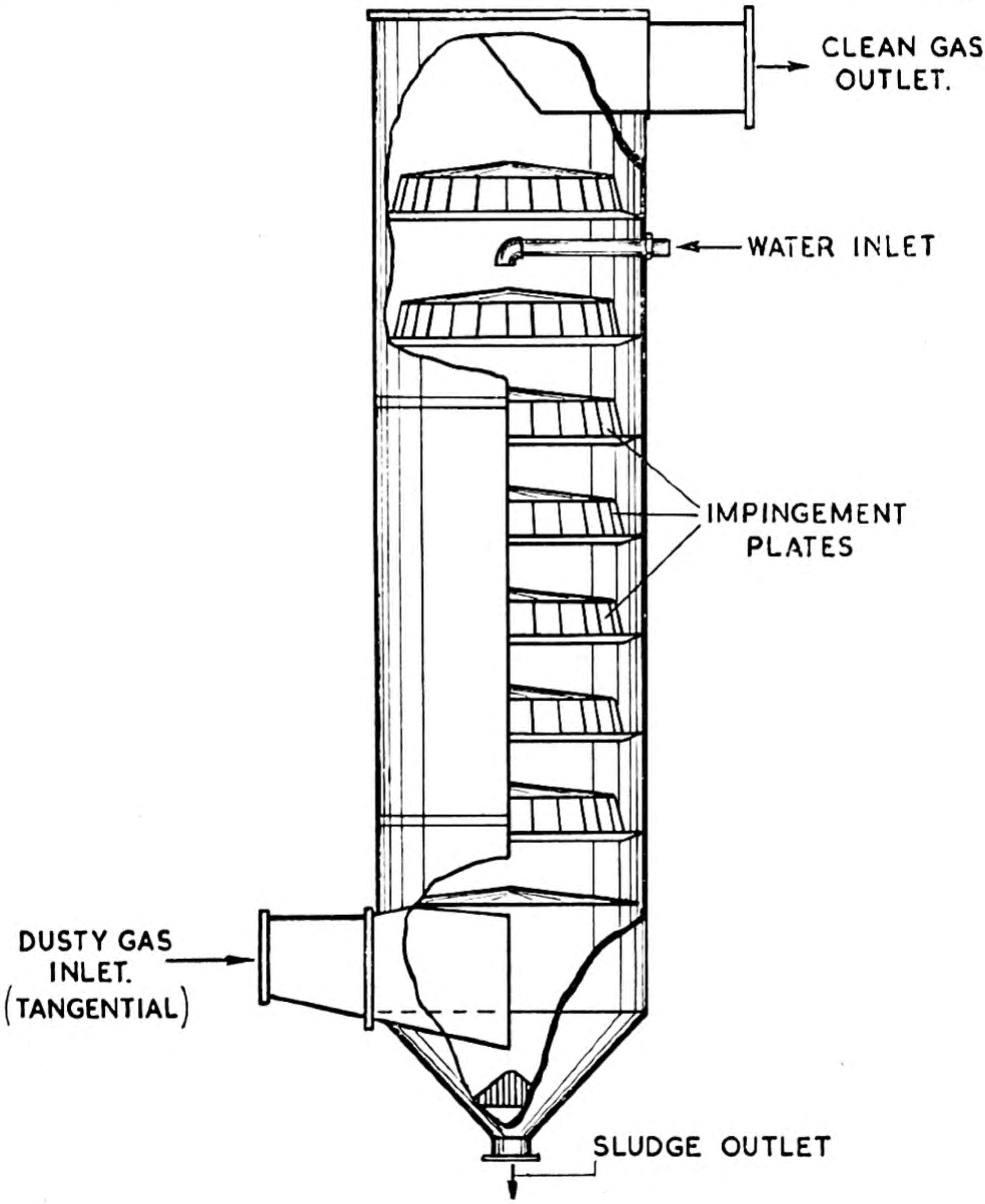

Fig. 2.—Spray Tower.

Typical size of unit : gas throughput—15,000 ft.^{3}/min. ; solids burden—20 grains/ft.^{3} wash liquid—50 gal./min. ; pressure drop—2—3 in w.g. ; height—32 ft. ; diameter—8 ft.

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disposal (or solids recovery if they have value). But their use is sometimes forced when in a dry method of separation it is not possible to keep above the dewpoint of the gas. They are also used to recover valuable dusts, sometimes simply by bubbling the gas through a liquid.

Gases are also cleaned in filters through the combined action of centrifugal force and diffusion[40]. There is little basic data[22] on how to dispose the filter obstructions. It must be remembered that in filtration the medium (cloth or packed beds of solids) must be cleaned frequently, the efficiency increasing as the dust deposits but the resistance to flow also increasing. There are possibilities in the use of a fluidized system which method has been applied with some success with acid mists[41].

Another method of gas cleaning, now beginning to be considered, is to induce agglomeration of the smoke or dust by sonic or supersonic waves[42-44]. The effects of particle size, shape and density on the nature of the induced vibrations have yet to be determined fully together with the role of hydrodynamic forces.

No reference has been made here to electrostatic precipitators, in the design and operation of which problems other than those of fluid flow arise, although the latter are important. It should also be mentioned that industry is also faced with problems of unwanted deposition, notably in the formation of deposits in boilers.

To sum up, three points are worthy of note. Firstly, the need for a physical attack on the properties of dilute suspensions of dust in gas has been realised (rather belatedly) in recent years. The work to date has shown the great importance of the pattern of gas flow and its inner structure on the motion of the dust particles. How to control this pattern of flow has, however, received little attention. Secondly, the commonest method of dust cleaning is to increase the relative motion of dust and gas, either by centrifugal force or by increasing the particle size with water droplets or through agglomeration. Are there, however, types of fluid flow containing aerodynamic barriers to the motion of the particles ? Can sprays be improved by " raining out " the particles with condensing steam ? Can use be made of the dust-free space adjacent to a heated surface ? This is of course the principle on which the thermal precipitator for the collection of dust samples operates. Thirdly, the extent to which model techniques can be applied in designing gas cleaners and dust suppression devices needs to be studied.

The efficiency of gas cleaning plant can be looked at from two points of view. In the first place, the size of particle which can be collected is important. This depends on the design ; there is a great need for improved methods of collecting very fine particles. The second aspect is the cost. So far as aerodynamic collectors are concerned, the problem is to impart sufficient velocity to the particles to bring them to a boundary in a short time and a small space. Ideally only the particles and the immediately surrounding gas need be so treated ; in practice all of the gas is treated and herein lies an (ideally) useless expenditure of energy, and a consequent

enhanced running cost. It would be useful to have a basis for assessing efficiency in this second sense.

FLUIDIZATION AND TRANSPORTATION

During the war years there were extensive developments in the catalytic cracking of oil using a fluidized bed of catalyst[45]. This successful use of the fluid-solids technique, as it is now called, has led to applications in other fields being considered, e.g., drying of finely divided solids[46], carbonization[47, 48], gas purification[49] and the collection of dust or mist[41]. These industrial developments have been accompanied by much laboratory work, culminating in a symposium on the dynamics of fluid-solid systems [50-69]. The material in these papers goes some way towards providing design data for industry, although much remains to be done, in particular on the case when the solids are consumed in the reactor, as in the gasification of carbonaceous materials[70].

Successful fluidization depends on a compromise between several factors. Excluding particles so small (say less than 10μ) that they cohere together appreciably, the various situations may be seen in terms of a Phase Diagram (Fig. 3) proposed by Zenz[71]. In a vertical tube containing a fixed bed of particles through which an upward current of gas is passed, the pressure drop and superficial gas velocity increase together, until the bed expands so that the particles are able to move relative to each other (Fig. 4). Further increase in velocity gives smooth or quiescent fluidization (Fig. 4C) the density of the bed decreasing and the unit bed resistance falling. Here there is rapid mixing, the main advantage of the fluidized system[52].

It has been suggested[50] from photographic studies that a portion of the gas forms with the solids a fluid system, through which bubbles of the remaining gas rise : these bubbles coalesce and in sufficiently long tubes grow to equal the tube diameter, when " slug " flow occurs.

Returning to the phase diagram for a vertical tube, there is a definite break between the pressure velocity relationship for a fluidized system and that for a dispersed suspension. At velocities above that for a dispersed suspension, there is upward transport of the solids (curves W_1, W_2, W_3 to the right of the dotted line in Fig. 3). In vertical co-current transport, as the velocity decreases, so does the pressure drop, until a minimum is reached ; further decrease in velocity results in hold-up or choking of the solids, analogous to slugging in the fluidized system. The curves (W_1—W_3) to the left of the dotted line in Fig. 3 are characteristic of downward movement of the solids against the gas flow.

Slugging and channelling, the two undesirable unstable situations found in fluidization, are less prominent in liquid-solid systems that in gas-solid systems, no doubt due in part to the smaller density difference (of solid and fluid) in the former case, but perhaps also because of compressibility effects in the latter case. The transition from fixed bed through fluidization to dispersed suspension in liquid-solid systems does not appear to have been studied in detail (see next section, however).

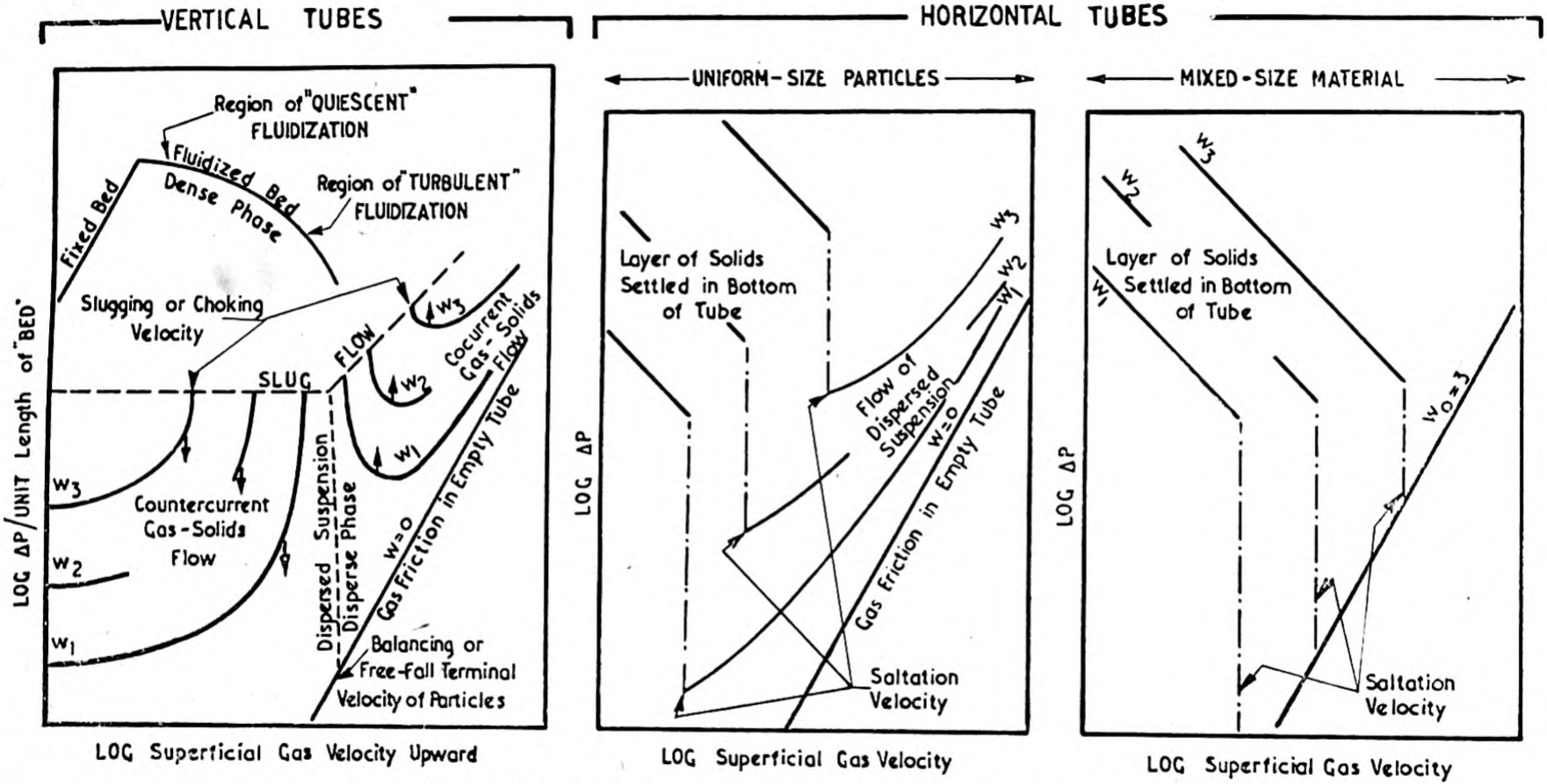

Fig. 3.—Schematic phase diagrams for particle-gas systems.

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In horizontal flow through a tube, the situation is quite different. As the velocity decreases, the particles begin to settle out, transport continuing in the upper part of the tube ; this gives a sharp break in the curves

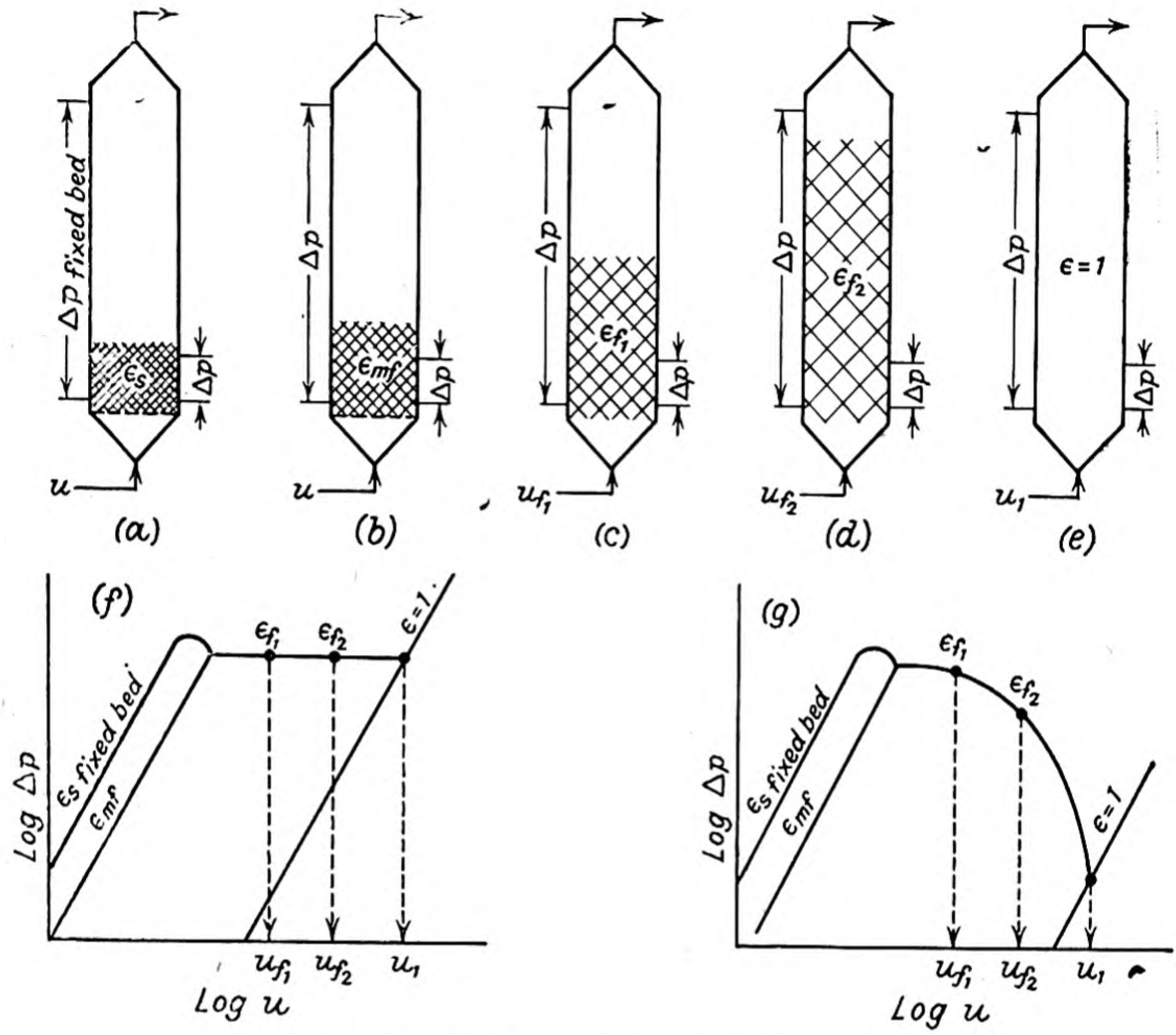

Fig. 4.—Graphical representation of fluidization data.
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relating pressure drop and velocity (Fig. 3). The velocity at which particles begin to settle out is the same as that at which particles begin to be picked up from the surface of the bed of material : since these particles rapidly fall back into the bed, the velocity has been named the " saltation velocity". The saltation velocity in horizontal flow and the choking velocity in vertical flow were the same in the experiments reported by Zenz on uniformly sized particles.

It is not suggested that this phase diagram is correct, but it serves to show the complexity of the phenomena encountered in fluidization and transportation[72-75]. There is little doubt that both applications will find further uses in industry, the former because of the good temperature control and rapid reaction rates that are possible and the latter for conveying all kinds of materials[76, 77] such as fly ash[78] coal and stone.

The transportation of coal in water up to 2 in. in size, raises further problems, for the solution of which existing knowledge of the effect of particle size, shape and density[50–70] may not be sufficient. Preliminary experiments[79] have shown that a high ratio of water to coal (4 : 1) is necessary; reduction of this ratio might well lead to a cheap and flexible method of transporting coal. Pneumatic and hydraulic stowing of waste rock in coal mines is another application in which there is great interest. In such transporting systems instruments for measuring and recording the flow of the various components are necessary (see examples cited in previous section). It may also be necessary to devise instruments for measuring the energy acquired by the material stowed ; this problem is complicated owing to the varied materials and sizes which are used.

In this field, as in all the problems of combined flow of fluids and solids, the scale factor is of great importance and in many cases the findings of small scale tests need confirmation on full scale equipment. The question of scale factor is however, discussed in another section of the Conference and will not receive detailed consideration here.

Physically the phenomena shown in the phase diagram may be related to the flow of water in flumes and estuaries, with the added complication of tidal effects and the influence of waves. Here model techniques have been developed with some success.

In conclusion reference must be made to the theoretical investigations of Burgers[80] and Brinkman[81], where an attempt has been made to develop mathematical relationships for fluids containing swarms of particles. This work is related mathematically to studies (based on the underlying assumption that sub-micron particles can be regarded as hydrodynamic entities) of the viscosity of suspensions[82], of determination of molecular weight in the ultra-centrifuge[83] and of pastes[84]. So far these theories are not adequate in their explanation of the experimental facts.

To sum up, divers industrial and natural processes embody the combined flow of fluids and solids in a two phase system. During the past two years, great advances have been made in collecting experimental data for such systems, but the range of particle characteristics (size, size distribution, shape, roughness and density), of fluids (gas and liquid) and of flow patterns that has been studied is still limited. Until a wider variety of conditions has received attention, the physical principles underlying such systems will remain debatable and adequate design data for industry cannot be provided.

GRAVITY CONCENTRATION

The density of a fluidized system is between that of the fluid and solid, so that a mixture of dense and light particles fed into a stationary fluidized system would tend to separate according to specific gravity. In coal washing plant use is made of this principle in a number of ways. The history of such plant dates back nearly a hundred years (primitive apparatus for concentrating ores was built on the same principle much earlier), but it is recognised that the principles of their construction are not yet clear[85]. Evidently the processes involved are complicated, the motion of any single

particle depending on its size, shape and density as well as the size, shape and density of the other particles present and their concentration in the fluid. Moreover, in coal washing, the process is further complicated by the means adopted for feeding in the raw coal and shale and for discharging the products—clean coal, middlings (material of high or moderate ash content) and dirt (refuse)[86].

In nearly all coal washing plant there is a zone, somewhat similar to a fluidized bed, where "hindered settling"[87, 88] takes place. By adjusting this zone in relation to the solid and fluid flow in the rest of the plant, separation by difference in density rather than in size can be achieved[89]. Remembering that the raw coal feed may contain particles ranging in size from several inches to the finest dust, the practical advantage of obtaining a good approximation of a float-sink process is obvious.

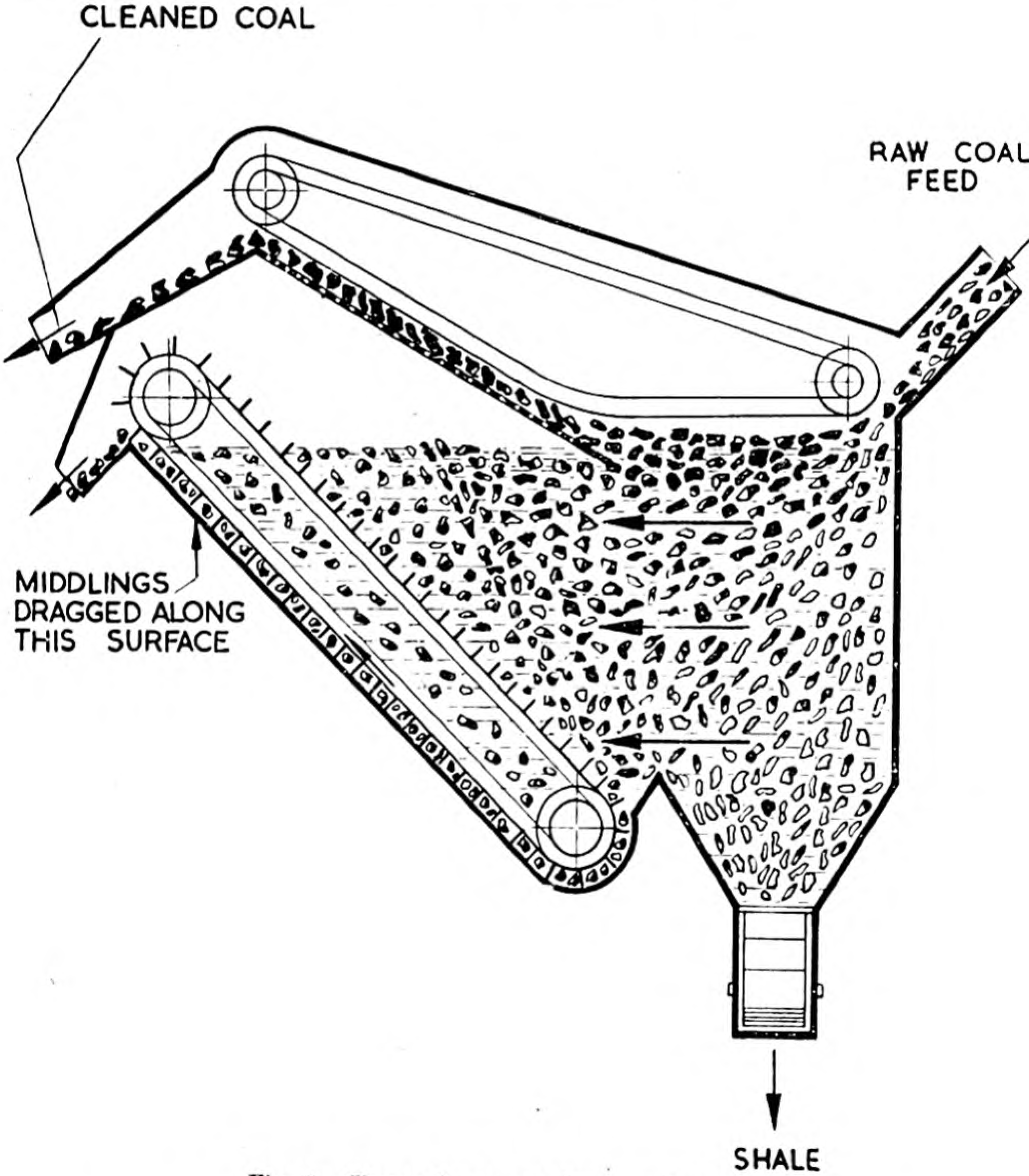

Fig. 5.—Tromp heavy medium coal washer.

Typical size of unit: solid throughput—100 tons/h; size of solids—5—¼ in.; materials wastage—3—4 lb./ton coal; depth of bed (estimated)—6 ft.

A physically simple form of coal washing plant employs a heavy medium of finely ground particles suspended in water(90). In the Barvoys plant a mixture of clay and barytes (in size all through 200 B.S. mesh) with water gives a nearly stable suspension and consequently an almost static separation. Finely powdered (less than 0·1 mm in size) magnetite or roasted pyrites gives, on the other hand, a rapid settling suspension ; in the Tromp process (Fig. 5) horizontal currents are passed through such a suspension—in which the density varies from top to bottom—to separate out the products. A third medium in use is loess—a naturally occurring material for which complete recovery for recirculation is not so necessary as with the first two. Loess is used in the Staatsmijnen process to give a heavy medium(91).

In other types of coal washer, a zone of " hindered settling " is formed from the feed material itself. The physics is here much more complex. To illustrate the various methods, three of the main types—trough, table and jig—can be described.

A launder or trough-washer, consists simply of a long narrow and slightly inclined trough to which raw coal and water are fed. The coal forms a bed, decreasing in mobility and increasing in density from top to bottom, through which the denser (and larger) particles sink to a draw-off point (Fig. 6) and over which the lighter (and smaller) particles flow. In

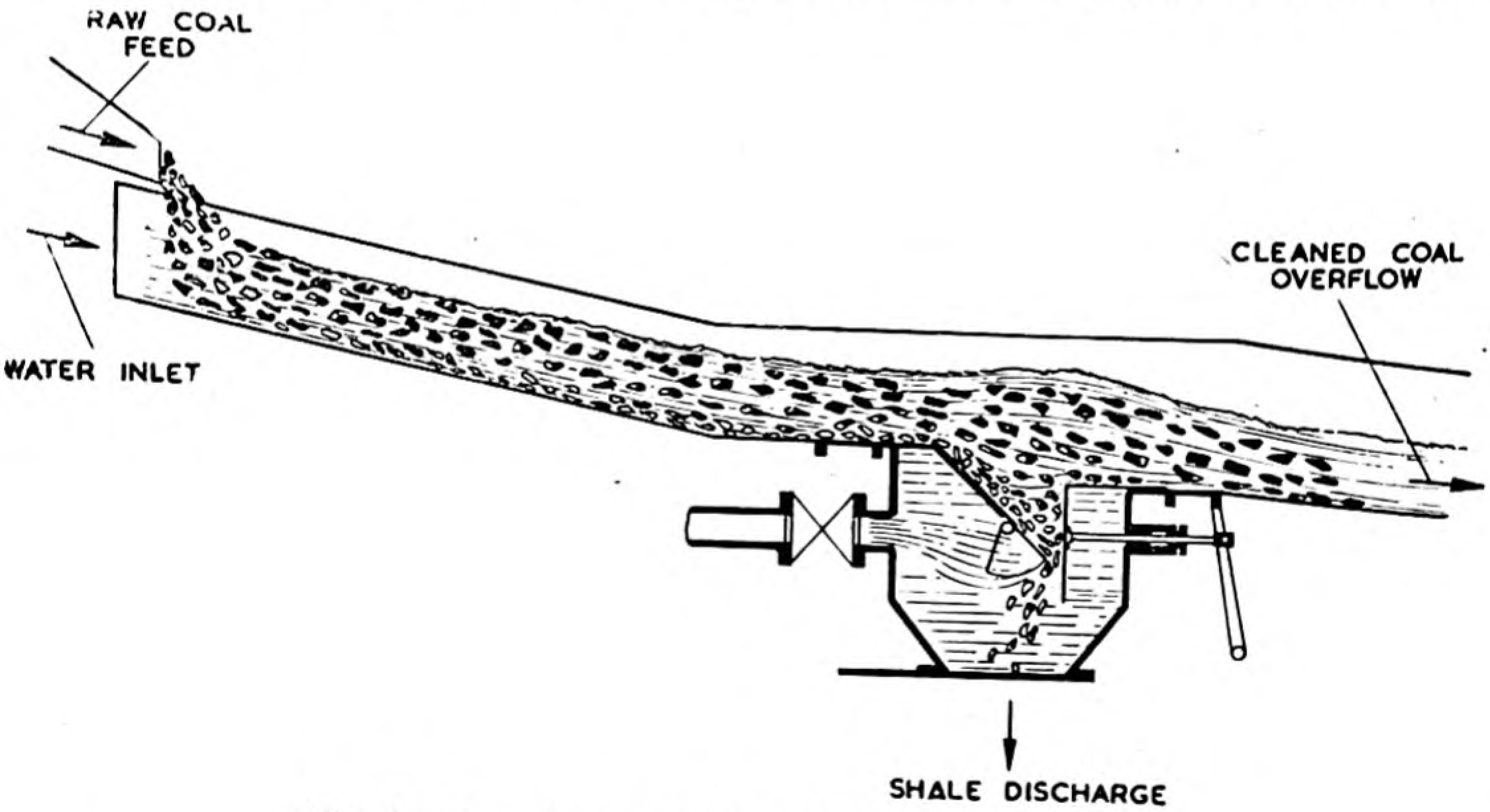

Fig. 6.—Trough coal washer (rheolaveur trough).

Typical size of unit : solid throughput—250 tons/h ; size of solids—4—½ in. ; wash water—250,000 gal./h ; trough width—4 ft. ; trough length (primary)—25 ft.

practice turbulence in the stream, disturbances due to drawing off products and trapping of particles in the bed, prevent exact separation from taking place. Sometimes the mobility of the bed is increased by upward currents ; also barriers may be fitted to control the bed depth. The essential(92) problem is to retard the flow of material sufficiently to enable a fluid or mobile bed to be maintained.

On the shaking table (Fig. 7) fitted with longitudinal riffles, the differential motion causes dense material to move along between the riffles and a cross stream of water carries the light material transversely. The bed

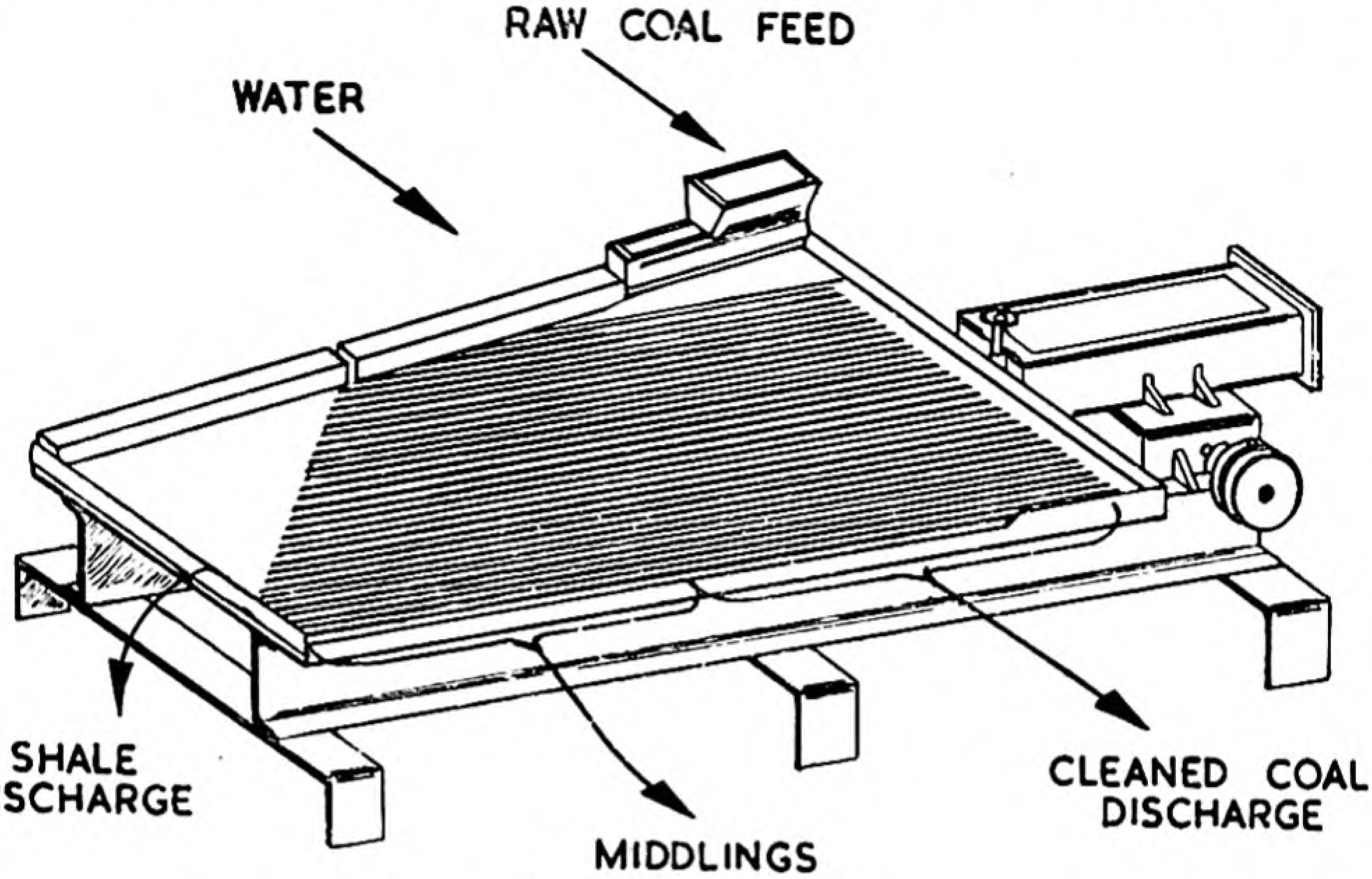

Fig. 7.—Shaking table.

Typical size of unit: solid throughput—2 tons/h; size of solids—$\frac{1}{8}$—0 in.; wash water—8 gal./min.; power required—1.0 h.p.; table width—6 ft.; table length—15 ft.

between the riffles is partly mobile and "hindered settling" occurs in it. The process is repeated many times as the riffles are crossed successively. Evidence has been given to show that stratification by density difference in the bed between two riffles, does not account entirely for the separation achieved(93), and that the cross flowing water, in addition to removing the top strata, flows through the interstices in the bed and thus aids separation in the lower strata. Since the riffles taper from the head towards the outer end of the table, conditions change progressively with the change in size and density of the particles present along the table. Such tables find their greatest use for cleaning fine coal.

The last type to be considered is the jig (Fig. 8). When a basket of material is pushed quickly downwards in a bath of water, the bed expands. On bringing the basket to rest some sorting of the particles takes place. Jigs—of which the Baum type will be considered here—are operated on this principle. They can separate a large range of sizes according to density. On one side of the U-shaped jig box is a perforated screen plate on which the material to be separated rests, and on the other a sealed chamber. Adjusting the pressure of the air in this chamber forces the water up through the plate and then sucks it back. The jig stroke is known to be important but the ideal is not yet clear(94). The refuse is withdrawn from the plate, as it collects, whilst the clean coal flows over. During the pulsion phase the bed opens, first at the top and then progressively towards the bottom, until the whole bed is loose. During the suction phase the particles are subject to hindered settling and since the bed closes from the bottom towards

the top, the density of the mobile bed increases upwards, thus ensuring good separation.

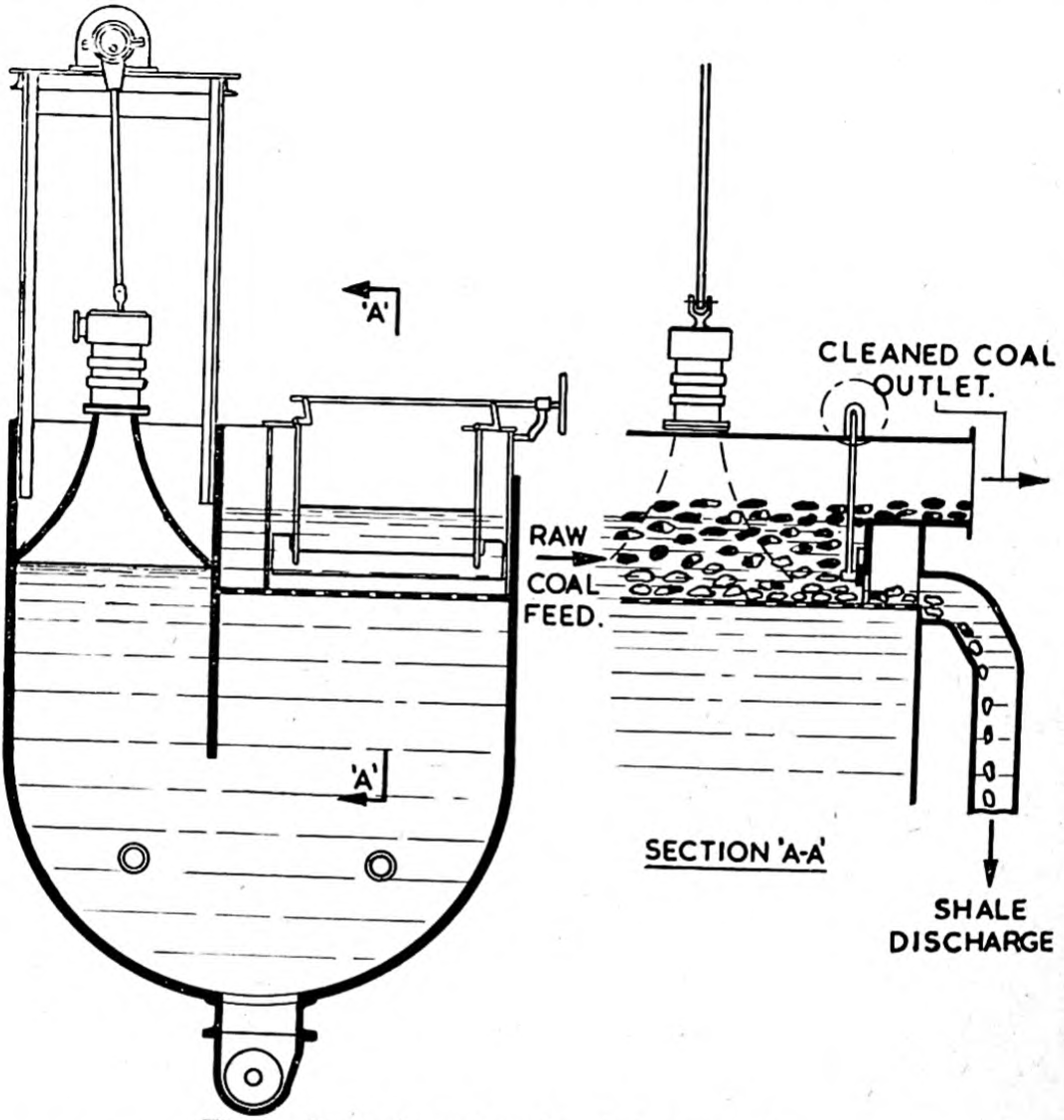

Fig. 8.—Baum jig coal washer (bed expanded).
Typical size of unit: solid throughput—120 tons/h; size of solids—5—0 in.; depth of bed—$1\frac{1}{2}$ ft.; screen area—80 ft.^{2}.

Many variations of these three types of coal washer are to be found in the coal industry. There is little doubt that a better understanding of the way in which they work would lead to material improvement in their design.

Finally, a major problem in coal cleaning at the present time is in the cleaning of fine coal less than 0·5 mm in size. This is needed not only to increase the yield of saleable product, but also to simplify waste disposal in coal washers in which many difficulties are experienced owing to the accumulation of fine material in the washery water. The necessity for cleaning dust or slurry may be partly obviated by improvements in the design of washers of the type already considered(95), but in general it is necessary to adopt froth flotation in which the fine material is agitated with

a small amount of reagent in the presence of water and air. Bubbles carry the coal to the surface as a froth whilst the slate and shale particles remain wetted and suspended in the water. Various types of cell are in use for froth flotation. It is desirable that the upper limit of size for the material fed to the flotation cells should be increased, or alternatively, the efficiency of cleaning finer sizes in other types of washer should be increased so that the gap between these two processes can be filled economically. For example, it is known that the recently developed cyclone coal washer[96] only works satisfactorily[97] if particles less than 0·02 in. are removed from the feed. In this washer a suspension of barytes or magnetite particles in water is fed into the cyclone with the coal to be cleaned[98-100]. An interesting point is that the density at which separation occurs in the cyclone is greater than is the density of the suspension fed in, indicating the collection of the suspended particles in the cyclone with (probably) a density gradient. The cyclone is able to deal with small particles because of the use of centrifugal force to accelerate their motion in the suspension; for this reason also it is compact, although this advantage is offset by the pumping cost.

To sum up, for many years, coal washers of various types have been in use and have been progressively developed. In the commoner types, the feed material itself is used to form suspensions (ranging from packed beds through dense suspensions to dilute suspensions) in which the remaining feed material is classified partly by size and partly by specific gravity. Whilst these processes are well established, a more detailed study of the principles on which they operate will undoubtedly lead to rapid developments leading to economies of considerable importance to the national welfare. It is encouraging that so much material has been made available in the past two years for the fluid-solid system (see previous section). It is now necessary however, to extend and amplify this material so as to be able to apply it to the (in this country) much more important field of coal washing.

COMBUSTION AND GASIFICATION

Recent fundamental work on the flow of fluids through packed beds [101-104] has served to show how the resistance of a bed of particles depends upon their size and size distribution, the voidage, the shape of the channels formed by the particles and the way they are packed into the container. This work is most relevant to the flow through packed towers (which find considerable application in the chemical industry) and to certain types of kiln or blast furnace where a deep bed of material is maintained, but in the general field of combustion and gasification other factors may have equal importance. Most boilers and furnaces have a comparatively shallow bed of material (sometimes only half a dozen particles in depth), so that accidental configurations leading to blow-holes and channels are of much greater significance than any average pressure drop[105]. In such plant the problem is still further complicated by the temperature gradient from the centre of the bed to the walls, which has a marked influence on the mass

of fluid flowing through different parts of the beds[105]. Also leakage past the walls or edges of the bed may be a major source of difficulty in controlling combustion.

Air is supplied to most appliances consuming solid fuel either through a grate or tuyeres. In the blast furnace, for instance, interest is focused on the formation of cavities in the burden close to the tuyeres. In gas producers, air and steam are delivered to the bottom of the bed and it is necessary to obtain a uniform distribution in order to control adequately the chemical reactions (Fig. 9).

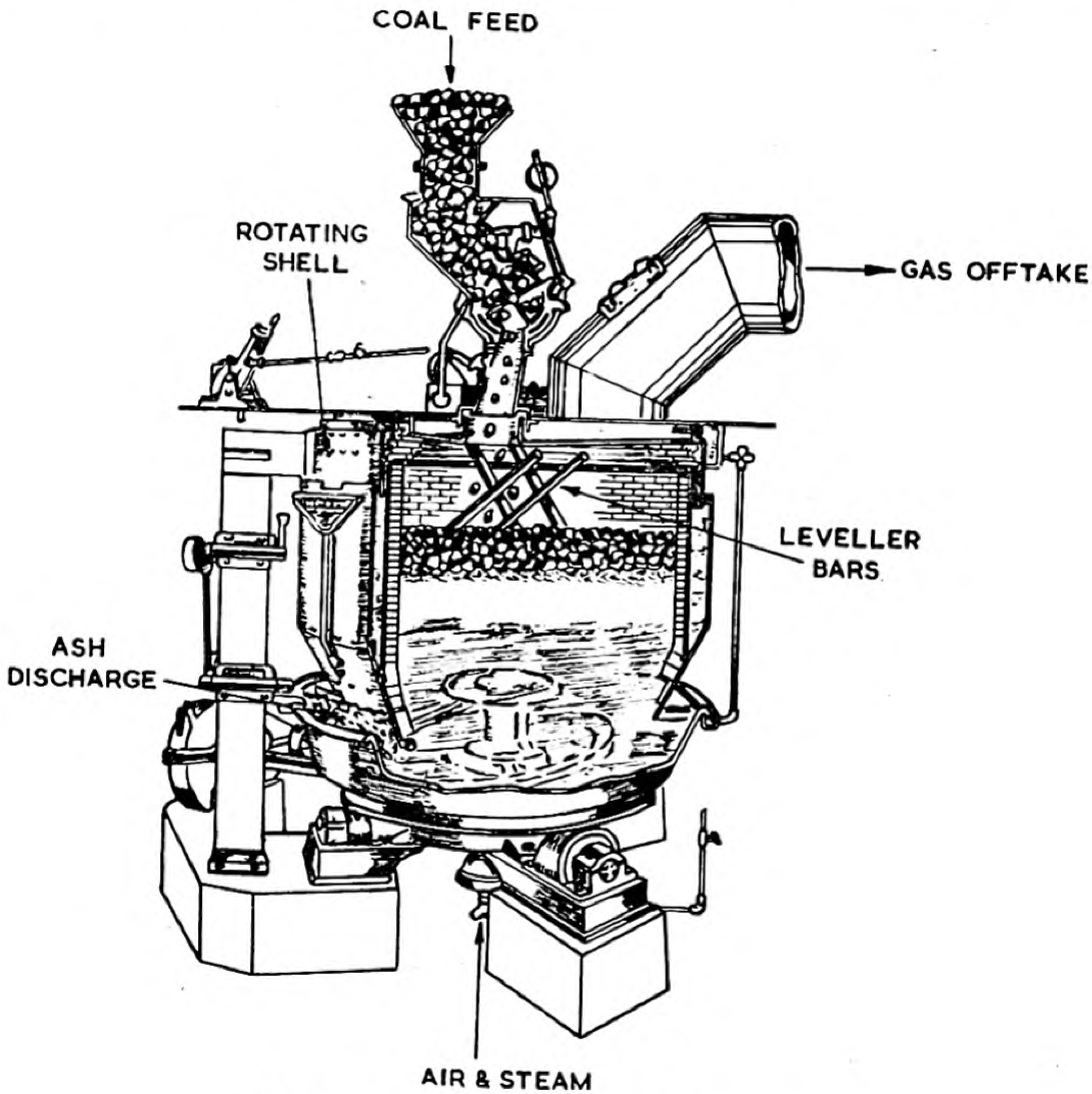

Fig. 9—11 ft. diameter Morgan gas machine.

Typical size of unit : solid throughput—3 tons/h ; size of solids—3—0 in. ; gas throughput—400,000 ft.^{3}/h ; depth of fuel bed—15 in. ; diameter of unit—11 ft.

In both gas producers and blast furnaces, the top feed of coal or burden determines the configuration at the top of the bed, whilst the build up of ash and the manner of its removal (e.g., by scraping) determines the situation at the base of the bed. Much work has been done on methods of reducing segregation in feeding[108] these plants[106, 109] with the aim of obtaining uniform conditions, and the avoidance of persistent channelling of the bed.

More work is needed, since the reactions in a blast furnace are by no means uniformly distributed throughout the bed of material(107) and the gas flow itself can be three times faster up the walls than through the centre of the bed(107, 107a). In some types of gas producer irregularities arising from the structure of the bed are partially counteracted by leveller bars which scrape the material on the top of the bed continuously during the operation of the producer. This method is adopted often with caking coals which cohere into irregular masses, with however, varying success.

It is clear therefore that when the fuel bed is relatively deep, the combined flow problems are complicated and difficult. On a chain grate stoker which has a very shallow bed, the upper portion of the bed can be in a teetering condition, arising in part from the velocity of gas flow and in part from the progress of combustion itself (Fig. 10)*. With the chain grate and many other types of grate used in boilers and furnaces, the resistance of the grate determines the magnitude of the effect of local fluctuations in the fuel bed resistance : here the flow patterns of primary and secondary air and the products of combustion in determining heat transfer to the boiler heating surfaces, are perhaps of greater significance.

In recent years, there have been developments in other methods of burning solid fuel. In the Downjet furnace(110, 111) combustion air is blown from a jet on to the surface of the fuel bed and the gaseous products of combustion leave the bed from adjacent areas of the same surface. When burning coke the fuel is consumed mainly in a zone a few particles deep beneath the surface. Flow conditions both in and above the fuel bed are complicated ; the jet or jets of air entrain some of the gases leaving the bed and therefore reintroduce them in the bed ; also some of the air is deflected before reaching the fuel bed and takes part in the gas phase reactions above the surface. Little is known about the depth to which a given amount of air penetrates (on the average) and the region within which this jet is effective. The jet, moreover, has a disruptive action on the bed of particles depending upon the shape of the jet, the air velocity and the angle of contact, but here again little is known.

Because of the technical difficulty of burning fine particles in a fuel bed some form of suspension burning is frequently adopted(112-118). To achieve satisfactorily complete combustion the particles must be retained in a hot combustion chamber for an adequate length of time and even with fine grinding this leads to large combustion chambers unless some method is adopted for retaining particles in the chamber for longer than the time of passage of the air and gases through the chamber. In the cyclone(119) furnace, in which a relatively coarse fuel is burned, those particles which do not burn in suspension are thrown outwards by centrifugal force and complete their combustion on slag coated walls. In the balanced vortex combustor the particles are retained in suspension by balancing the outward centrifugal force and aerodynamic drag due to the inward flow in the vortex (120, 121). Such methods may in time prove suitable for operation at the high pressures required for the open cycle gas turbine and the cyclone

*The figure given is slightly unusual in having a chain grate in a shell boiler.

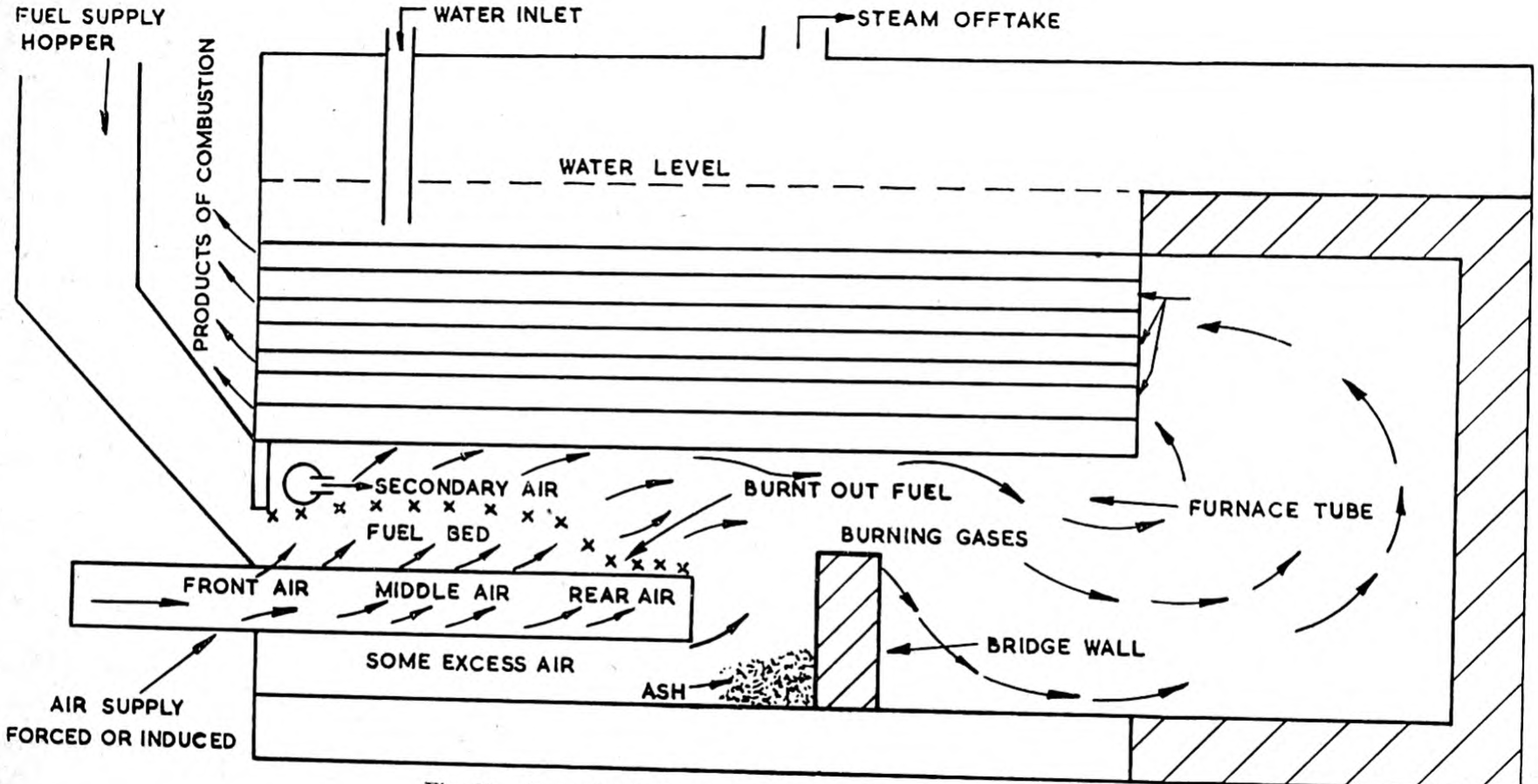

Fig. 10.—Gas and solid flow; chain grate stoker in a shell boiler.

Typical size of unit: solid throughput—0.3 tons/h; size of solids—$\frac{1}{2}$—0 in.; gas throughput—2,300 ft.^{3}/min.; depth of fuel bed—5 in.; grate temp.—270° C.; grate width—3.5 ft.; grate length (effective)—6.5 ft.

furnace in particular may provide a much needed method for burning low grade fine coal. Apart from practical difficulties the design of such units depends on a knowledge of the trajectories of burning particles. Enhanced combustion rates can be obtained by increasing the relative motion of gas and particle, as for example in a sonic field; some preliminary investigations have shown improved rates of combustion[122, 123, 124].

It will be appreciated that in all combustion and gasification appliances successful operation depends on control of the relative rates of gas and solid feed and that in part these influence each other. It is for this reason that gravity feed of solids to such appliances has so often proved least troublesome: unfortunately, gravity feed is limited to certain classes of fuel and to plant having relatively low output.

The paucity of work of direct applicability in the fields of combustion and gasification may be largely due to the fact that the fluid flow through the bed is only one of several factors such as the influence of temperatures and local channelling and the heat transfer from the combustion products, affecting the design of plant. In designing these appliances however, it is necessary to make use of the physical principles underlying the structure of fuel beds and the manner in which gases pass through them, particularly having in mind the need to feed through the appliances the correct proportion of solids and gas. There is no doubt that physical investigations would lead to advances which would be of material benefit to British industry.

ANCILLARY PROBLEMS

This survey would not be complete without drawing attention to the need to characterise the particles entering into the combined flow, by size and size distribution, by shape and by density. In some cases surface roughness also may be of importance. Apart from the classical investigations of Heywood[17, 125] little attention has been given to this arduous (in the experimental sense) task. Perhaps the development of automatic instruments for measuring particle profiles and for counting fine particles with improved methods of density measurement will lead to a greater body of information in the future.

CONCLUSION

Scientifically, four fields of study are connected: determination of molecular weight in the ultra-centrifuge; viscosity of suspensions; deposition of alluvial material in rivers and estuaries; and the field of dilute suspensions, fluidization and flow through packed beds. It is extraordinary how few connexions have been established between these four fields and it is extremely probable that careful scrutiny of published material would lead to valuable conclusions. In this connexion it should be noted that the references quoted in this survey are not presented critically; they have been given solely for the benefit of those unfamiliar with the field, as examples of some recent work, on which judgment is yet to be passed.

On the industrial side, empirical progress in the development of coal cleaning plant has been temporarily over-shadowed by the interest attaching to the catalytic cracking of oil by the fluid solids technique. It has been the purpose of this survey to bring out the connexion between the physics of the phenomena taking place in such diverse industrial fields as combustion and gasification plant, gravity concentration, fluidization and gas cleaning. This connexion of course becomes less close when considerations of mass transfer and heat transfer are introduced. These last have been excluded under the title of the Conference but it should be remembered that both in fluidization and in flow through packed beds, mass and heat transfer are equally important with the combined flow of the fluids and the solids.

Generally it may be said that the work which has been done has not been presented in a form easily assimilated by industry. The literature presents an incoherent mass of material and much further work is needed if some order is to be brought into it. Some of the papers to be presented later in this Conference will be concerned with this question. It may be hoped that the present Conference will enable the industrialist to see where immediate benefits can be gained by the application of present knowledge.

ACKNOWLEDGMENTS

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3

Survey of Industrial Problems involving the Pattern of Fluid Flow

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ABSTRACT. This paper reviews and collates the fundamental fluid flow problems met with in some twenty industries. First those problems involving basic flow phenomena are examined under the headings flow in boundary layers, flow under entry conditions, and scale effects. Secondly problems about steady and varying flow in circuits are grouped and discussed. Finally problems concerning flow patterns in spaces are examined. The methods available for studying these different problems and their advantages and disadvantages are briefly mentioned.

1. INTRODUCTION

The industrial problems requiring fundamental work on fluid flow reviewed in this paper are those in which the fluid flows in tubes or spaces and where the state of the flow, its pattern or the measurement of the flow are important. Problems involving granular suspensions are not discussed, nor are those concerning flow through jets and the subsequent mixing of different fluids.

An analysis of the fundamental problems in fluid flow met by a representative sample of over twenty research organisations interested in various aspects of fluid flow showed that the problems falling into the field demarcated above could be grouped as follows. First, there are a number of problems connected with three basic flow phenomena, viz., flow in boundary layers, flow under entry conditions and scale effects. Secondly there are problems concerned with flow in circuits and various elements of these circuits. When the flow is not in a circuit it will be in a space and another group of problems is connected with such cases. Some problems also occur in connexion with flow under the particular conditions when the fluid is very viscous. The full list of the problems in the field covered is, of course, very varied. Although the classification suggested is not rigid, it has been found to be convenient when the problems which occur frequently or which appear to be most important have been allocated to the most appropriate group.

2. PROBLEMS INVOLVING BASIC FLOW PHENOMENA

From the evidence available it appears that important fluid flow problems occur in connexion with three general flow phenomena. The first of these is flow in boundary layers and in particular the transition between stream line and turbulent states, the second is flow while affected by entry conditions and before the steady state natural to the conditions has been attained, while the third subject is scaling effects.

2.1. *Flow in boundary layers.*

According to Prandtl's theory of fluid friction, when a uniform fluid stream flows past a smooth flat plate the fluid in contact clings to the solid surface without slipping, but slipping occurs between successive fluid layers.

In any plane normal to the surface the velocity varies from zero at the solid surface to a maximum in the free stream. The layer of fluid in which most of this velocity variation occurs is found by measurement to be relatively thin and is called the boundary layer. The type of flow within the boundary layer depends on the conditions, and assuming that at low velocities this is laminar, then as the velocity increases the flow in the boundary layer changes gradually in a transition region and becomes turbulent and eventually, the boundary layer may separate from the surface, in which case fluid from downstream moves up along the surface in the opposite direction to the stream so that eddies are set up. During the turbulent regime there is a very thin layer near to the surface in which the flow is laminar; this region is called the laminar sub-layer.

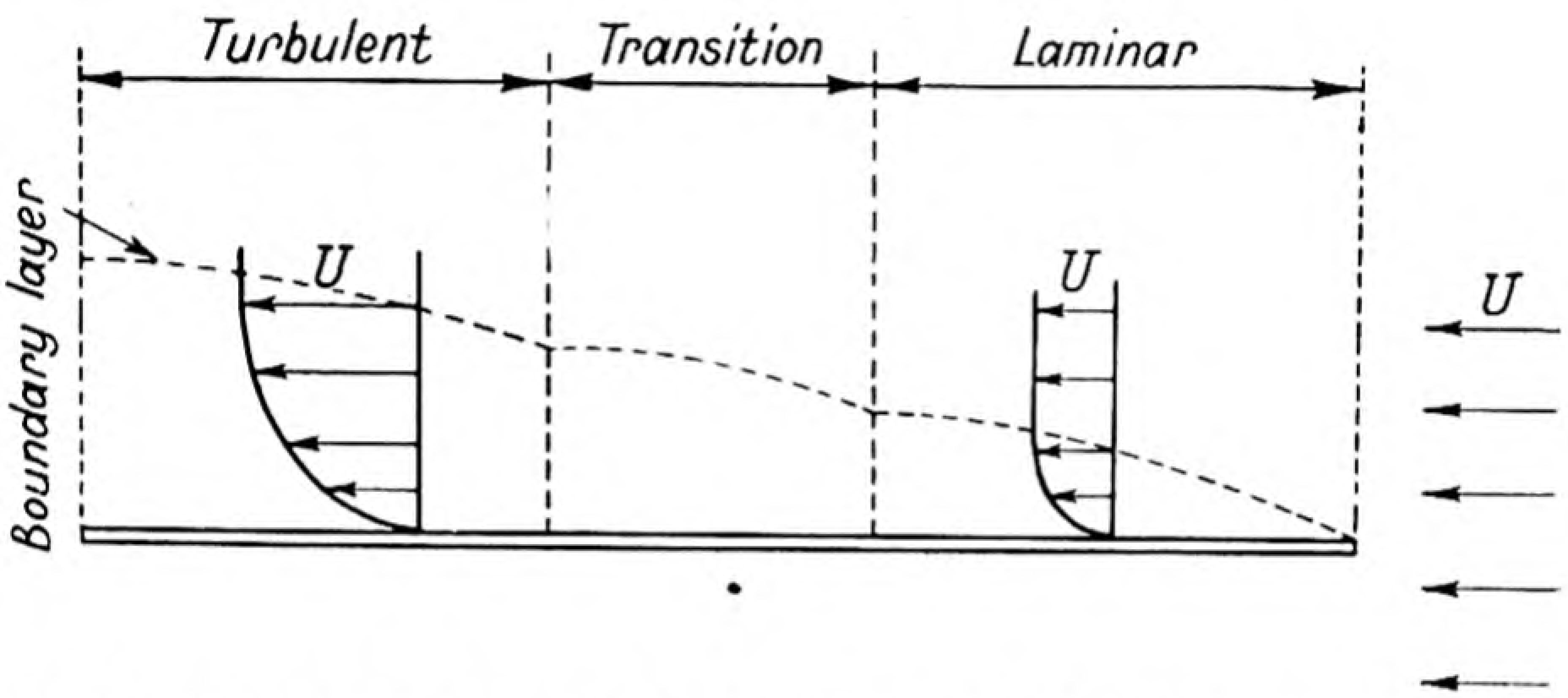

Fig. 1.—Boundary layer and velocity distribution along a plank (diagrammatic only).

Flow in the boundary layer is illustrated in Figs. 1, 2, and 3, which demonstrate the boundary layer and velocity distribution along a plank, the effect of flow regime on the frictional resistance coefficient and a typical velocity distribution around a streamlined body.

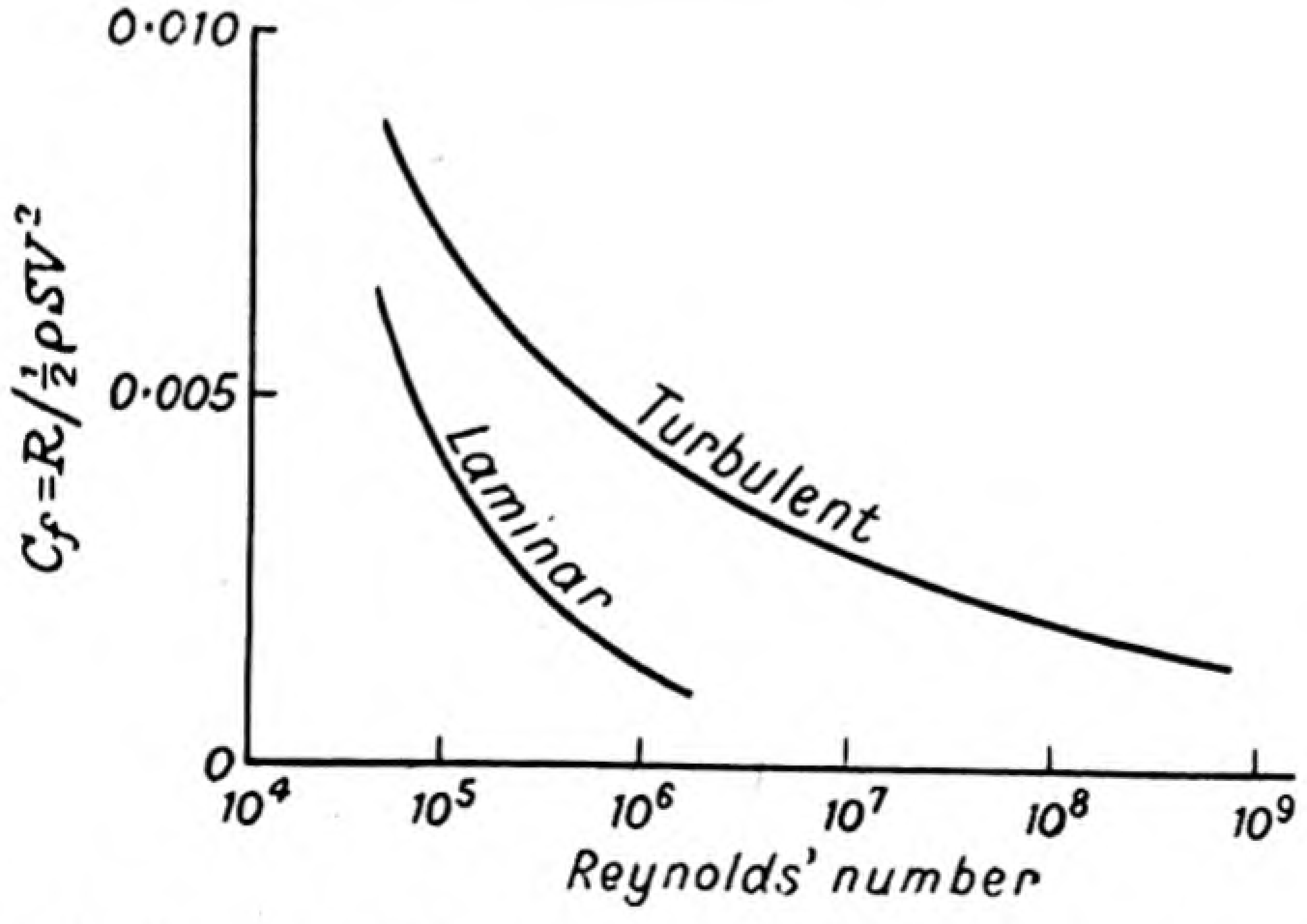

Fig. 2.—Frictional resistance coefficient as affected by Reynolds' number and flow regime.

Computational methods exist which give satisfactory results for the initially laminar boundary layer velocity distribution. These methods can be extended to two dimensional curved boundaries and to the case of axial symmetry but in the case of the general curved boundary three dimensional effects arise which cannot at present be solved—an example is the flow in a pipe bend.

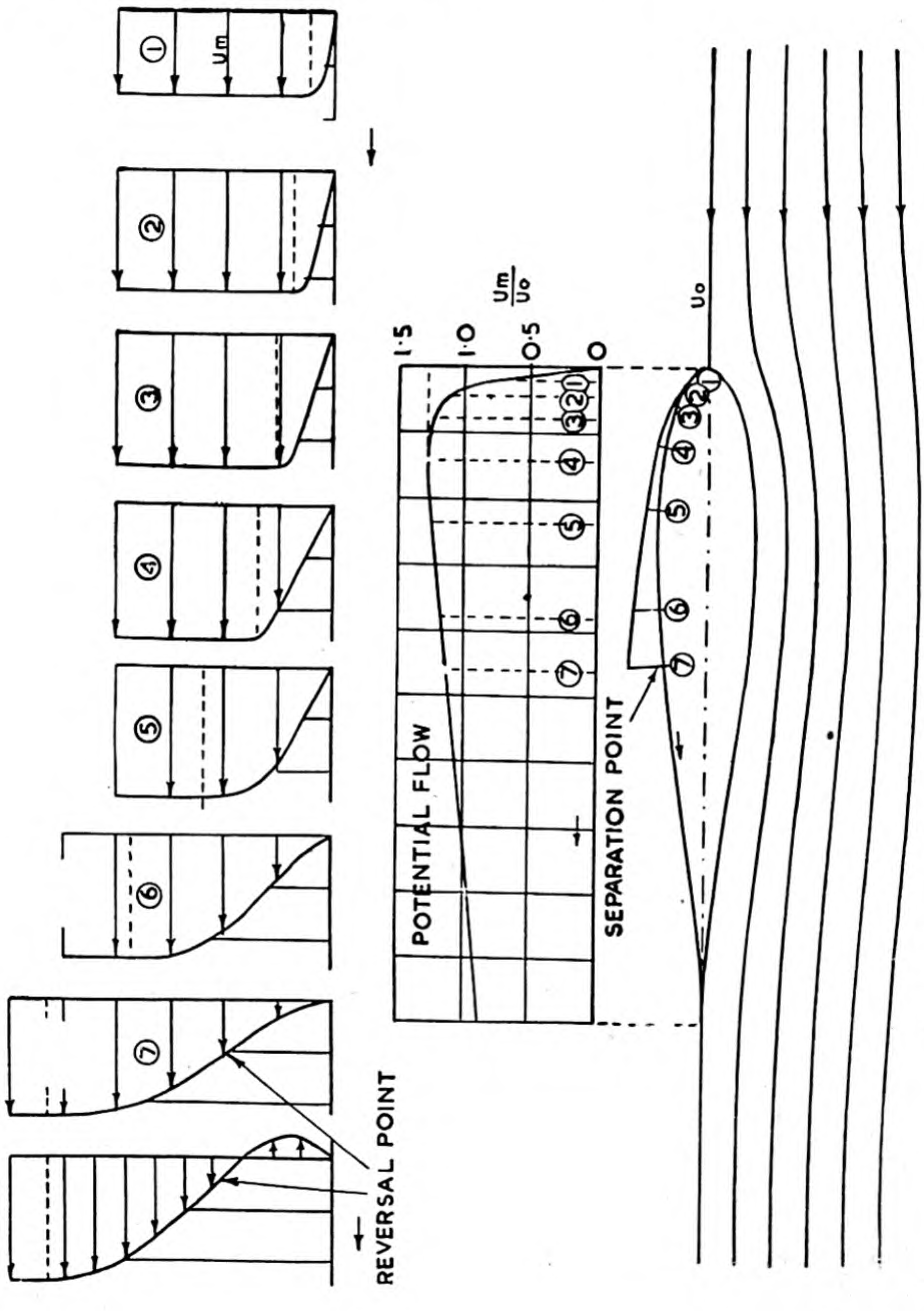

Fig. 3.—Typical velocity distribution around a streamlined body.

While further investigation of boundary layer phenomena occurring in connexion with ideal surfaces is required, especially directed to providing solutions to problems involving three dimensional curvature, there is another very important factor to consider—the effect of boundary roughness on fluid flow.

This is of importance in most pipe systems but it has other aspects since such roughness affects the drag of stream line bodies like aerofoils and ship hulls. In the case of aircraft, for example, it is obviously advantageous to maintain extensive regions of laminar flow in order to reduce the skin friction drag. Much can be done to preserve laminar flow by the design of favourable shapes for the wing and fuselage and a considerable amount of theoretical study has been devoted to the subject but, in practice, premature transition from laminar to turbulent flow is often caused by random roughness and waviness as well as by blemishes in the construction of the aircraft surface.

The standard method of determining the point of transition in a boundary layer is a tedious process since it involves the comparison of boundary layer profiles at different points along the surface. This method has been replaced by an interesting technique which gives a rapid and visual measurement of the relative dispositions of laminar and turbulent boundary layer flow and which can be used in flight to show the origin of disturbances which lead to early transition. The method depends on the fact that the diffusion of fluid particles through the boundary layer is much greater when the boundary layer is turbulent than when it is laminar. Thus, a volatile substance spread on the surface of a moving body will evaporate more quickly from the parts of the surface where the boundary layer is turbulent than from those parts where it is laminar; alternatively, a chemical substance attached to the surface of the body will react with some other active substance injected into the main part of the flow at a greater rate when the boundary layer is turbulent. Both principles have been applied to the determination of transition on an aircraft wing in flight, the first taking the form of a solution of hydroquinine diethyl ether in petroleum sprayed on to the aircraft surface to form an extremely thin film of crystals which evaporate first from the part where the boundary layer is turbulent. In the second method a solution of starch, potassium iodide and sodium thiosulphate is applied to the surface under observation and the aircraft is flown into a trail of chlorine vapour laid in the atmosphere by another aircraft. The chlorine reacts first with the potassium iodide on the part of the surface where the boundary layer is turbulent, and the released iodine discolours the starch leaving a distinctive pattern to show the areas of laminar and turbulent flow. A number of variants of these methods exist, the choice of any one depending on such factors as flight speed and atmospheric humidity.

These methods can also be applied to wind tunnel models, and, in particular, to tests at high speeds. This latter application is of great importance when shock waves are present in the flow field, since the interaction between a shock wave and a boundary layer may depend critically

on whether the boundary layer is laminar or turbulent. In these conditions interest is not confined to the drag, but includes the complete pattern of flow about the body.

Boundary layer flow is also important in ship model experiments undertaken with the object of evolving the optimum form of hull for specified conditions. It has been found that laminar flow can occur over an appreciable part of the length of the waxed models used in the experimental tanks, whereas it is known that full-scale conditions are certain to be fully turbulent. It is essential, therefore, that measures should be taken to avoid laminar flow on the models. The usual method is to fit a wire to the model near the bow. A similar precaution is often taken when wind tunnel tests are made on aircraft models.

Another problem in this field is the determination of the magnitude of the boundary layer present in the flow of gases through tubes. There is particular interest in the case when the flow in the boundary layer is streamline. Measurements have so far been made by techniques which must have caused some disturbance of the flow pattern and it is desirable to make direct measurements of the boundary layer thickness by techniques of flow visualisation which do not disturb the flow pattern.

In addition to the required extensions of the computational methods for determining laminar boundary layer velocity distribution and predicting the change to turbulence and the separation point already referred to, more data on turbulent boundary layer flow and other methods of treating this are needed. A digest of experimental results on turbulent boundary layer velocity distribution obtained from work on diffusers, flow through pipes and along flat plates has been given by Goethals(1). At present such flows cannot be analysed satisfactorily and the best that can be done is to form convenient organizations of experimental data. As more data become available a better approach should be possible to such problems as the reduction of energy losses and the development of more efficient flow channels and machines.

2.2. *Flow under entry conditions*

Very often in cases of fluid flow through tubes and ducts, the nature of the flow is known both qualitatively and quantitatively once the disturbances arising at the entrance to the channel have died away in the first few diameters length of the channel. When long channels are being considered these entrance effects may often be neglected. However, in many industrial operations, the process occurs in short channel lengths while the flow is still determined by the entry conditions.

A good example of this type of problem occurs in the heat exchanger of an instantaneous gas water heater. Here the exchanger is often of the radial finned type, the spacing between fins being of the order of 0·1 in., the width of the gas way being about 1 in., and the length of this approximately rectangular channel being about 4 in. as in Fig. 4. Most of the heat exchange occurs in the part of the path first traversed and, in fact, the shape of the leading edge of the fins can have an important effect on the

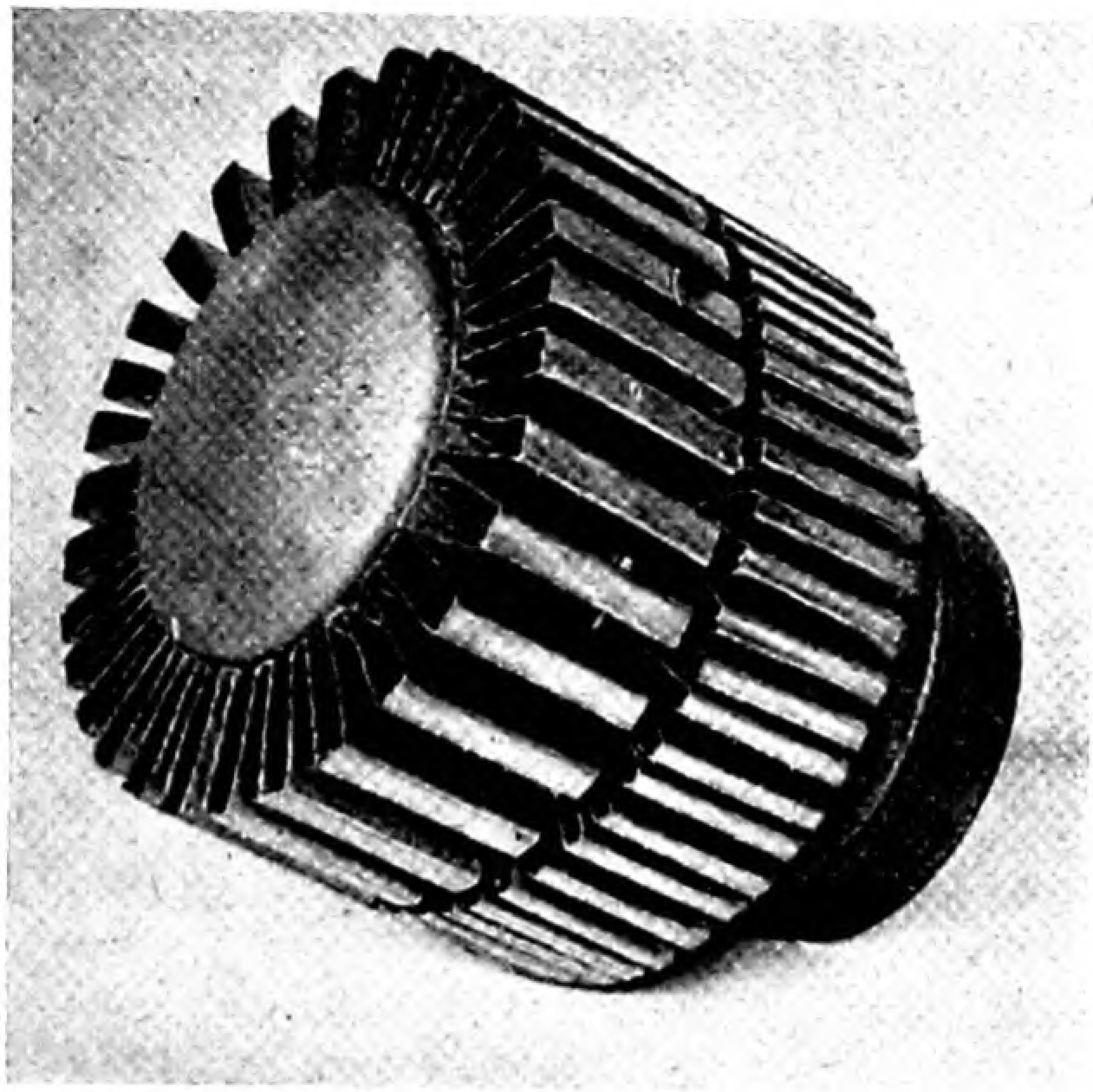

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Fig. 4.—Water heater heat exchanger. Length of fins about 4 in.

heat transfer. Clearly the flow in the entrance, and, say, the first inch length of path must have a controlling influence on the heat transfer and so on the efficiency of such exchangers.

Some work has been done by Bennett(2) on the heat transfer occurring under such conditions, but very little appears to be known about the flow under entry conditions, although techniques for investigating this would appear to be available, such as Lewis & Von Elbe's powder method(3) of flow visualisation.

2.3. *Scale effects.*

As is well known, amongst the many contributions of Osborne Reynolds to the study of phenomena connected with fluid flow is the famous principle of dynamical similarity of flows in two systems which are geometrically similar when conditions are such that a dimensionless parameter, called Reynolds' number in honour of its discoverer, has the same value for the two systems. This discovery enables scaled experiments to be made in conditions where the study of the full scale phenomenon is difficult or impossible. Model experiments are being used in increasing numbers and are continually being applied in new fields. However, when the results of model experiments are converted to the full scale conditions, there is always some degree of uncertainty about the accuracy of the scaling. It is therefore often necessary to check this by carrying out some measurements at full scale and thus determine this accuracy. It should perhaps be pointed out that such effects in industrial applications are small on the whole, so that at the worst the number of measurements required at full scale is considerably reduced, while it may always prove possible to show that scale effects are negligible in any particular case so that in future work

along similar lines the results of model experiments may be safely scaled to the full size. However, the necessity of determining scale effects does constitute a serious disadvantage in the application of model techniques.

There are many examples of the use of model experiments and it is perhaps hardly necessary to mention the most common of these—the wind tunnel—except to indicate one of the major problems of such work. It is often possible to obtain a close approximation between the Reynolds' number of the model and full size conditions by control of the model size and the speed and density of the working fluid, but it is also necessary to arrange that all of the other experimental conditions are the same. In comparing wind tunnel experiments with full size phenomena occurring in the atmosphere (the most common case), the intensity of turbulence encountered in the atmosphere is very low, and it is therefore necessary to reproduce this condition in the tunnel ; this state is very difficult to achieve since the flow in the working section is influenced by wakes shed from various parts of the tunnel structure. Very considerable progress towards reaching the desired state has been made in low speed tunnels by making use of fine mesh screens in the circuit and refinement in the design of the diffuser and bends. The residual turbulence is largely contributed now by noise generated at the driving fan and inside the boundary layers on the tunnel walls. The scaling problems due to stream turbulence, and in particular the effect of this on the transition from laminar to turbulent boundary layer flow, are being attacked by obtaining information about the effect of free stream turbulence on the laminar boundary layer in order that the results obtained from tunnels in which the degree of free stream turbulence is relatively high may be correctly interpreted and applied to the full scale.

Another example of the difficulties caused by scale effects occurs in the development by model testing of new pumps for large projects. In such work complete similarity does not obtain and some doubt arises in scaling up the model data to the full sized machine. Generally the differences that have been found are only of the order of two or three per cent. in efficiency and perhaps a little more in the actual pump characteristic. This degree of uncertainty is very undesirable since large pumps are guaranteed to achieve definite efficiencies. Naturally hydraulic, leakage and mechanical friction losses are relatively larger in models than in the final machine. Because the first is primarily a Reynolds' number problem, the second a mechanical one and the third a lubrication one, the whole problem cannot be satisfactorily treated as simply a Reynolds' effect which has been suggested in the past. To provide adequate data of this problem it is necessary to know separately the variation of bearing and gland friction with size, and perhaps operating pressure, the variation of leakage or volumetric efficiency with size and pressure and finally the variation of hydraulic efficiency (or losses) with Reynolds' number. The first is probably sufficiently well-known but the latter two need considerable further study. This summary of required pump design and performance data indicates that systematic tests on the effect of varying about six basic design variables are required. Little published information is sufficiently comprehensive to be of much use, and published design methods are based on

elementary theory which is not reliable and are not used by manufacturers who work simply from past experience of individual machines.

Another instance of the difficulty occurs in connexion with problems of convective flow in food stores. Here a reliable model technique is required whereby full scale effects could be produced by experiments under controlled conditions on such models.

An example of the magnitude of the difficulty of assessing the size of scale effects in a particular case is provided by a problem occurring in the gas industry. The standard type of low pressure gasholder now being constructed is well known to everyone by sight. This is called the spirally guided gasholder, and is interesting to the engineer since it is a floating structure. The various problems involved in its design and structure have been solved, if not finally, at least with sufficient accuracy to obtain satisfactory performance. Once such a structure is erected however it is subjected to a varying wind load the magnitude of which (in common with many other wind loads on structures) is not known accurately. Wind tunnel experiments on models of spirally guided gasholders have been carried out by the National Physical Laboratory for the gas industry to determine the wind pressure distribution and the drag and lift occurring. To check the magnitude of scale effects some measurements of wind pressure distribution on an actual gasholder are being made. When it is realised that the height of the particular holder chosen for the experiments is 120 ft. and its diameter is 120 ft., and also that an essential part of such measurements is the determination of the static pressure of the atmosphere which must be made in a part of the space around the holder in which the wind is not disturbed by the presence of the obstacle (i.e., the holder) some idea of the difficulty of the undertaking can readily be obtained.

While it is possible that with such a wide variety of applications and with many factors effecting the size of scale effects it will always be necessary to study them mainly on an empirical basis, it would appear that the gravity of the problem would justify further fundamental investigation of scale effects. An attempt at a theoretical approach to this subject has been made by Decius(4) whose aim is the treatment of scaling laws for model experiments by the application of dimensional analysis and who gives a new criterion for the possibility of satisfying the scaling laws when certain restrictions are imposed on the variables.

3. FLOW IN CIRCUITS

Whenever fluids are used, whether in the laboratory or industrially, flow occurs through various valves, vessels, pumps, etc., which are connected by tubes or ducts according to the conditions. Many problems arise in connexion with the flow through such a circuit. The flow in the circuit may be steady or varying and different problems are important in each case. The term steady flow is used here to include turbulent flow providing the rate of flow past any section of the circuit is constant when averaged over a time great enough to eliminate variations in this rate due to turbulent eddies. It may be worth emphasizing that cases do occur in

which apparently difficult problems involving the steady flow of gases can be shown by using a suitable measuring technique to be in reality problems in pulsating flow.

3.1. *Steady flow in circuits.*

3.1.1. *Frictional loss and pressure drop.*

Many of the problems occurring in connexion with steady flow in circuits are concerned with the frictional losses and corresponding pressure drops in various of the circuit elements, in particular pipes, valves, orifices and Venturi plates. In this field there is available a large amount of empirical data, very often collected from the engineering aspect.

Such problems in the flow of gases are of common occurrence but often of a trivial nature. Frictional losses become more important in some industrial processes such as steel making and iron making on account of the very large volumes of gas required, and also in the flow of producer gas through flues, orifices, Venturi plates, etc., when used in the melting of glass. Similar losses, of course, affect the distribution of town gas. Frictional losses are also important to the gas industry in aerated burners. In such burners, the energy of gas issuing from an orifice is used to provide a gas/air mixture by entraining air, and one of the main ways in which the available energy may be wasted is by friction in various parts of the burner.

A physical investigation of the flow resistance in circuit elements of shapes other than the standard ones usually dealt with, is needed. There is also the possibility that the comprehensive fundamental study of this subject may lead to new methods of treatment which would enable the flow in a particular case to be deduced from general basic data. Many attempts to do this are being made, such as McClain's book[5] on fluid flow in pipes which endeavours to present a treatment of the theory of such flow from the dimensional viewpoint. Recent work on fluid flow in pipes has been presented by Bull[6].

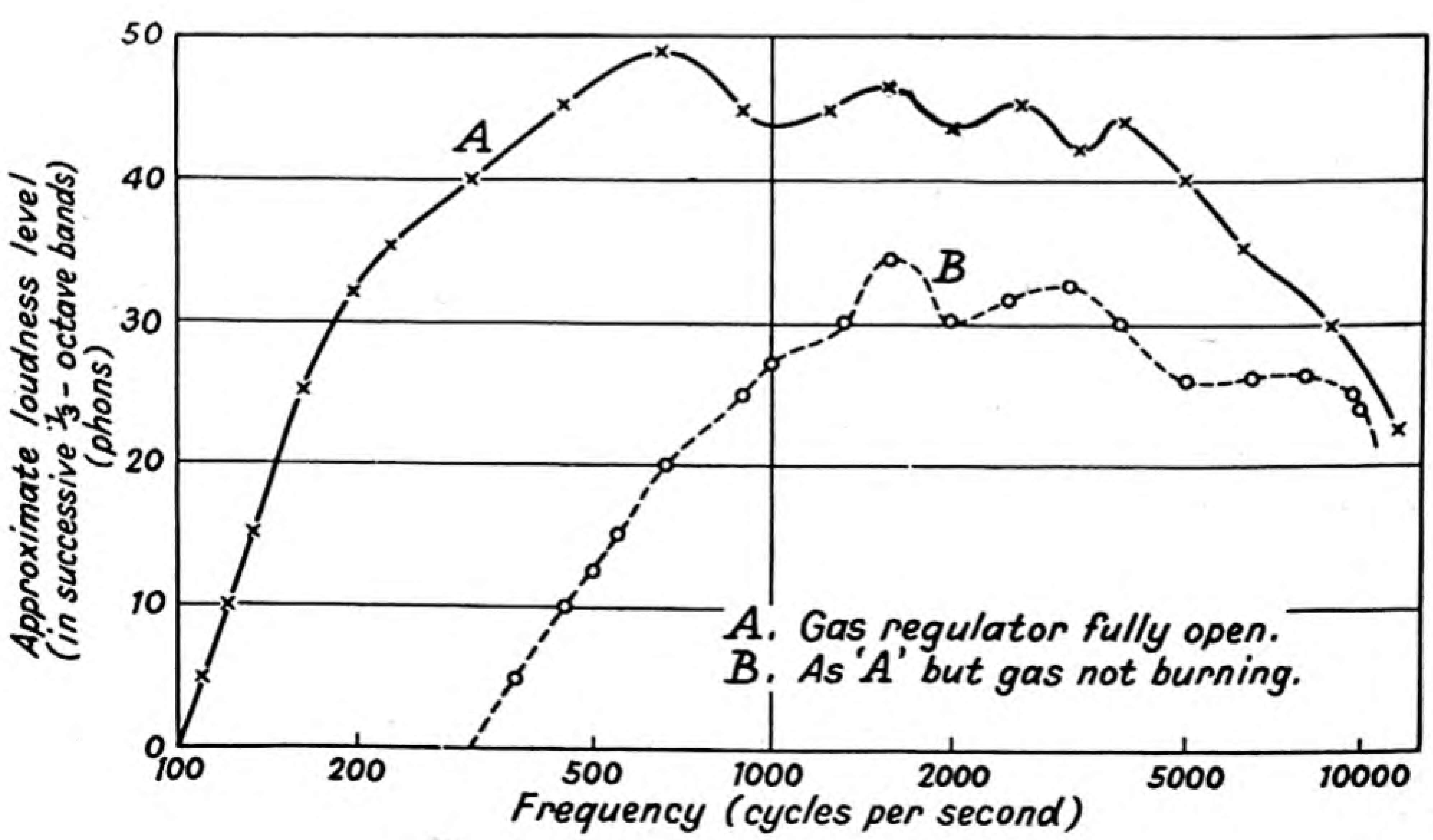

Fig. 5.—Analysis of noise by boiling ring.

3.1.2. *Noise.*

Another quite different aspect of steady flow in circuits is sometimes important. This is the noise produced by such flows and experimental investigation of this subject is required with the object of eliminating noise. This work would involve the detection of the circuit elements causing the noise, the determination of the mechanism by which the noise is produced and the collection of data to provide the basis for the design of noiseless circuit elements. Fig. 5 illustrates the results of a preliminary investigation into the noise emitted by a gas cooking burner.

3.1.3. *Flow in wide tubes and ducts.*

Problems of a different type concerning flow in circuits arise when the flow pattern is important. This is likely to happen when the velocity of the flow (along the tube or duct) is such that velocities across the stream become appreciable. An example of this type of problem occurs in the coal industry where ventilation is an important element. This can be readily appreciated from the fact that in some cases as much as 11 tons of air must be pumped for each ton of coal raised. Moreover, with the advent of mining at greater depths, ventilation becomes more and more important and may well be the factor which limits the maximum depth at which working is possible. The basic problem is the design of air circuits to provide an adequate air replacement rate for a minimum expenditure of energy. The factors to be considered are the size of air passage, the wall roughness and the effect of both temporary and permanent obstructions in air passages. The possibility exists of using electrical analogues on which to base the design of circuits to provide adequate air changeover where desired with minimum horsepower requirements. To do this, it is first necessary to determine the nature of the impedance to air flow of various circuit components encountered underground such as bends, supports, airways of different wall roughness and airways of variable cross-section.

Another aspect of this problem of ventilation in mines is the danger due to certain gases released into the airways. The information required is, assuming point or line sources, how long will stratification of the dangerous gases exist for given air velocities in the circuit components? It is possible to investigate this by the use of a model technique if the difficulty of coping with the different gas densities can be overcome. On the instrumentation side, instruments to measure ventilation parameters but suitable for use underground are needed. Again in connexion with boiler trials and with dust deposition in stacks at power stations the measurement of total flow and the flow distribution in large ducts under turbulent conditions is necessary.

The major requirements for solving the type of problem that arises from steady fluid flow in wide ducts is the further development of techniques for dealing with such circuits—such as reliable model methods or electrical analogue methods and possibly the application of flow visualisation techniques since the difference between flows in relatively narrow pipes and those in wide ducts is mainly that transverse currents are important in the

latter. In addition the development of methods of measurement of flow is needed which can be applied to hot, dirty gases which deposit tar on orifices and pitot tubes and which tend to burn out hot wire anemometers.

3.1.4. *Flow at very low pressures.*

Problems concerning flow in circuits also occur in connexion with the flow of gases in pipes at low pressures—say below a few m.m. of mercury. These conditions now occur in large scale industrial plants and for instance in vacuum drying. Both theoretical and experimental information is available for this type of flow at subsonic velocities ; Mellen[7] has contributed a general article which gives references to other more rigorous and detailed papers, and Brown et al.[8] gives data on flow through pipes and capillaries and also elbows, valves and short tubes in both the molecular flow and slip flow regions, and a method of correlating this data. Information on supersonic flow under these conditions is lacking.

3.2. *Varying flows.*

Problems arising from fluid flow when the flow is varying may be divided into two groups according to whether the flow is transient or cyclically repeated.

3.2.1. *Transient flow.*

An example of a problem concerning transient flow is the elimination of coal dust explosions in mines. The main requirement in this connexion is the design of an air circuit so as to inhibit or control the transient air blast in the circuit, when the transient flow may have velocities up to about half the speed of sound. The stages in the investigation of such a requirement would be (*a*) the measurement of characteristics of transient flow initiated in an air circuit of known components by a real ignition and the deduction of a parameter describing equivalent transient motive force (on an electrical analogy), followed by (*b*) the determination of transient flows in various types of air circuits from equivalent motive force sources described by the parameters deduced in (*a*) and then (*c*) the investigation of the design of air circuit components and combination of components to see if any system of inhibiting the range of transient air flow is practicable.

In addition to this, the dust explosion problem also involves the interaction of a transient air flow and the dust over which it passes. In particular the following points require investigation :

(*a*) The tangential and normal forces acting on dust deposit due to a transient air flow.

(*b*) The rate of flow of air into dust deposit of known permeability during transient flow.

(*c*) The effect of the shape of the dust on the forces investigated under section (*a*).

(*d*) The effect of the surface roughness of the deposit on these forces.

3.2.2. *Regularly varying flow.*

Problems concerning periodic flows are mainly ones of instrumentation, and involve the development of pressure gauges which will indicate the true mean value of a varying pressure. As an example, a problem arising

in the internal combustion engine industry may be quoted. This is the measurement of the air entering internal combustion engines or air compressors of the reciprocating type. This measurement is difficult since the air velocity varies throughout the cycle of operations, these variations being greatest in the case of single cylinder units. The method usually adopted smooths the flow by means of a large vessel or series of small vessels connected by passages resistant to the flow so that the variation in pressure drop is reduced. This has the disadvantages that it is inconvenient, that the residual error is unknown and that this error depends on the form of the pressure-time diagram of the cycle. The Alcock viscous air flow meter was developed as an improvement on the above method. The principle of this is to arrange that the flow is viscous and streamline so that the pressure drop is proportional to the flow and hence the time mean pressure drop gives a true measure of the mean flow. This method has the drawbacks that the calibration is seriously affected by clogging, due to dirt, of the small passages in the meter and that it is difficult to obtain a correct measure of the true mean pressure drop.

The necessity of measuring mean pressures also occurs in the same industry when assessing the merits of various types of air compressor used for super-charging internal combustion engines, since considerable pressure variations occur during the working cycle at both inlet and outlet with the positive displacement type of compressor which is widely used. The recording and analysis of the pressure diagrams is impracticable and so some form of direct reading mean pressure gauge is required.

4. FLOW PATTERNS IN SPACES

As previously mentioned, most of the fluid flow problems concern the frictional losses occurring in circuits. These are due of course to the boundaries limiting the flow. If these boundaries are not important the focus of interest is transferred to the distribution of the flow and its pattern. This state of affairs arises when the flow occurs in a space. The space may be either open or confined—for example the flow past obstacles such as aerofoils and the flow inside furnaces. It is sometimes convenient to study flow patterns on the full scale but very often model techniques are used. In this case the difficulties associated with scaling effects must be overcome. This subject has already been discussed in Section 2.3.

Various techniques for observing or measuring flow patterns are in use. The normal method of introducing a probe into a fluid stream cannot be applied very satisfactorily since it is directional and therefore not suited to investigate patterns and moreover it is very laborious.

One group of methods makes use of the variation of the refractive index of a fluid stream which may be caused by variations of pressure, temperature or composition of the stream. Examples of these occur in flow past obstacles in wind tunnels where the obstacle causes pressure changes in the stream near it, in the flow of gas through an orifice into the atmosphere when changes in the composition of the gas stream occur due to mixing of the air and gas and in the study of flames when both temperature

and composition vary across a section of the stream. There are three methods of using this variation of refractive index. The simplest is the shadow photography technique which merely depends on the fact that the light passing through, for example, a stream of gas, which is entraining air and therefore has a variable composition across its section, will be deviated different amounts in different parts of the stream ; so the light intensity on a screen behind the stream with respect to the light source will vary. This method, as compensation for its simplicity, has the drawback that it is not very sensitive and the further disadvantage that the position of an edge of a shadow on the screen does not correspond to the position of the physical phenomenon causing the shadow owing to the deviation of the light. The principle of this method is shown in Fig. 6

Fig. 6.—Diagram showing principle of shadowgraph method.

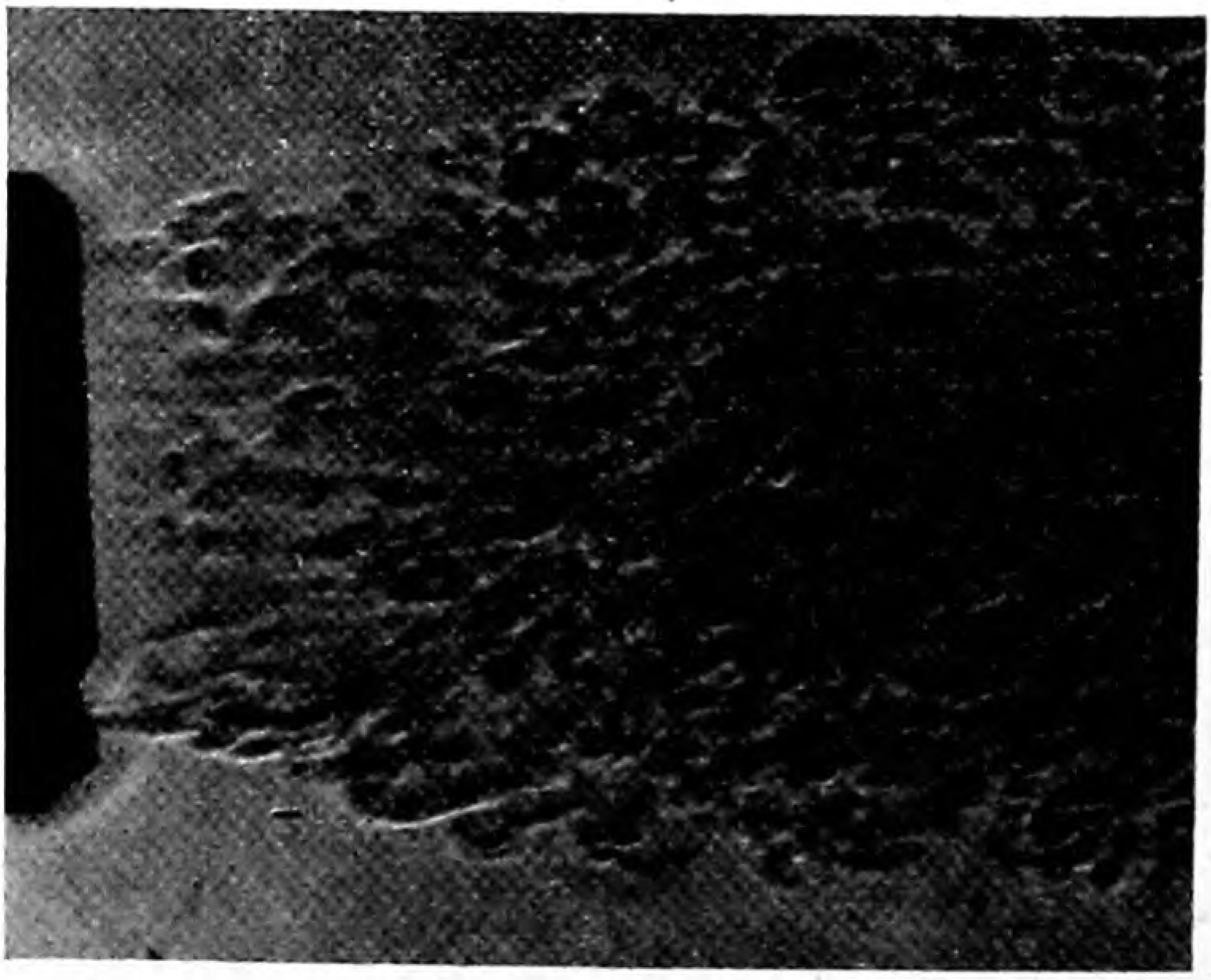

Fig. 7.—Shadow photograph of a stream of heated air.

and the picture obtained by this method of a stream of hot gas emerging from a tube is given in Fig. 7.

The next in order of complexity is the Schlieren technique(9). In principle the Schlieren method consists of an optical system containing two apertures and arranged so that no light passing the first aperture can pass the second unless it has been deviated between the two. If the light is deviated by some variation in refractive index, it passes the second aperture

and is brought to a focus on a screen. In practice there are many variations of this technique—one or two mirrors or lenses may be used, the apertures may be knife edges or a circular opening or the Ronchi modification may be used, in which case the method is semi-quantitative. This method can be highly sensitive and also suffers little from disadvantages caused by the deviation of the light since this is brought to a focus. There are many methods of applying the Schlieren technique. One of these is shown diagrammatically in Fig. 8. The appearance of a stream of hot gas emerging from a tube photographed using this method is shown in Fig. 9. The subjects treated in Figs. 7 and 9 are very similar.

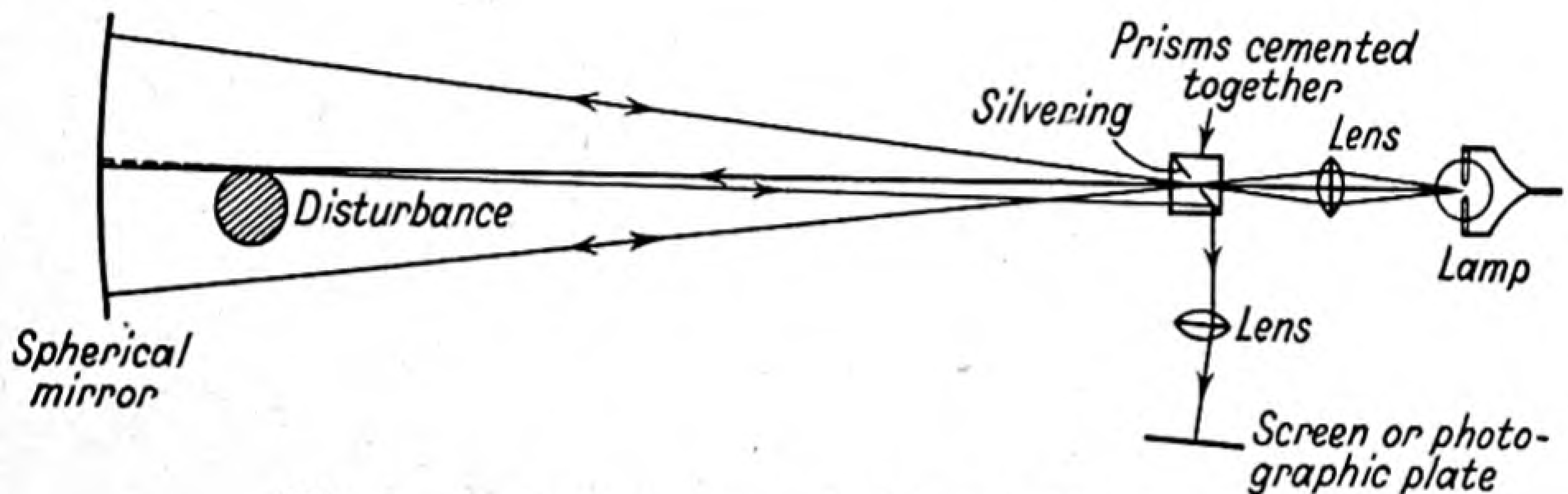

Fig. 8.—Diagram showing principle of Schlieren apparatus.

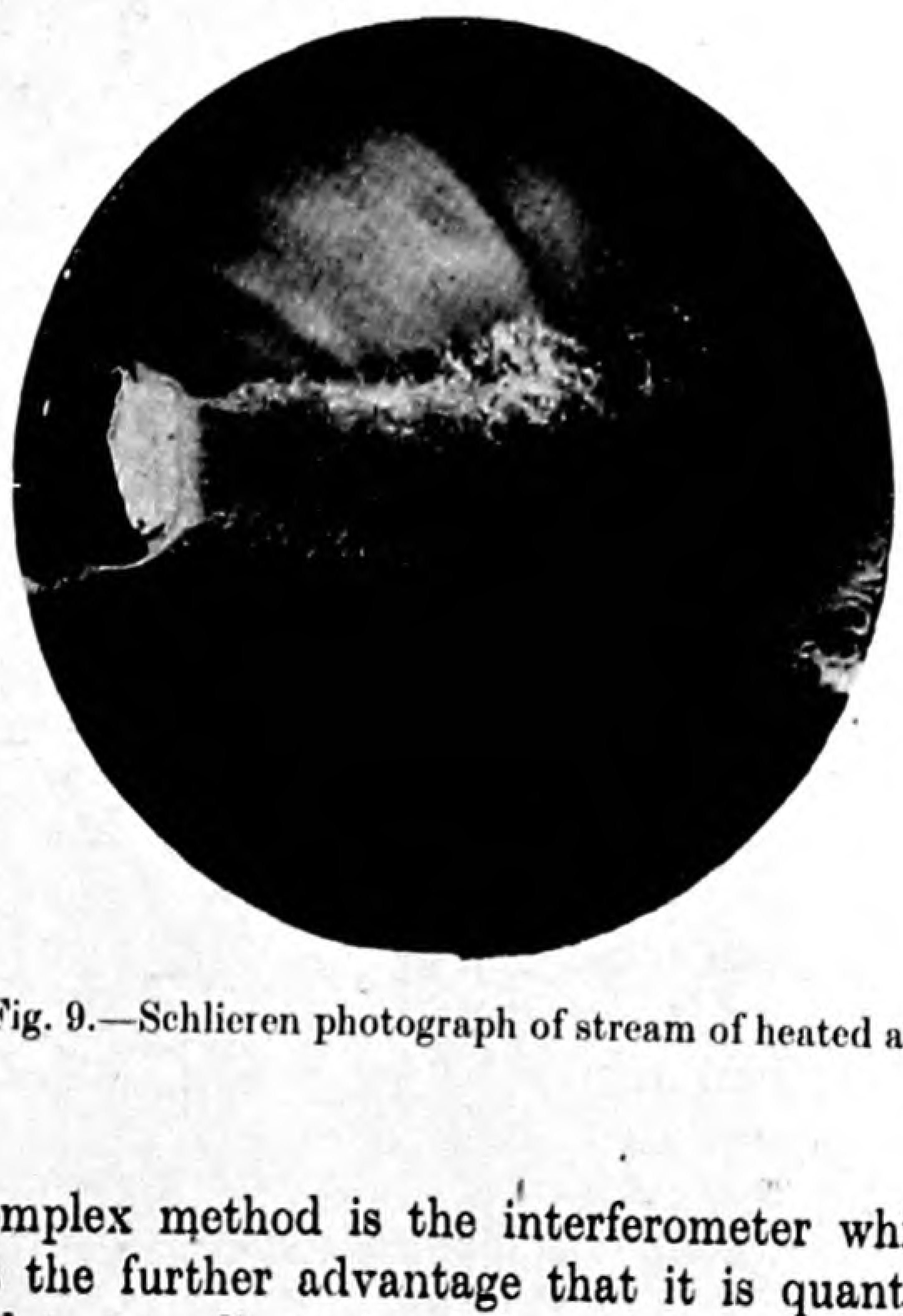

Fig. 9.—Schlieren photograph of stream of heated air.

The most complex method is the interferometer which is extremely sensitive and has the further advantage that it is quantitative although only when applied to two dimensional flows or to three dimensional flows possessing rotational symmetry.

Another method—of which there are many variations—uses the entirely different principle of introducing small particles into the stream. Using a suitable form of illumination, the tracks of these particles are then photographed as lines, the length of which are readily varied by adjusting the exposure time. If intermittent illumination is used, the track of a particle is obtained as a series of dashes from which the particle velocity

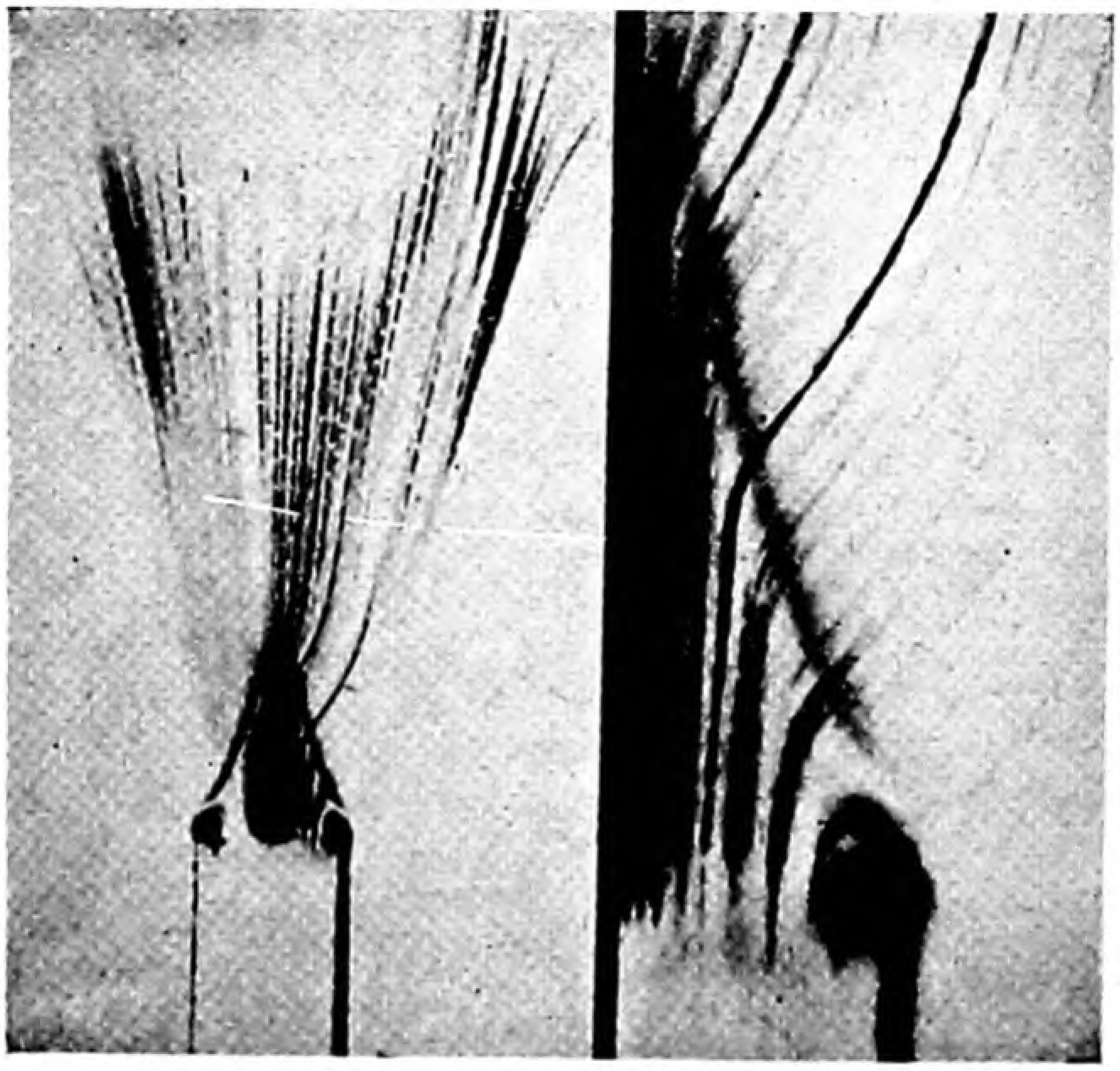

Fig. 10.—Flow of gas through a flame (due to Lewis and Von Elbe).

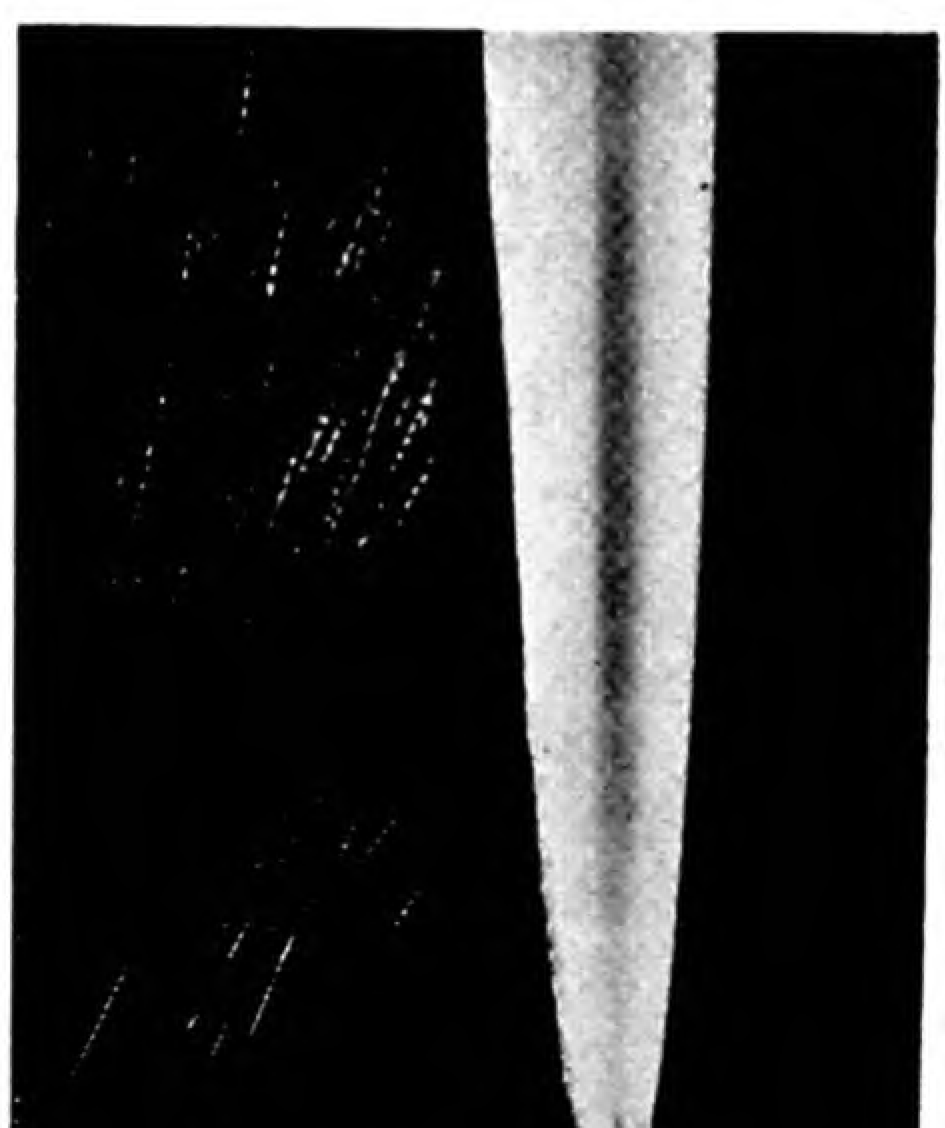

Fig. 11.—Flow of air into a flame.
(*photograph by Gas Research Board*).

may be measured. A further refinement of this technique has recently been used whereby the direction of motion is indicated by using different times of illumination repeated regularly. Originally this method was used to show the streamlines in liquid flow by the introduction of solid particles into the flow. Subsequently it was extended to gas streams and later liquid droplets were used. When this is possible a very great increase in the amount of light reflected by a particle is obtained. Recently in model experiments, using water as the fluid, small air bubbles have been employed as the particles. Figs. 10 and 11 show two applications of this principle. The first, due to Lewis and von Elbe[3], shows the path of gas through a flame whilst the second shows the movement of air towards a flame to provide the secondary air for combustion.

Another method for the qualitative study of gas flows at low density has been developed recently. This uses the afterglow in a gas of low density when excited by an electrical discharge[10]. The flow pattern of the glowing gas is recorded photographically. Although the phenomenon has been applied it has not yet been fully investigated, but enough work has been done to show that this method may serve the function at low density served by smoke-flow observations at low speeds and Schlieren observations at high speeds in streams of higher density.

As mentioned, problems involving flow in spaces occur in many industries. One large group of these problems involves the determination of the flow pattern in furnaces—using that term in its widest sense. In particular, study is needed of the flow pattern from jets into confined spaces and the effect of the geometry of the confined space on the flow past obstacles. For example, the iron and steel industry needs to know the flow pattern of gases in the various types of furnace that it uses. Again in the gas and coal industries, information regarding the flow patterns inside ovens and furnaces is required. This problem is obviously related to the last with the difference that there is a solid object being heated in the space.

Another aspect of flow in spaces is provided by the flow of air from the atmosphere into a flame to provide the secondary air for combustion. This is a subject that has been neglected and very little quantitative data is available. The subject has some importance since the movement of the air outside a flame can exert considerable influence on the form and behaviour of the flame.

Another problem concerning flow in large spaces which awaits investigation is the efficient ventilation of industrial buildings such as melting shops. Large convection currents must occur in these and clearly these should be used to produce the ventilation required.

A problem in this field also occurs in connexion with convective flow in food stores. A valuable aid to the attack on this problem would be the development of a reliable visualisation technique which could be applied on a model scale and which would deal with the movement of air by natural and forced convection, with the effects of surfaces maintained at specific temperatures and with solids producing heat at specified rates. In

addition an instrument is required to measure the speed and direction of feeble currents of air and to give a remote indication of these measurements.

The main requirements in the attack on the problems involving flow patterns in spaces are the development of further methods of flow visualisation and the application of existing or new methods to three dimensional flows.

In addition, since so many of the problems arising from gaseous flow in spaces concern furnaces and burners, information is also required concerning the modification of the flow occurring when a very hot stream is considered (up to about 2000°C.) compared with the flow occurring at normal temperatures. This can be particularly important when model scale experiments are used to determine the flow, since the model experiments are often carried out at low temperatures. The data required is the value of the various physical quantities determining flow and heat transfer at temperatures up to about 2000°C.

5. CONCLUSION

There are many industrial fluid flow problems in which the flow pattern is either the important factor or else one of the major controlling factors. The scope for the application of methods of the type described in this paper is therefore large. Some of the papers being presented to the conference will deal with such methods in detail. At the same time, it is clear from the survey of industrial problems that it is necessary to develop the experimental techniques for investigating patterns of fluid flow so that they may be applicable over wider fields. One of the directions in which this development is most necessary is that of dealing with three dimensional problems, either directly or by the integration of results obtained in two dimensions.

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Survey of Industrial Problems involving the Hydromechanics of Fluid Flow

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ABSTRACT. The problems that arise in the design of hydraulic equipment and the methods available for dealing with them are indicated by detailed reference to two special cases : the design of a centrifugal pump and of a pipe line.

While the crude principle of a centrifugal pump is quite simple its behaviour does depend on flow at high speeds within complex boundaries which can involve large or small losses depending on the choice of type and detail dimensions of the pump to suit the required performance. The special problems of gland seals and cavitation erosion are also outlined.

Reliable estimation of the friction in commercial pipes depends on a knowledge of wall roughness effects. The present state of this problem is reviewed together with the additional losses introduced by pipe fittings and the dangerous pressure surges that can be set up in pipe lines by changes in flow. Further research is urgently required in many of these fields.

INTRODUCTION

Some of the problems in fluid mechanics which concern the industries served by the British Hydromechanics Research Association may not be generally appreciated, and the purpose of this note is to call attention to such of these that are possibly amenable to physical research.

The field of interest may be roughly defined as the behaviour of fluids that are contained within boundaries, as distinct from aerodynamics, ballistics or ship design, and may be classified broadly into :

(*a*) Machines involving transfer of mechanical energy by fluids such as rotodynamic and positive displacement pumps, turbines and motors, presses, fluid couplings, compressors, fans, injectors ;

(*b*) transport and control of fluids by pipes, channels, valves, meters ;

(*c*) processes involving fluids such as fluidization, absorption, spraying, mixing and separation.

The flow phenomena involved in these various practical applications include viscous and inviscid flow, turbulence, cavitation, compressibility, pressure surges, and surface tension effects. The problems are normally considered to be within the domain of the applied mathematician rather than the physicist, but the simplifying assumptions which sometimes have to be made (such as the neglect of viscosity or the conception of an ideal fluid) often lead to misleading results for real fluids. It is possible that if the basic physical processes involved in the practical problems of fluid mechanics were better understood the engineer might be able to design more from first principles rather than having to rely on analogies or model experiments.

As is so frequently the case in engineering research, one of the most difficult aspects is the definition of the problem, and it may be of assistance therefore if typical simple examples are considered, and those various relevant design problems which need more study are examined in detail. The examples chosen are the design of a centrifugal pump and a pipe line.

CENTRIFUGAL PUMP DESIGN

A centrifugal pump consists of a rotating impeller which gives angular momentum to the fluid, some of which appears as a pressure rise from the central inlet to the peripheral outlet and the remainder as a great increase in whirl velocity at outlet. The casing is provided with diffuser vanes or a volute in which the high whirl velocity is reduced and partly converted into further pressure energy.

It is generally possible to design and manufacture any machine over 100 h.p. with an overall efficiency of from 80 to 85%, depending on the type. Improvements on these efficiencies and reductions in size (e.g., by increasing speed) can be effected but there is at present very little systematic data other than that which each manufacturer has culled from his own experience. Some important points needing systematic study will now be discussed.

(*a*) *Effect of the number of Impeller Vanes.*—The total head is generated by the impeller giving angular momentum, or whirl, at its discharge. Fig. 1 shows that if the fluid left the impeller, relative to it, at the blade angle β, the whirl component would be $u—f\cot\beta$; but because of the relatively few blades (usually 5 to 9) it would be surprising if all the exit water did come off exactly at this angle and, in fact, it does not: the actual whirl is usually only about 0·7 of this theoretical amount.

This " whirl-slip " is reduced by having more pump blades so that a greater head is generated, but the discharge at best efficiency point is reduced by the blockage of the additional blades. Since head and delivery make up the two output terms in efficiency it is rather likely that there is, for a given set of other conditions, a definite number of blades corresponding to optimum efficiency. Practical means of estimating this effect have been given by Pfleiderer and Stodola and theoretical solutions of the flow of a perfect fluid through a simplified impeller have been made in general terms. These are not particularly reliable and the latter do not apply to the usual form of double curvature impeller which is found in practice to be the most efficient, and in any case they neglect boundary layer and other effects. Little data is available on which the matter can be studied further and the choice of the number of blades is a matter of practical experience.

In the case of the modern axial-flow compressor it is possible to use cascade data with a good chance of obtaining a reliable solution, but similar reliability is not found with the axial pump because of its much smaller hub-tip ratios which do not permit the safe application of two dimensional flow theory.

(*b*) *The Effect of Impeller Vane Angles.*—As was pointed out above, simple theory is of little use since the fluid leaves at something like one half the blade exit angle and this factor is the basis of the " number of blades effect". Quite a large proportion of the delivery pressure head is attained by recovering at least a part of the kinetic energy of the high absolute exit velocity v (Fig. 1) so that we should expect a greater head from the same machine if β is larger since v is then larger. There are considerable losses in

any diffuser, and especially in the types used in centrifugal pumps, so the overall efficiency falls if too large a proportion of the total head has to be recovered from the impeller exit kinetic energy. On the other hand if β is especially small, the whirl component, w is small, and the total head generated for a given impeller diameter is small : in that case, other losses such as disk friction, become large in comparison to the total energy transferred to the fluid and efficiency is reduced. Efficiency therefore varies from a low value at very small exit angles, to a maximum at an angle which will depend on the pump type, and then falls again due to excessively high exit velocity at large blade exit angles. It might be interesting to outline

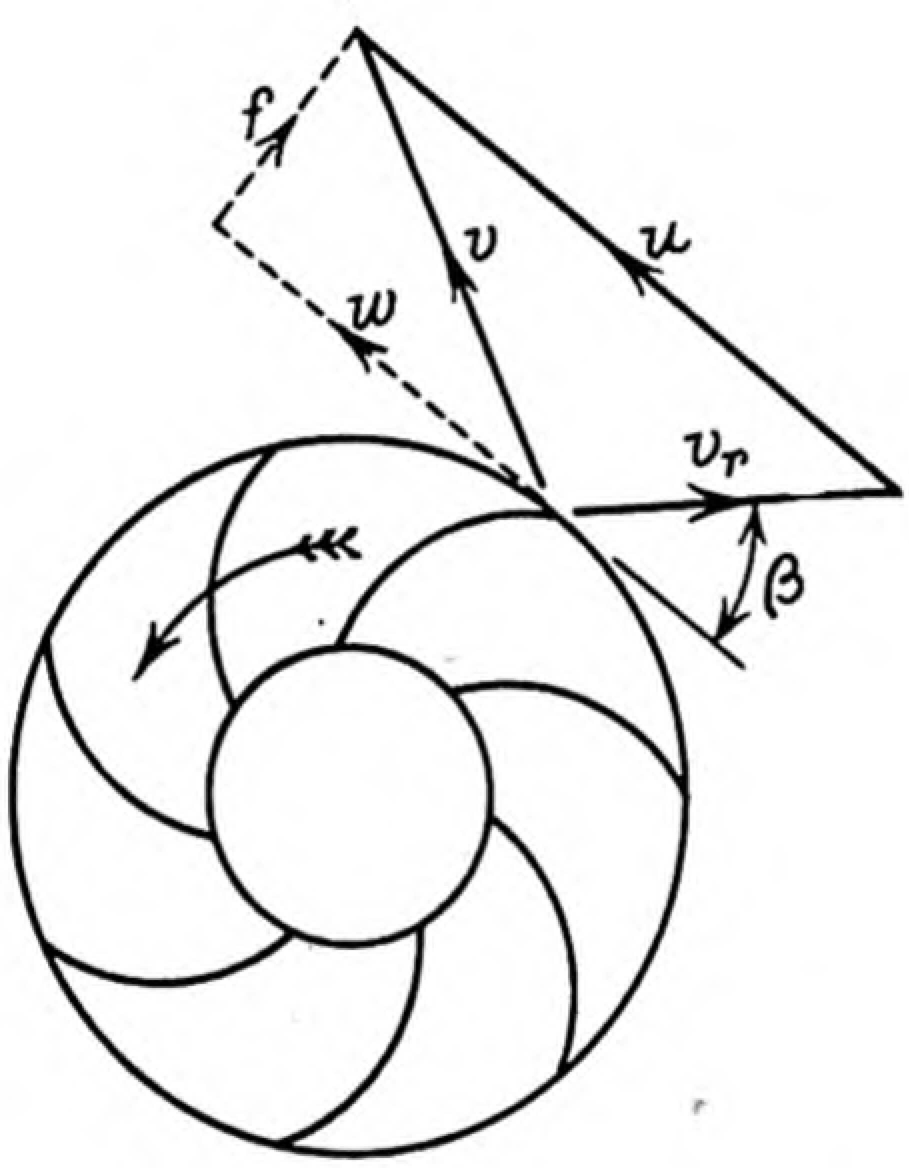

Fig. 1.—Simplified velocity triangle at exit of impeller.

very roughly how the efficiency of a pump is affected by these considerations: The total theoretical head (in the form of both pressure and velocity energy) generated by the impeller is given (neglecting whirl-slip) by simple momentum considerations as $uw/g = H_{th}$. From the exit velocity triangle in Fig. 1, $w = u - f\cot\beta$. If we take $f/u = 15/100$ then we can calculate the variation of $H_{th}/\frac{u^2}{g}$ with β. That is, the theoretical head generated for a given impeller peripheral velocity, u, which is plotted in Fig. 2.

The exit kinetic energy $v^2/2g = (w^2 + f^2)/2g$ can also be put in terms of u^2/g and β.

If we assume that only 0·5 of $v^2/2g$ is converted into pressure energy and the remainder is a diffuser loss then the total head appearing from the pump would be (H_{th}—0·5 $v^2/2g$) while the shaft horse power put into the pump is proportional to H_{th} so efficiency on this basis is (H_{th}—0·5 $v^2/2g$)/H_{th}. This has been calculated over a range of blade angles and, quite naturally, this

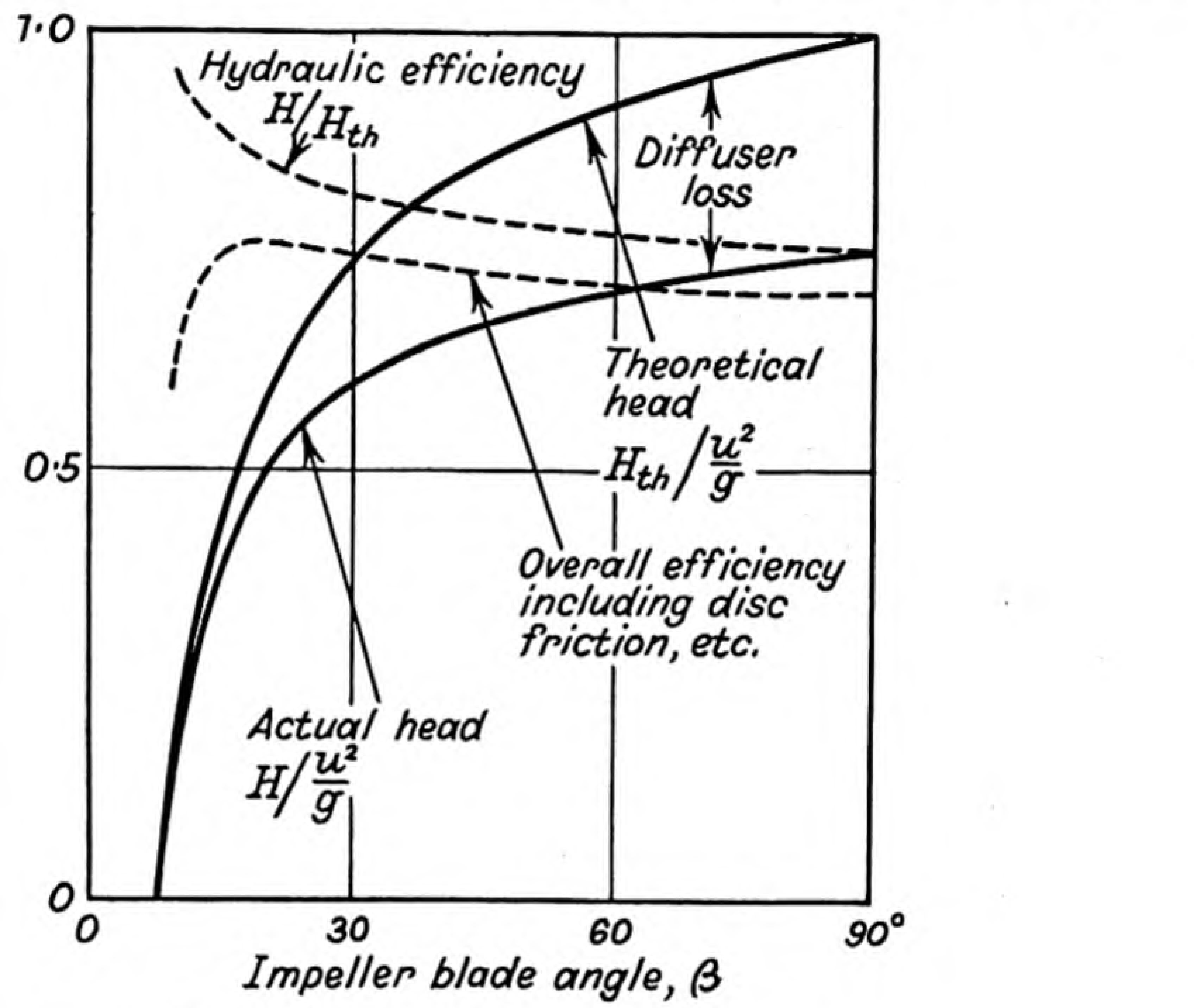

Fig. 2.—Effect of blade angle on head and efficiency.

efficiency increases with reducing blade angle for the reasons given above. However, the fixed loss terms which do not vary with blade angle have been neglected. Perhaps the most important is the skin friction on the two impeller shrouds, to overcome which the shaft must supply a constant additional horse power. The equivalent head of this disk friction loss may be about 0·07 u^2/g. So the pump efficiency including disk friction is roughly (H_{th}—$\frac{1}{2}v^2/2g$)/(H_{th} + 0·07 u^2/g) which is also plotted. As predicted, this efficiency rises with decreasing blade angle until it appears a maximum at about 20° and then begins to fall because of the overwhelming effect of disk friction and is zero at the β corresponding to zero head.

This shows roughly how blade angle affects efficiency. In addition it also considerably alters the shape of the head—discharge characteristic. It is not possible to calculate either of these effects reliably from straightforward analysis.

(*c*) *Pump Volutes and Vaned Diffusers*.—It was pointed out in Section (*b*) above that quite a large proportion of the total energy imparted by the impeller to the fluid is the high kinetic energy at the impeller exit. Unduly large losses arise if these exit velocities are too high so here also is a limitation on speed and head per stage. In a volute casing part of the high velocity

is dropped as the fluid enters the volute where a constant, fairly high velocity is maintained and any further deceleration is effected in the conical expansion leading from the volute to the discharge flange. The flow in the volute is particularly complex and their cross sectional shape appears to be a matter mainly of fashion : formerly they were made of circular cross section but a pear shape is more popular nowadays. Instead of fitting a volute collector it is possible to decelerate the fast whirling discharge of the impeller with a set of stationary vanes which collect the fluid at its exit angle and then gradually straighten it out to mainly radial flow. Such a vaned diffuser is quite efficient at normal delivery but at other deliveries the entry angle of these fixed vanes does not correspond with the direction of the impeller discharge and high losses result. Vaned diffusers are generally used in pumps where space is limited as in multistage pumps or where the mechanical strength of the casing is important.

The number of vanes in a diffuser has a considerable effect on its performance characteristic and much further study is required before it is possible to predict the best arrangement for a given new set of requirements.

(*d*) *Cavitation.*—The total energy of the fluid entering the suction side of a pump is usually low, so that any hydraulic losses and the kinetic energy required to arrive at that zone of the impeller at which the fluid begins to pick up mechanical energy can only be obtained at the expense of a further reduction in the already low pressure. The normal limit of minimum pressure is usually determined by the onset of cavitation which occurs when the absolute pressure drops to the vapour pressure of the fluid, with resultant formation of vapour bubbles. If machines are run under cavitating conditions there is usually excessive noise and a loss in efficiency and output, while erosion frequently occurs on the surfaces where cavitation is taking place.

The avoidance of conditions leading to cavitation may increase the cost of a pump considerably, sometimes necessitating placing it at a lower level to increase the static pressure at the suction, with a corresponding increase in costs of excavation and civil engineering work. It also limits the design speed resulting in a larger machine and a more expensive driving motor. This problem also affects hydraulic turbines and ship propellers and considerable research on the distribution of pressure on blade profiles has been undertaken to avoid localised areas of abnormally low pressure. There are, however, certain applications, such as steam condensate extraction pumps, where a certain amount of cavitation must be tolerated and attention is then given to materials which have proved to be more resistant to the consequent erosion. As the ability to run at higher speeds is becoming of greater economic importance the problem of mitigating the nature and effects of cavitation is increasingly urgent and it is probable that if the fundamental physics of the process were understood there would be more chance of finding a solution to the practical engineering problems.

Theories of cavitation generally fall into two groups : mechanical and electro-chemical. The mechanical theories argue that the very rapid bubble collapse results in very high local pressure due to impact which

subject the solid boundaries to mechanical fatigue. High temperatures due to adiabatic compression and corrosion may also enter into the problem. Several theories have been advanced based on electro-chemical effects associated with the tensile fracture of liquid molecules. There is little physical data on which to investigate the fundamental nature of the cavitation and erosion problem and considerable further study is required. The British Hydromechanics Research Association is at present investigating this problem with a magneto-striction vibration apparatus by measuring the amounts of metal lost by erosion and by passing into solution by corrosion.

(e) *Seals*.—A normal centrifugal pump requires two types of seal. One is an internal one to prevent the discharge from the impeller short circuiting back to the suction and the other is a seal to prevent leakage of fluid, or ingress of air along the driving shaft.

The internal seal is frequently merely a close running fit between the casing and replaceable wearing rings. Leakage is not externally obvious and wear is not detected until there is an appreciable reduction in delivery. For continuously running pumps, however, the financial cost of the unnecessary power loss can be large and its reduction would warrant considerable expenditure on research. The shaft seal, although not necessarily an important power waster, has received somewhat more attention as its failure causes an obvious nuisance. The conventional seal for this purpose is a gland packing which can be tightened or repacked easily but needs relatively frequent attention.

The problem of preventing leakage between moving parts of machinery is at present almost entirely one of lubrication. Up to about 1930 leather or impregnated hemp materials were used for seals and gave quite satisfactory service—the seals were porous and could provide their own lubricant and were not very critical to the state of the shaft—but increases in rubbing speeds and process temperatures created the need for a more robust and stable material.

In these fields synthetic rubber is now usually used, but in addition to the many advantages which the change has brought, since rubber can be made resistant to oils, or to corrosive or volatile fluids, one disadvantage has yet to be satisfactorily overcome. Without lubrication, rubber has a very high coefficient of friction so that when rubbed on the continuous track of the shaft, high local temperatures are produced and the rubber soon fails; on the other hand since the rubber is homogeneous it is almost impossible to keep a layer of lubricant between the surfaces without allowing the working fluid to escape. The present solution to the problem is to make the seal with a a very narrow lip, kept in contact with the shaft by a spring, but though this arrangement restricts the rise of temperature the efficiency depends upon the maintenance of continuous line contact of the lip, and the seal can hardly be said to be robust. The shaft, too, must be true and very smooth, a condition which is difficult to fulfil in a machine which has been in service for a long while.

An alternative to the lip seal, which has come into use in recent years, is the radial face seal employing two rigid mating surfaces. Here again

the difficulty is to provide lubrication without leakage, and synthetic carbon has been employed with some success though it tends to wear rapidly.

It would seem that these are by no means permanent solutions to the problem, but merely more or less satisfactory expedients.

(*f*) *Scale Effect.*—The preceding sections have indicated some of the difficulties in designing a good pump for a new set of requirements, it is therefore common practice, in all cases of economic importance, to carry out model tests to determine the best design. Of necessity, complete similarity can never be achieved and doubt arises about the exactness of scaling up the model results because hydraulic losses, mechanical friction losses and clearance leakage loss from the delivery side of the impeller back to the suction are relatively larger in the smaller model than in the final machine. The model's performance is somewhat inferior to that of the full size machine. Generally the difference is of the order of 2 or 3% in efficiency and perhaps a little more in the head-delivery characteristic.

Because the first is primarily a Reynolds' number problem, the second a mechanical one and the third another Reynolds' number effect, the whole problem cannot be satisfactorily treated as a single Reynolds number which has been suggested in the past. To provide adequate data of this problem it is necessary to know separately the variation of bearing and gland friction with size, and perhaps operating pressure ; the variation of leakage or volumetric efficiency with size and pressure, and finally, the variation of hydraulic efficiency (or losses) with Reynolds' number for different types of machine. The first is probably sufficiently well known but the latter two need considerable further study.

THE LOSSES IN PIPE SYSTEMS

(*a*) *The Roughness Problem.*—When it is required to pipe a quantity of fluid from one place to another it might appear a simple matter to install a pipe of sufficient size to carry the flow at a reasonable velocity. Friction loss plays a large part, however, in determining the most economic velocity and for this reason it is of great importance that the flow losses in commercial pipes and fittings are known to a high degree of accuracy.

In all pipe flow the velocity at the walls is taken to be zero, a basic physical assumption which has not yet been rigorously demonstrated, although it is very reasonable, and all the available evidence is in its favour. Purely viscous flow in straight pipes is a simple matter unless it is complicated by a varying viscosity due to temperature changes, large pressure changes or non-Newtonian properties and will not be mentioned further.

Turbulent flow in pipes is, however, both a difficult and important problem to which there is no universal solution. It has long been known that if the friction loss, h_f, in a length l of pipe of diameter d is given by the Darcy formula

$$h_f = f\frac{l}{d}\frac{V^2}{2g}$$

then the friction coefficient f is in general a function of Reynolds' number and the dimensionless wall roughness. When the roughness, which is usually put non-dimensionally as k/d where k is a measure of roughness size, and the Reynolds' number are small, the pipe behaves as if it were smooth and f is then a function of Reynolds' number alone. For Reynolds' numbers above 10^5 the best form for the function for "smooth" pipes is that due to Prandtl and Nikuradse:

$$\sqrt{1/f} = 2 \log \frac{R_e\sqrt{f}}{2{\cdot}51}$$

where R_e is Vd/ν, the pipe Reynolds' number, ν being the kinematic viscosity. This form for the function was derived from several weak premises and is not likely to be the exact and final relation between friction coefficient and Reynolds' number. No great deviations from this law have yet been found except at Reynolds' numbers less than 10^5 when the Blasius Law, $f = 0{\cdot}3164/R_e^{\frac{1}{4}}$ is more accurate.

We can expect that wall roughnesses begin to have an effect when the protuberances begin to shed an eddying wake. There are obvious similarities between the behaviour of a wall roughness protuberance and a single bluff obstruction in a uniform stream. In this case an eddying wake commences when the Reynolds' number of the obstruction Vl/ν is about 40, where V is the velocity past the obstruction whose size, from the point of view of obstruction, is l, so a wall roughness might be expected to affect friction when its Reynolds' number $V_k k/\nu$ is also about 40, where V_k is the velocity at its crest and k is the height of the roughness. If we assume a velocity distribution near the walls given by $v = a_1 y + a_2 y^2 + \ldots.$ where y is distance from the walls, then the so-called friction velocity

$$V_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{\mu}{\rho}\,(dv/dy)_0} \triangleq \sqrt{\nu a_1}$$

on the assumption that the flow is laminar right at the walls. Hence

$$V_k k/\nu = (k^2/\nu)\,(a_1 + a_2 k + \ \ldots) = (a_1\ k^2/\nu)\left(1 + \frac{a_2}{a_1}\,k + \ldots\right)$$

$$= \frac{V_*^{\,2}}{\nu}\frac{k^2}{\nu}\left[1 + \frac{a_2}{a_1^{3/2}}\,\nu^{\frac{1}{2}}\left(\frac{V_* k}{\nu}\right) + \ldots\right]$$

Neglecting all terms greater than the square leads to $V_* k/\nu = \sqrt{\dfrac{V_k.k}{\nu}}$ which is a much more useful form of the roughness Reynolds' number because it no longer contains the unknown velocity V_k at the crest of the roughness. The critical value of $V_* k/\nu$ should therefore be about $\sqrt{40} \triangleq 6{\cdot}5$. By definition we have $V_* = V\sqrt{f/8}$ where V is the mean velocity, so this new roughness Reynolds' number can be written as $\dfrac{Vd}{\nu}\cdot\sqrt{f/8}\cdot\dfrac{k}{d}$.

It is interesting that this approach gives approximately the same result as that suggested by Schiller(1), whose opinion was that roughness began to have an effect on friction when the protuberance was of the same order of size as the laminar sub-layer. The thickness δ of the laminar sub-

layer is, according to Schlichting(2), given approximately by $V_*\delta/\nu = 5$† which is roughly the same value as the critical value of V_*k/ν estimated for the commencement of roughness effects. Initially, when roughness is only beginning to have an effect, it might be expected that fairly small losses due to the protuberances' eddying wakes would take place in addition to the ordinary smooth pipe friction, i.e., a small amount of loss proportional to V^2 would be added on to the smooth pipe friction which is, at moderate pipe Reynolds' numbers roughly proportional to $V^{1\cdot 75}$. We should therefore expect the apparent exponent of V to increase gradually with increasing roughness effects to a limiting value of V^2 for very rough pipes when the laminar sub-layer has been completely destroyed by strong eddying wakes shed by the roughnesses. When this so-called rough stage is reached all the energy dissipation is caused by eddying and there is no part of friction that depends directly on viscosity, i.e. Reynolds' number. This state is reached when $V_*k/\nu \geqslant 50$ or so.

The weakness in such an analysis is that while it is quite plausible for a single roughness protuberance on a smooth wall it seems hardly a reliable approach in the case of a surface that is all roughness. Tests on smooth pipes artificially roughened with uniform sized sand or other bodies have been made by Freeman(5), by Fromm(6), by Nikuradse(7) and by Colebrook and White(8). Simpler tests in rectangular channels with only one wall roughened have been made by Fritsch(9), Schlichting(10), Tripp(11) and Skoglund(12). Schlichting also studied the effect of roughness spacing and of odd shaped roughnesses such as angle irons, cones, grooves, slots, etc.

The behaviour of all such uniform sized roughnesses is very similar: the friction loss eventually becomes independent of Reynolds' number and it is then possible to use the Karman-Nikuradse rough law $\sqrt{1/f} = 2 \log \dfrac{3{\cdot}7d}{k}$ for uniform sand roughness, while for roughness types which are different from sand an " equivalent sand roughness " can be assigned to the roughness by fitting the Karman-Nikuradse rough law to available results and for all other pipe sizes this law will be satisfactory.

Since both the beginning of roughness effects and the beginning of fully rough flow is determined by the value of the roughness Reynolds' number $V_*k/\nu = Re\sqrt{f/8}.k/d$ it is reasonable to assume that the whole course of the transition from smooth to fully rough flow conditions should be, for a given type of roughness, uniquely determined by this number. Nikuradse plotted the deviation of the actual values of $\sqrt{1/f}$ from that given by the rough law against the roughness Reynolds' number and found that all his test results fell on a single curve given in the first part by an inclined straight line equivalent to the smooth pipe law for all V_*k/ν up to about 3; this corresponds to hydraulically smooth conditions. Then follows a transition

† It is not possible to define exactly the extent of this laminar flow region close to a smooth wall in turbulent flow. Fage(3) gives 2.5 for this Reynold's number and Schlichting's earlier(4) estimate was 7.

curve (Curve II, Fig. 3) corresponding to the region where both Reynolds' number and roughness control friction, and finally when $V_* k/\nu$ is greater than about 60, a horizontal straight line indicating zero deviation from the rough law, i.e. the fully rough region.

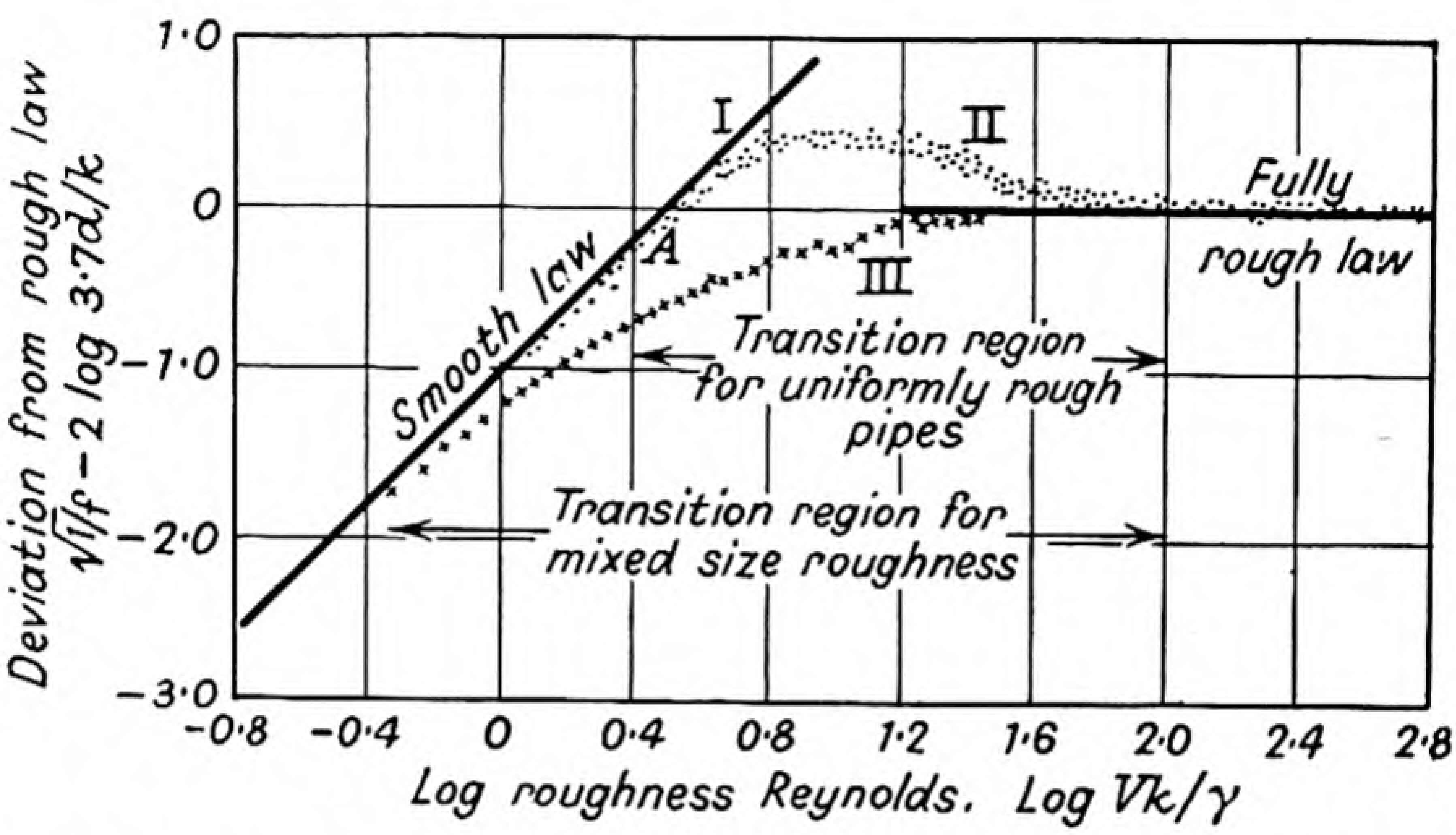

Fig. 3.—Transition from smooth law (curve I) to rough law for uniform size roughness (curve II) and mixed size roughness (curve III).

When Colebrook and White[8] carried out their tests on pipes roughened with mixed sizes of sand and tried to plot these on the transition diagram their transition curves for the mixed size roughness were quite different from Nikuradse's uniform roughness size results. All commercial rough pipes have a very wide range of roughness and since the fully rough region usually corresponds to velocities much higher than those generally used it is not possible in practical calculations to use the simple rough law and formulae based on the transition curves for commercial pipes must be used (e.g. Curve III, Fig. 3). This is the main reason why accurate friction calculation for commercial pipes is so difficult since both Reynold's number and roughness have important effects. In addition there is the problem of assigning an equivalent roughness size, k, to the mixed roughness of commercial pipes, which can only be done by extrapolating the transition curve for a commercial pipe until Reynolds' number effects vanish and then fitting the result to the rough law. Since quite different transition curves obtain for uniform size roughness and mixed roughness the actual form of the transition curve will vary with roughness size-distribution. For instance; most commercial pipes seem to deviate from the smooth law when $V_* \bar{k}/\nu$ is about 0·3 instead of Nikuradse's 3, where $\bar{k}$ is the mean effective k at fully rough conditions. From this it appears that the maximum size of roughness present in a commercial pipe is of the order of 10 times the mean effective roughness. This is hardly likely to be the same for all types of commercial pipes, so the point at which a pipe begins to deviate from the smooth law can vary. The transition curve for commercial pipes is a simple change from the smooth law $\sqrt{1/f} = 2 \log \dfrac{R_e\sqrt{f}}{2.51}$ to the

rough law $\sqrt{1/f} = 2 \log \frac{3.7d}{k}$ and is roughly asymtotic to each, Colebrook and White therefore proposed their function $\sqrt{1/f} = -2 \log\left[\frac{2{\cdot}51}{R_e\sqrt{f}} + \frac{k}{3{\cdot}7d}\right]$ which should cover the whole region of commercial rough pipe flow since it is asymtotic to the smooth curve when $k/3{\cdot}7d$ is small compared to $2{\cdot}51/Re\sqrt{f}$ and vice versa. The agreement at the ends of transition is quite good but because all commercial pipes do not follow the same curve errors arise in the transition region which sometimes amount to about 2% to 6% in friction loss.

Thus the analytical approach to pipe friction calculation is sometimes subject to serious uncertainties nor does it lead to simple calculations. Empirical methods are still very popular and although they give accurate results as good as the Colebrook-White formula in particular cases, they do so only over limited ranges and extrapolation is very dangerous.

Considerable further study of artificial mixed size roughness and more accurate data on the friction in modern commercial pipes, together with detailed mechanical measurement of their roughness, will be necessary before complete and reliable explanations of the rough pipe problem are possible.

(*b*) *Pipe Fittings*.—Many pipe systems include a large number of fittings such as elbows, bends, branches, reducers, etc., each of which introduces additional energy losses, which, in the case of complex networks such as cooling water circuits in power stations, may often be larger than the straight pipe friction. The flow in a pipe bend is characterised by the

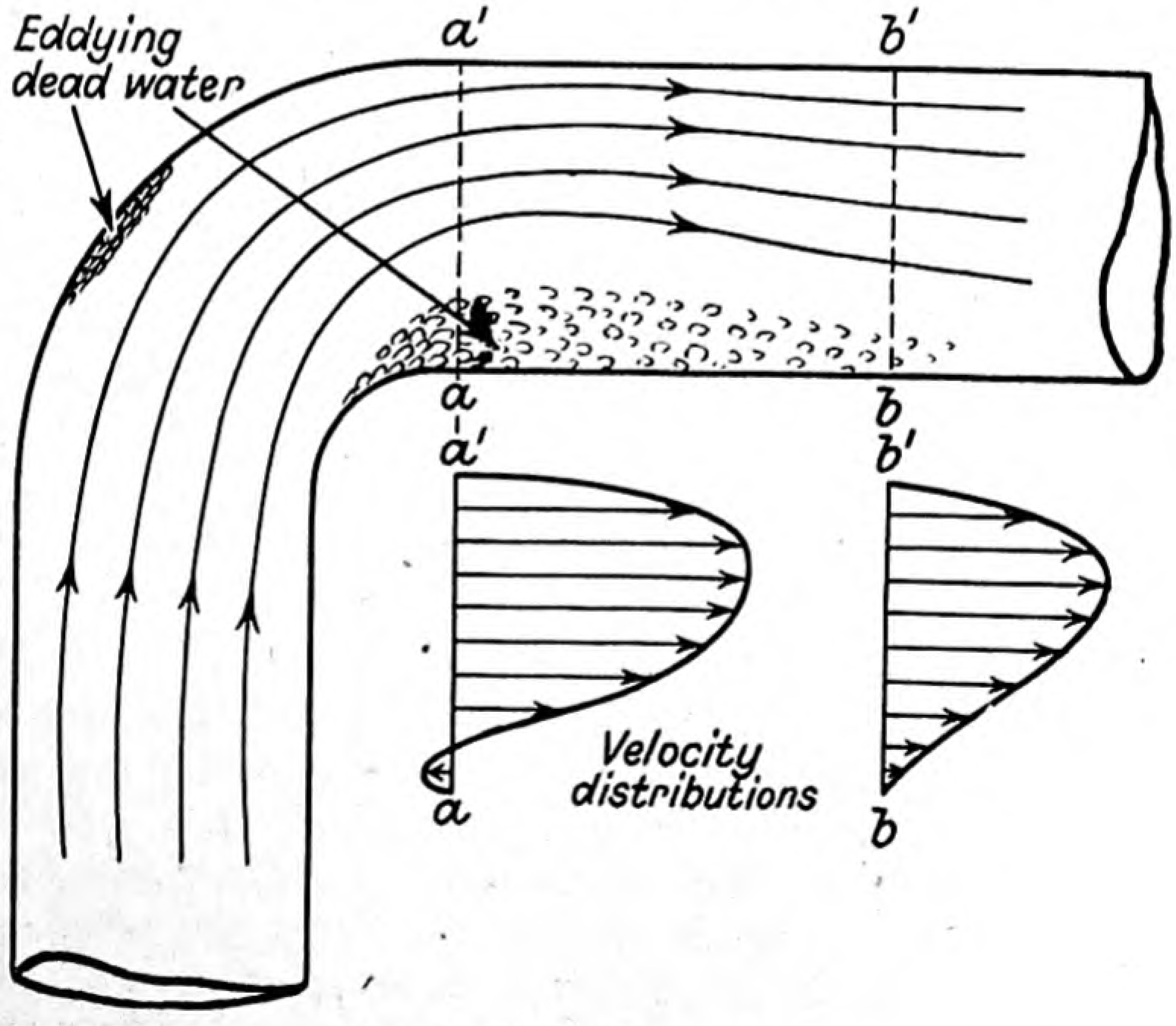

Fig. 4.—Flow in a pipe bend.

" flow separation " at the inner and outer walls shown in Fig. 4. At the inner wall there is a falling pressure in the first half of the bend which

corresponds to an increasing velocity, while in the second half of the bend, if the fluid were perfect, there would be a corresponding rising pressure back to the normal value shortly after the end of the turn. The boundary layer of relatively slow moving fluid near this inner wall is unable to proceed against the adverse pressure gradient and the flow separates, leaving a " dead water " space in about the inner quarter of the pipe section area. Thus, the discharge of the bend is made up of a jet in the outer part of the bend that may have a velocity about double the mean, and a " dead water " region in the inner part. This jet finally diffuses in the downstream straight pipe until the normal pipe velocity distribution is re-established : in doing so considerable energy losses arise due to the Carnot impulse effect. There are other components in the loss caused by a bend but this one which takes place after the bend is believed to predominate. Reductions in bend loss can be made if the exit velocity distribution is improved, which can be done by the several methods of boundary layer control that are now available. Many other pipe fittings, such as reducers, expansions, valves and some pipe joints, cause losses of a similar nature. If the re-establishment of normal velocity distribution after a bend is prevented, for instance by another bend shortly after the first, then some of the energy loss that would have been associated with the first may not take place and it is often found that the energy loss of two bends close together is less than twice that of one : in fact, adding a 30° mitre bend on to a 60° so as to make a total turn of 90° actually reduces the total loss coefficient from 0·47 to about 0·4.

The difficulty of estimating the friction loss in commercial fittings is even greater than that of pipe friction : very little is known about the effects of roughness ; the actual internal shape of commercial pipe fittings varies appreciably from one to another and, again, little is known about this beyond the obvious fact that sharp corners have an adverse effect.

It would generally be fortuitous if the head losses due to the fittings in a pipe network were estimated closer than 20%.

(*c*) *Ageing.*—When a pipe system is constructed and is found to deliver the required flow the hydraulic difficulties are not necessarily over. If the pipes were installed in good condition they will often increase their roughness with age, which will reduce their carrying capacity. In ferrous pipes this increase of roughness is often due to the growth of " tubercles " which have recently been found to be caused by and largely consist of sulphate-reducing bacteria. Considerable research is at present in progress on this subject.

(*d*) *Pressure Surges.*—The maximum stress to which a pipe line may be subjected will almost certainly arise from pressure surges caused by fairly rapid changes of flow. In the case of pipe lines feeding turbines this is likely to be caused by throttling at the delivery end ; while in the case of pumping mains it can be caused by shutting down the pump. Various remedial devices such as air vessels, surge chambers, relief or bypass valves can be used and the problems involved are of a more practical than theoretical nature. The fundamental principles underlying the propagation of pressure changes in pipe lines are known and it is possible, at least in theory, to evaluate the consequences of any changes in the steady state of flow, either

graphically or by an analytical method. The application of this technique to practical cases, however, is limited to that of a uniform pipe line without branches or other discontinuities, since the calculations involved soon become too complex to yield useful results in a reasonable time.

It will thus be necessary to make use of models, which may be either hydraulic or electric since the hydraulic problem is analogous to the propagation of voltages surges in electrical networks. The major difficulty is one of measuring equipment and technique ; in order to reduce the physical size of the model, a much shorter time scale has to be adopted so that instruments for recording rapidly changing pressures and voltages have to be developed.

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GROUP II

FUNDAMENTAL PROBLEMS

5

Problems in the Atomisation of Liquids

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ABSTRACT. Fundamental experimental and theoretical work on the dispersion of liquids as fine droplets is reviewed with special reference to swirl chamber, air blast and centrifugal types of atomisers. The importance of the boundary layer in the swirl chamber atomiser and its effect on droplet size is stressed. The mechanism of dispersion of jets in a high speed air stream is discussed and an account is given of equations covering the size-distribution of the droplets. Problems for further study are indicated. Attention is drawn to the need for improved methods for determining the size-distribution of the droplets and for expressing their statistical parameters.

INTRODUCTION

The atomisation of liquids is a subject which has received a great deal of attention, particularly in the past two decades. It has become of increasing importance in a number of diverse industrial applications where the dispersion of a liquid into droplets by a spraying device is an essential requirement. The spraying of liquid fuels for internal combustion engines and gas turbines is by itself an application of prime importance, but there are numerous other examples of the commercial and technical use of atomised liquids, such as in chemical processes involving reaction, absorption, evaporation, etc., the spraying of metals, agricultural spraying, and the production of aerosols for insecticidal, disinfectant and therapeutic purposes. In the majority of industrial uses of atomised liquids it suffices to produce droplets not smaller than 100 or 50μ diameter, but, in the last three examples quoted above, clouds of droplets not greater than 10μ diameter are often required. The main theme of this paper is to consider problems involved in the production of these fine particles, the emphasis being on the basic physics and not on questions of design of atomising devices.

The physical aspects of atomisation include a description of the fundamental methods and mechanism of the dispersion of liquids and of the quantitative theory of the observed phenomena. As a corollary to this discussion it is necessary to consider requirements for improved experimental methods for estimating the size-distribution of the droplets and for expressing their statistical parameters.

The literature of the subject is voluminous. Recently, Muraszew(1) completed a critical review of the available information on the phenomena of fuel sprays for internal combustion engines and this includes a comprehensive account of atomisation in general. Many of the papers deal only with empirical design data because often practical necessity has not permitted more than a superficial study of what is, after all, a very complex phenomenon. It is proposed in the present paper to mention only those papers which throw high lights on the mechanism and theory of atomisation.

METHODS OF ATOMISATION

There are two main types of atomisers in common use. The first is the hydraulic injection or hydrodynamical type in which the liquid is forced through a nozzle and breaks up into droplets. Here the disintegration depends more upon the physical properties of the liquid and the

conditions of ejection from the nozzle than upon reaction between the liquid and the surrounding atmosphere. Probably the most successful hydraulic device, and indeed the only one that need be considered for fine atomisation, is the swirl chamber atomiser used in agricultural spraying machinery, oil fired furnaces, internal combustion engines and gas turbines. The swirl is produced by leading the fluid tangentially into the chamber and allowing it to spray out through a central orifice of small diameter.

The second type is the air blast or aerodynamical atomiser in which compressed air or other gas is used at high velocity to break-up the jet of liquid from the nozzle and plays a predominating part in obtaining a fine degree of atomisation. This kind of break-up is met in the conventional spray gun and in venturi atomisers.

Mention should be made of a third method, which is effective although not in such general use as the other two. This depends on centrifugal action, the liquid being fed on to the centre of a rotating disc and centrifuged off the edge. The spray is characterised by homogeneity of the main droplet size in marked contrast with the heterogeneity of sprays produced by the other methods.

SWIRL CHAMBER ATOMISERS

The usual type of swirl atomiser consists essentially of a conical chamber with a small exit orifice at the vertex. Liquid is forced through tangential ports at the base of the chamber and swirls. If the pressure is sufficient and the liquid is not too viscous, a vortex with a hollow air core is established, the liquid issuing as an unbroken film which is tulip shaped at low pressures and conical shaped at medium pressures, whilst at still higher pressures the cone of liquid breaks up into discrete droplets close to the orifice. The design parameters for swirl chamber jets have been derived on the assumption that the liquid is a perfect fluid. An account of this work has been summarised by E. A. Watson[2] and he also gives graphs for the mean diameter of the droplets produced by atomisers as a function of the flow number and pressure. It should be noted that the term " mean diameter " used to characterise sprays usually refers to the " Sauter mean diameter " ($d_\circ$) which is defined as the diameter of a single droplet having the same volume to surface ratio as the total sum of the drops in the spray. Thus

$$d_\circ = \frac{\Sigma x^3 \Delta n}{\Sigma x^2 \Delta n}, \tag{1}$$

where Δn is the number of drops of diameter x.

The conditions inside the swirl chamber can only be truly described if the viscosity of the liquid is taken into account, and the problem then becomes one of considerable hydrodynamical complexity. In a recent paper G. I. Taylor[3] has shown that neglect of viscosity leads to conditions within the swirl chamber which could not possibly hold, but in another paper[4] he has worked out a simplified boundary layer theory which corresponds more closely with reality. This theory is worth considering in some detail.

Following Taylor, Fig. 1 is a diagrammatic representation of a conical flow chamber. The liquid enters tangentially the chamber of radius R_3 and emerges through an orifice of radius R_2 after passing down a converging cone whose vertex subtends an angle 2α. When the liquid is swirling, the main body may be taken as moving with velocity Ω/r in circles round the axis, where r is the distance from the axis and Ω is the circulation constant. At the surface of the cone, there will be a boundary layer of retarded liquid, which is slowed down by viscosity, and consequently has not sufficient centrifugal acceleration to hold it in a circular path against the inward pressure gradient which must exist throughout the fluid in order to balance

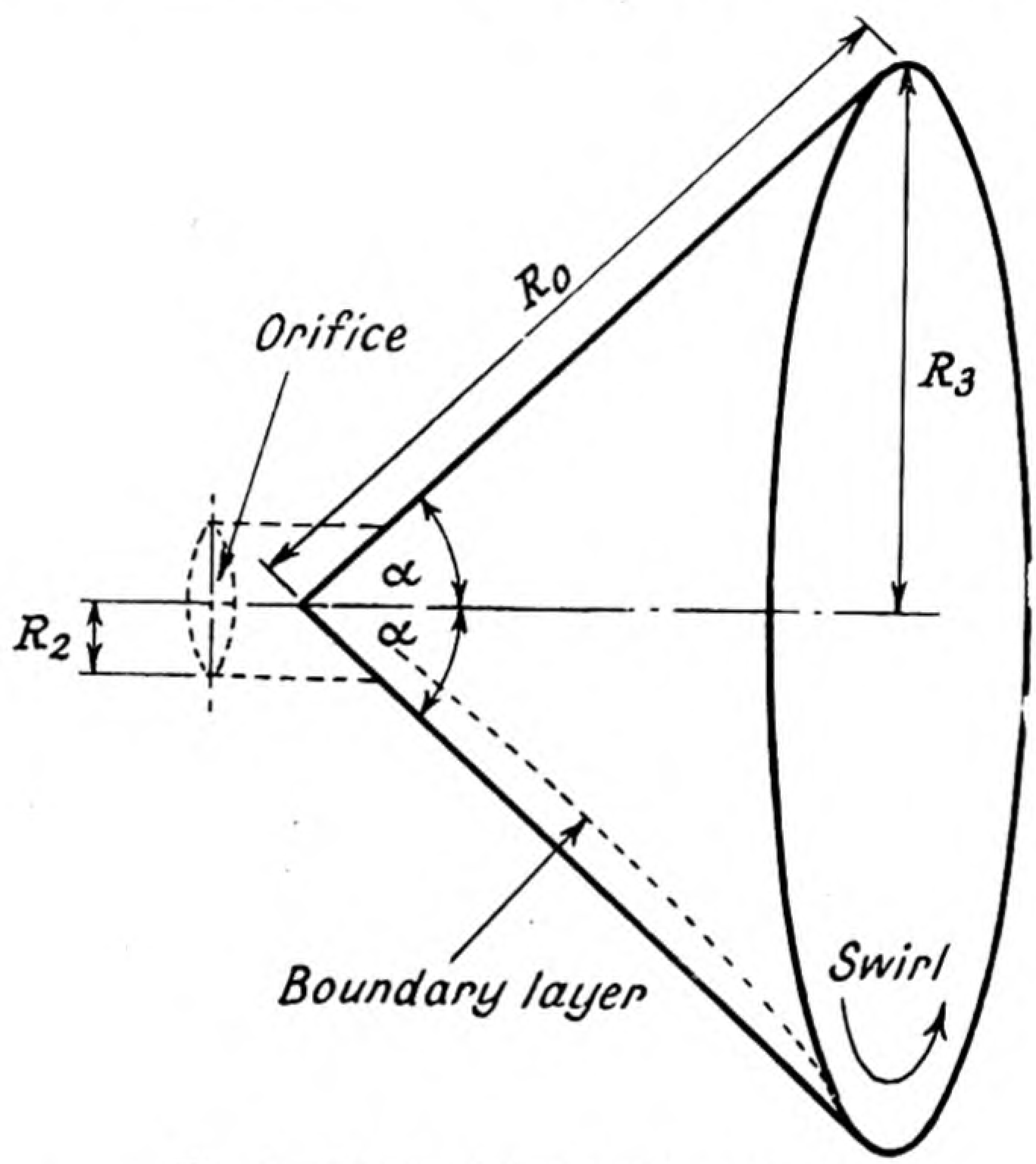

Fig. 1.—Diagram of conical flow chamber.

the swirling motion outside the boundary layer. A current directed towards the point of the cone must therefore set in over a layer close to the surface of the cone. The liquid issuing from the orifice is derived mainly from this boundary layer, but there must also be a boundary layer surrounding the core which comes from the other side of the swirl chamber. In all cases so far explored the latter layer must have overlapped the outer boundary layer and all the fluid coming from the atomiser must have come through one or other of the boundary layers.

Taylor has calculated the thickness of the boundary layer by an approximate analysis based upon Pohlhausen's use of the momentum integrals through the boundary layer. He has discussed the growth of the layer from the edge of a cone immersed in a fluid swirling in a free vortex and calculated the velocity with which the fluid flows towards the vertex, neglecting the axial component of velocity outside the layer.

He derived the expression

$$\frac{\delta}{R_2} = \frac{\delta_1}{R_1 \sin \alpha} \left[\frac{\nu \sin \alpha}{\Omega}\right]^{\frac{1}{2}}, \tag{2}$$

where δ is the thickness of the boundary layer, δ_1 is a non-dimensional parameter, R_1 may be taken as equal to R_2/R_3, and ν is the kinematic viscosity.

The velocity U of a liquid, density ρ, driven under a pressure P is given by $U^2 = 2P/\rho$. In the case where $R_2/R_3 = 0{\cdot}1$ and $\alpha = 45°$, it follows from values of δ_1 computed by Taylor that

$$\frac{\delta}{R_2} = 23{\cdot}8 \sqrt{\frac{\nu}{\Omega}} \text{ so that } \frac{\delta}{R_2} \geqslant 23{\cdot}8 \sqrt{\frac{\nu}{UR_2}},$$

since Ω is less than UR_2. If water is driven through a nozzle, say, 2 mm. diameter at 10 atmospheres pressure, $U = 45$ m./sec., and $\delta/R_2 \geqslant 0{\cdot}11$.

T. G. Hodgkinson(5) has attempted an approximate solution to the difficult problem of determining the contribution of the core flow and concluded that, under certain conditions, it might equal or even exceed that from the boundary layer at the surface of the atomiser. He has also calculated the dimensions of a low output atomiser to produce droplets of size of the order of 10μ diameter by making use of Taylor's analysis, as shown below.

Equation (2) can be transformed to give the expression

$$\delta \geqslant \left[\frac{12Q\nu}{2\pi U^2 \sin \alpha} \cdot \frac{\delta_1^2}{ER_1^2}\right]^{\frac{1}{3}}, \tag{3}$$

where Q is the quantity of the liquid issuing per second, and E is a variable quantity, dependent on R_1, values of which Taylor has computed. If the atomiser has a double orifice, then there is no back pressure from the rear surface of the chamber, therefore the core contribution is less and the thickness of the issuing layer of liquid is reduced to a minimum. Consider a double orifice atomiser, with a total output of 20 cm.^{3} of water per minute, operated at a pressure of 100 atmospheres conveniently obtainable from an air bottle. Taking the value of R_1 as $\frac{1}{4}$ and sin α as $\frac{1}{2}$ and remembering that $U^2 = 2P/\rho$, it follows from equation (3) that

$$\delta \geqslant \left[\frac{12 \times (\frac{1}{6}) \times 0{\cdot}01}{2\pi \times 200 \times 10^6 \times 0{\cdot}5} \cdot \frac{(2{\cdot}28)^2}{5{\cdot}35 \times (0{\cdot}25)^2}\right]^{\frac{1}{3}}$$

$$\geqslant 0{\cdot}0008 \text{ cm. } (8\mu).$$

If the size of the droplet produced is of the same order as the thickness of the boundary layer, then droplets of about 8μ diameter may be expected The size of the swirl chamber may now be calculated from equation (2) which can be put in the form

$$R_3 \leqslant \frac{\delta^2 U \sin \alpha\, R_1}{\nu \delta_1^2} \leqslant \frac{0{\cdot}0008^2 \times 1{\cdot}4 \times 10^4 \times 0{\cdot}5 \times 0{\cdot}25}{0{\cdot}01 \times (2{\cdot}28)^2} \leqslant 0{\cdot}0215 \text{ cm.}$$

The corresponding value of R_2, the radius of the orifice, is 0·0054 cm.

Thus it is seen that the dimensions of a swirl atomiser to produce very fine droplets at the required output become too small for practical purposes. A somewhat larger double orifice atomiser designed to produce 10μ droplets did not prove successful, owing to the difficulty in obtaining the necessary highly polished interior surfaces and the high precision required in drilling the holes, so that the predicted mean size of the drops was not obtained. In addition, clogging and erosion by minute suspended particles in the liquid would tend to prevent steady operation of the device for any lengthy period.

From equation (3), Hodgkinson has shown that Watson's data(2) conform approximately to the expression

$$d_{\circ} = 225 \left[\frac{F}{\sqrt{P}}\right]^{\frac{1}{2}},$$

where $d_{\circ}$ is in microns, F is the flow number $= Q/\sqrt{P}$, the volume flow, Q, is in gallons per hour, and P in pounds per sq. in.

Although it seems unlikely that swirl atomisers will be adopted for very fine spraying, there still remain problems the solution of which will eventually lead to the prediction of the final drop size-distribution from the conditions of atomisation. The extension of Taylor's boundary layer theory to include the core contribution should enable the thickness of the issuing cone of liquid to be predicted with greater confidence than at present, and this might well be followed by experimental test to verify the conclusions that have been reached. Further study of the mechanism of the break-up of the liquid film is also highly desirable. Flash photographs of the cone of issuing liquid in the stage when the cone is large show that it has transverse ripples, and that ligaments break away from the edge of the film and disrupt into droplets. The phenomenon is complicated by the liquid being in a state of shear due to each particle in the fluid having a tangential velocity appropriate to its path as it leaves the orifice. In many ways the break-up has much in common with that occurring in air atomisation, to which reference is made later, and it may be that theories developed in the latter connexion will cover the swirl atomiser as well.

AIR BLAST ATOMISERS

The main hope for the production of fine atomisation lies in the application of aerodynamic forces. Comparatively fine atomisation has been achieved in the conventional type " scent spray " in which the jet of liquid is met by a high speed turbulent stream of air. Many years ago, it was discovered that by the use of a baffle(6) it was possible to remove from the spray all the coarse droplets and produce a cloud consisting only of droplets less than 5μ diameter. It was thought at one time that a certain amount of shattering of the liquid occurred at the site of impingement of the droplets and air on the baffle, but recent experiments(7) have shown that this does not occur and the proportion of fine droplets released in this type of atomiser is only 1/1000th part of the volume handled by the spray nozzle. Moreover, the spraying process is in itself very inefficient. Calculations show that

the energy required to sub-divide a given volume of liquid into droplets of 5μ diameter is very small compared with that actually expended in practice. The process is wasteful and little progress has been made in the design of air blast atomisers since the first methods were developed. Despite claims to the contrary, no atomiser has yet been made which disperses all the liquid in particles less than 10μ diameter without the use of baffles.

As in the case of swirl atomisers, the literature on air blast atomisation is profuse, but again it is only necessary to mention here a few papers which have a bearing on the physical aspects of shattering by high speed air blasts. The mechanism was discussed by Castleman(8) in a noteworthy paper. He examined spark photographs by Scheubel(9) of the break-up of jets of water and alcohol in a high speed air stream and came to the conclusion that a necessary step in atomisation is the tearing of ligaments from the stream of liquid. These ligaments eventually break up into drops. In the photographs at relatively low air speeds (less than 60 m./sec.) the ligaments are clearly visible, but at comparatively high air speeds (100 m./sec.) the ligaments have largely vanished and the smaller drops appear to be torn directly from the main mass.

Applying the analysis developed by Rayleigh(10) to predict the sizes of drops produced by the collapse of a liquid cylinder under the influence of surface tension and the rate at which collapse takes place, Castleman calculated that for water a ligament with a length to diameter ratio (Z) of 4·5 will have the fastest rate of collapse. He took from data by Sauter(11) a value of the *radius* of 5μ as the mean drop size of water in a high speed air stream. Undoubtedly this was an underestimate, as shown by Lewis and colleagues(12), and 15μ is a more likely figure. However, considering a 5μ drop he found that the time for this drop to form will be of the order of 10^{-5} sec. He postulates that as the air speed increases, the sizes of the ligaments decrease, their life periods become much shorter and much smaller drop sizes result. This decrease in size soon reaches a limit at which the ligaments collapse practically as soon as formed and that is why they are not visible in the photographs. He considers that

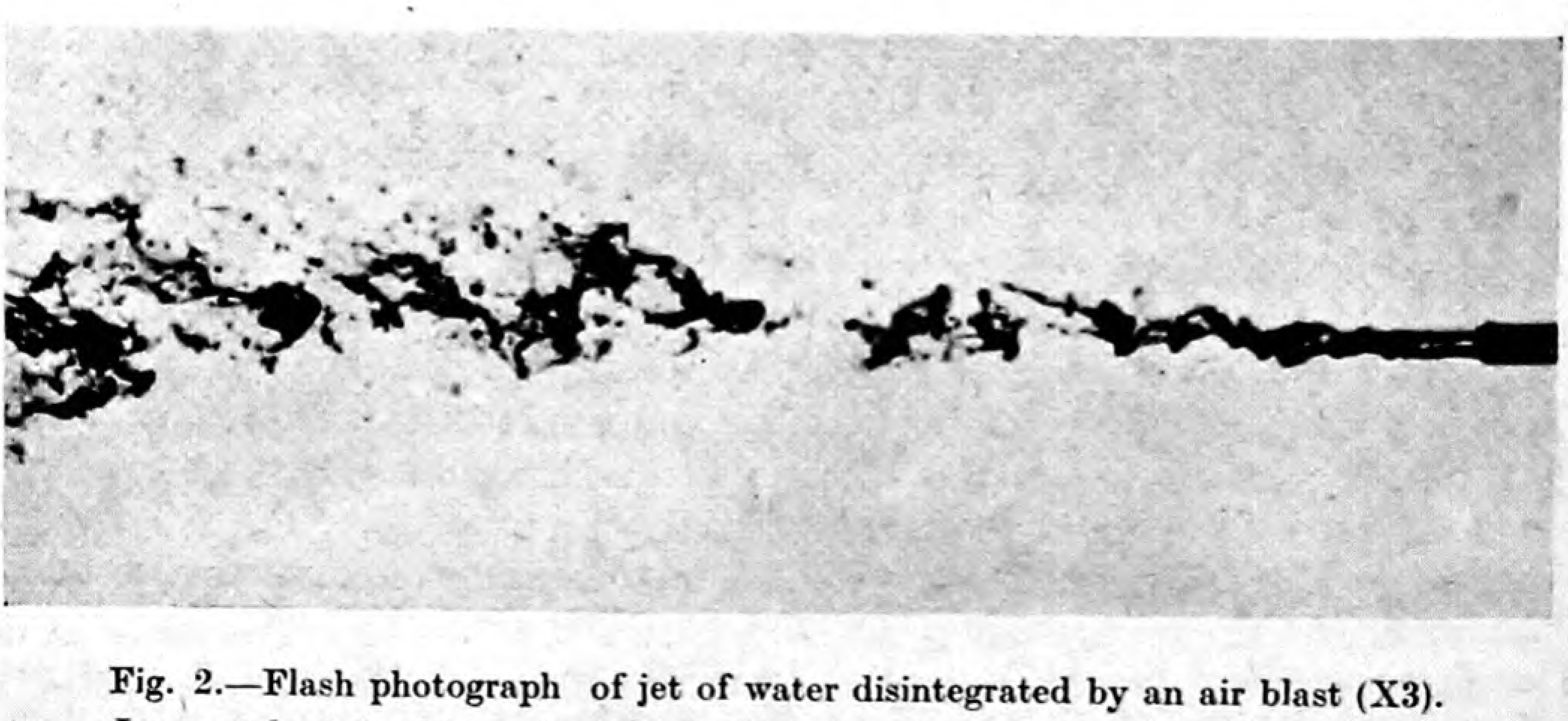

Fig. 2.—Flash photograph of jet of water disintegrated by an air blast (X3). Jet speed : 4.5m./sec. Air speed : 30m./sec. (approx.). Exposure : 2μsec.

there is a limit to which break-up can proceed and is of the opinion that the data of Sauter and the photographs of Scheubel confirm this view. Whilst this provides a picture of the mechanism, it cannot be complete because it is known that drops much smaller than 15μ or for that matter, 5μ, can be formed in sprays. Moreover, as revealed in some of the photographs in low speed air streams (see Fig. 2) films of liquid, which are stretched and burst, contribute to the drop formation besides the ligaments. This phenomenon has been demonstrated by Lane and Edwards[7] in the case of bursting single drops. Also no reasons are adduced to explain the spray heterogeneity which must arise from variation in the sizes of the ligaments and in their break-up.

It is of interest to calculate the time for a droplet to take up the speed of the air stream. The equation of motion of a spherical droplet is

$$\frac{\pi}{6} d^3 \rho \frac{dv}{dt} = \frac{\pi}{8} d^2 C_D \rho_o (u-v)^2, \tag{4}$$

where v is the droplet speed, u is the air speed, t is the time, d is the droplet diameter, ρ_o the air density, ρ the liquid density and C_D is the drag coefficient.

Integrating (3) gives

$$\frac{1}{u-v} = \frac{1}{u} + \frac{3C_D\rho_o}{4\rho d} t \cdot \tag{5}$$

Putting $u = 200$m./sec., $\rho = 1$ g./cm.^{3}, $\rho_o = 0{\cdot}001225$ g./cm.^{3}, and taking appropriate values of C_D, the time for various sizes of droplets to attain 90% of the air speed would be approximately as follows:

Diameter (μ)	Time (secs.)
5	9×10^{-5}
10	3×10^{-4}
15	5×10^{-4}

It is seen that the droplets rapidly take up the speed of the air so that, if the ligaments also collapse quickly, atomisation must be complete within a very short space of time. This is in accordance with the observations of Nukiyama and Tanasawa[13] who showed that atomisation of the jet ends completely at the air vent at a relative speed of 221 m./sec. The maximum size of droplets which could just survive, if suddenly introduced into an air stream, may be calculated by equating the aerodynamic pressure to the surface tension pressure [1, 7, 23, 24]. Under certain limiting conditions, $(u-v)^2 = kT/\rho d$, where T is the surface tension and k is a constant determined experimentally. Thus a 15μ water droplet would be on the point of breaking when $u-v$ is 200 m./sec.

The ligaments and films stripped off the jet, and the jet itself, must be accelerated by the air blast, so that the differential velocity between liquid and air is not maintained. This would account for the heterogeneity of the spray since the differential velocity of liquid derived from the inner part of the jet would be lower than that from the outer part.

A different approach has been made by G. I. Taylor in connexion with spray formed by the release of a jet of liquid into the slip stream of an aircraft. He suggests that ripples are formed and these can be resolved into harmonic disturbances of various wave lengths and amplitudes. There is a wave length of maximum instability which breaks up into corresponding drops, so that it is possible to calculate the predominating size of drop. C. H. B. Priestley[14] has applied Taylor's mathematical concepts to calculation of the size-distribution of the drops. He finds that the wave length, λ, of the most rapidly growing wave is given by

$$\lambda = 2\pi \left[\frac{4}{3}\right]^{\frac{1}{3}} \rho^{-\frac{1}{3}} \rho'^{-\frac{2}{3}} T^{\frac{1}{3}} \mu^{\frac{2}{3}} V^{-\frac{4}{3}}, \tag{6}$$

where ρ is the density of the air, ρ' the density of the liquid, T its surface tension, μ the viscosity of the liquid and V the velocity of the jet. Experiment showed that the diameter of the predominating drop could be taken as being roughly equal to the wave length. The sizes of the other drops were calculated by a step by step method taking into account the stripping off of the liquid from the jet and the gradual retardation from the speed of the aircraft to zero. Fair correspondence in the size-distribution was obtained between theory and practice. This treatment has not been applied to atomisers in general, but it has undoubted possibilities. It must be noted, however, that the assumption is made that the air is non-viscous, whereas the thickness of the air boundary layer might prevent the small wave lengths from being unstable and thus limit the smallness of size of drop which could be produced at high velocities.

Another investigation of importance was made by the Japanese workers, Nukiyama and Tanasawa[13], who determined the size-distribution of droplets by air jet atomisers under a variety of physical conditions. Their work is the most comprehensive that has yet been done on the subject. Although they did not investigate the underlying mechanism of atomisation they developed by dimensional analysis semi-empirical formulae proved to hold over a wide range.

If d_o is the Sauter mean diameter in microns, v the difference between the velocities of the air and liquid in m./sec., and σ, μ and ρ the surface tension, viscosity and density of the liquid respectively, all in c.g.s. units, Q_1 the volume flow of liquid and Q_a that of the air, then

$$d_o = 585 \, \frac{1}{v}\sqrt{\frac{\sigma}{\rho}} + 597 \left[\frac{\mu}{\sqrt{\sigma\rho}}\right]^{0.45} \times \left[1000 \, \frac{Q_1}{Q_a}\right]^{1.5}. \tag{7}$$

H. C. Lewis and colleagues[12] have made an analysis of data in the literature, which have revealed substantial agreement with the Japanese equation. They criticise the values observed by Sauter, which are roughly one third of those predicted by the equation, and there is little doubt that there are errors in the photometric deduction of drop sizes.

The formulae for the size-distribution derived in the course of these investigations will be considered later but, so far, it has not proved possible to predict the distribution merely from the design characteristics of the atomiser. There is need for further fundamental study of the break-up

of jets of liquid by aerodynamic forces. The eventual aim would be to obtain sufficient information to calculate the dimensions and operating conditions of an atomiser to give any desired size-distribution. Such a research would naturally take into consideration all relevant physical properties of the fluid and, in this connexion, the influence of dissolved gases would be worth investigation.

One difficulty in the way of obtaining a more exact knowledge of the mechanism of break-up in very high speed air streams is the rapidity of motion of the droplets. Even under low magnification, the droplets appear as streaks when photographs are taken with a flash of the order of one microsecond, and all fine detail is lost. Development of tubes to give flashes of much shorter duration would help progress.

The conditions in the air jet are also of importance and further work should be undertaken on supersonic flow particularly with converging-diverging or Laval nozzles, and on the effect of the stationary shock waves obtained with ordinary jets[15]. Another aspect is the possibility of coalescence and re-combination of droplets in the turbulent gas stream and this might well deserve attention.

PRODUCTION OF HOMOGENEOUS DROPS

The production of clouds of homogeneous drops is a still more difficult problem and has not been effected by conventional mechanical means, but recently Walton and Prewett[16] have succeeded in generating highly dispersed mists of almost uniform droplet size by feeding liquid on to the centre of a high speed air-driven top. The authors estimated that all droplets were within 8% of a mean size ; some detailed figures for the drop size-distribution have been given by May[17]. The physics of the process has also been discussed by Hinze and Milborn[25]. This method of drop formation is particularly useful for calibrating sampling instruments and further research, both experimental and theoretical, to secure even greater homogeneity in the spray would be worth while. For many purposes droplets finer than 10μ, the present limit with the air driven top, are required, necessitating much higher peripheral speeds than yet employed.

SIZE-DISTRIBUTION OF DROPLETS FROM ATOMISERS

Nukiyama & Tanasawa[13] have derived an empirical formula having the merit that it enables d_0 to be determined easily and quickly. They selected at random about 50 tests with air atomisers and found the results could be fitted by the following expression :

$$dn/dx = ax^p \exp(-bx^q), \tag{8}$$

where dn is the number of drops with diameters between $(x - \Delta x/2)$ and $(x + \Delta x/2)$ microns, and a, b, p and q are constants for a given nozzle.

It was found that, although q is constant for a given nozzle over wide ranges of operating conditions, it is quite sensitive to variations in the type and size of nozzle and must be determined experimentally in each case. On the other hand, p was found always equal to 2. The applicability of the equation has been demonstrated by Lewis and colleagues[12], who also showed how the mass median diameter, a parameter of importance in many applications, can be derived from d_0.

If the volume of air is large in comparison with that of the liquid then the formula takes the form

$$dn/dx = 0{\cdot}5\, nb^3x^2 \exp(-bx). \tag{9}$$

Another empirical formula is the Rosin-Rammler relationship[18] which has the form

$$R = 100 \exp\left[-(x/\bar{x})^n\right], \tag{10}$$

where R is the percentage cumulative volume of droplets of size greater than x, the droplet diameter, $\bar{x}$ is the "absolute size constant" and n is the distribution constant. Muraszew[1] has pointed out that there is no theoretical basis for this formula. Nevertheless it has been freely used since the spray parameters can be derived from it relatively easily.

The frequency curves for many heterogeneous particle dispersions are of the skew cocked-hat type and the size-distribution may be expressed in terms of a logarithmic probability law having the form[19] :

$$F(d) = \frac{\Sigma n}{\log \sigma_g \sqrt{2\pi}} \cdot \exp\left[-\frac{(\log d - \log d_g)^2}{2\log^2 \sigma_g}\right], \tag{11}$$

where $F(d)$ is the frequency of observations of diameter d, Σn the total number of observations, σ_g is the geometric standard deviation, and d_g the geometric mean diameter. If the cumulative percentage of the number of spray particles is plotted against diameter on logarithmic probability paper, a straight line is often obtained. A number distribution giving a straight line usually plots as a mass distribution having an increasing slope towards the larger diameters. The log probability method appears to be basically sound, but it is not widely used in connexion with sprays.

Hatch and Choate[20] have considered the various parameters characterising particle size-distribution and how they are related. There is, however, still scope for work on the application of statistical concepts to the expression of size-distribution and examination of existing data along these lines. Rapid methods for converting one parameter to another would also prove very useful.

METHODS FOR DETERMINING SIZE-DISTRIBUTION

Existing techniques for determining the size-distribution of droplets in sprays are for the most part slow and laborious. This appears to be a field in which electronic aids might find a useful application in speeding up the counting of droplets and assessment of their size-distribution. A start has been made in the United States where F. T. Gucker and C. T. O'Konski[21] have used a photo-electric cell in conjunction with a scaler counter for counting small particles. In a later paper[22] they describe an apparatus for counting at considerably higher rates ; this has a differential pulse amplitude selector designed to count pulses within a predetermined voltage range and thus allow the rapid determination of size-distribution.

Optical methods based on the light scattered or transmitted by a volume of the spray are fraught with so many inherent difficulties due

to diffraction, and anomalous scattering in the case of very fine particles, that this may not be a profitable line of research with industrial applications in mind.

CONCLUSION

Some of the problems arising from the study of the atomisation of liquids have been indicated. The importance of basic theory in a subject having such wide applications in the industrial field cannot be over-stressed. Contributions like those of Sir Geoffrey Taylor are of immense value in providing the background to the design of efficient atomising devices. There can be little doubt that, if a generalised theory were available, much of the effort expended in *ad hoc* investigations would be saved and it would be far easier to appreciate the significance of the data with which the literature abounds.

ACKNOWLEDGMENTS

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6

Boundary Layers and Skin Friction in a Compressible Fluid Flowing at High Speeds

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ABSTRACT. The introductory remarks describe broadly the special features of boundary layers in compressible flow, viz., the existence of both thermal and velocity layers and their interdependence, the sensitivity of the external flow to the layers, and their interaction with shock waves. The results of importance arising from the theory of the laminar boundary layer and of its stability to small disturbances are then discussed, followed by a summary of the present inadequate state of our knowledge of turbulent boundary layer characteristics. It is noted that progress in the latter must await the production of more experimental data. The paper concludes with a discussion of scale effects and the allied problem of boundary layer—shock wave interaction.

NOTATION

c_p	specific heat at constant pressure,
c_v	specific heat at constant volume,
c_f	local skin friction coefficient $(2\tau_w/\rho_1 U_1^2)$,
C_f	overall skin friction coefficient $= \frac{1}{x}\int^x c_f.dx$
i	T/T_o,
J	mechanical equivalent of heat,
k	coefficient of thermal conductivity,
M	Mach number,
p	pressure,
R	Reynolds number,
$\overline{R}$	gas constant,
t	time,
T	temperature,
u	velocity component parallel to surface in boundary layer,
v	velocity component normal to surface in boundary layer,
U	velocity outside the boundary layer,
x	distance measured along surface,
y	distance measured normal to surface,
α	perturbation wave length parameter $= 2\pi/\lambda$,
γ	c_p/c_v,
δ	boundary layer thickness,
λ	perturbation wave length,
μ	coefficient of viscosity,
ρ	density,
σ	Prandtl number $= \mu c_p/k$
τ	viscous stress,
θ	momentum thickness of the boundary layer,
ω	exponent in viscosity—temperature relation.

Suffix 1 refers to quantities measured just outside the boundary layer, suffix 0 to quantities in the undisturbed stream, suffix w to quantities measured at the surface, and suffix i to quantities measured in incompressible flow.

INTRODUCTION

THE ideas underlying the concept of the boundary layer and the approximations involved in boundary layer theory for incompressible flow, first developed by Prandt[1] are now well understood. These ideas can be readily extended to the flow of a compressible fluid, but in this case thermal effects play an essential part and complicate both the physical picture and the theory. Thus, adjacent to a body moving in a compressible fluid there is not only a velocity boundary layer in which the velocity changes rapidly from that of the boundary to that of the free stream outside the layer, but there is also a temperature boundary layer in which the temperature likewise changes rapidly from that of the boundary to that of the free stream. Experimentally, these layers are usually well defined ; for analytical purposes we may exclude from the velocity layer regions in which the viscous forces are less than some arbitrary quantity of a small order compared to the inertia forces, and similarly from the temperature layer we may exclude regions in which the energy changes due to heat conduction and viscous dissipation are less than some arbitrary quantity of a small order compared to the energy changes produced by convection and the inertia forces. Inside boundary layers and wakes viscous and heat conduction effects are important, as they are also in the interior of shock waves, elsewhere the flow behaves as if it were inviscid and non-conducting.

We find that for values of the Prandtl number $(\mu c_p/k)$ of the order of unity the thicknesses of the two boundary layers are of comparable magnitude. This is of considerable practical interest as the value of the Prandtl number for air is close to unity being about 0·72. Further, the boundary layer thicknesses decrease relative to the body size with increase of Reynolds number and for Reynolds' numbers of the order normally considered in aerodynamic applications the thicknesses are small compared with the linear dimensions of the body. The approximations of boundary layer theory then readily follow, since the layers may be visualised as regions in which the rates of change in a direction parallel to the surface of the velocity components and their derivatives as well as of the temperature and its derivatives are all small and may be neglected compared with the corresponding rates of change normal to the surface, except when the curvature or rate of change of curvature of the surface is large. With these approximations the equations of motion and energy of a compressible viscous fluid reduce to the boundary layer equations, the errors of which can be shown to be inverse functions of powers of the Reynolds number. As in incompressible flow, our problem then is to solve these equations with conditions determined at the outer edge of the boundary layer by a solution of the equations for the flow of an inviscid, non-conducting fluid outside the boundary layer.

The complications of compressible boundary layer theory derives from the interdependence of the equations of motion and energy, due to the

variations of density, viscosity and conductivity with temperature. The temperature field is itself a function of the heat transfer at the wall and of the Mach number outside the boundary layer. The theory has consequently not been developed as yet to the same extent as for incompressible flow. Experimentally, it is now well established that the flow in the boundary layer can occur in either the laminar or turbulent states and the same factors appear to affect the stability of the former and its transition to the latter as in incompressible flow. The theory of the laminar boundary layer on a flat plate at zero incidence is reasonably complete, but that for the laminar boundary layer on a cylinder is incomplete, except in special cases, or is approximate involving assumptions whose validity has yet to be checked. Much less developed is the theory of the turbulent boundary layer, which has at present little experimental data to guide it and which consists mainly of hopeful extrapolations of incompressible flow theory and empiricisms. We can seek some consolation for our ignorance of the characteristics of turbulent boundary layers at high speeds from the fact that at the great heights at which these speeds are likely to be attained the Reynolds' numbers in many cases may be relatively small (see Fig. 1) and the chances of extensive laminar boundary layers are correspondingly enhanced.

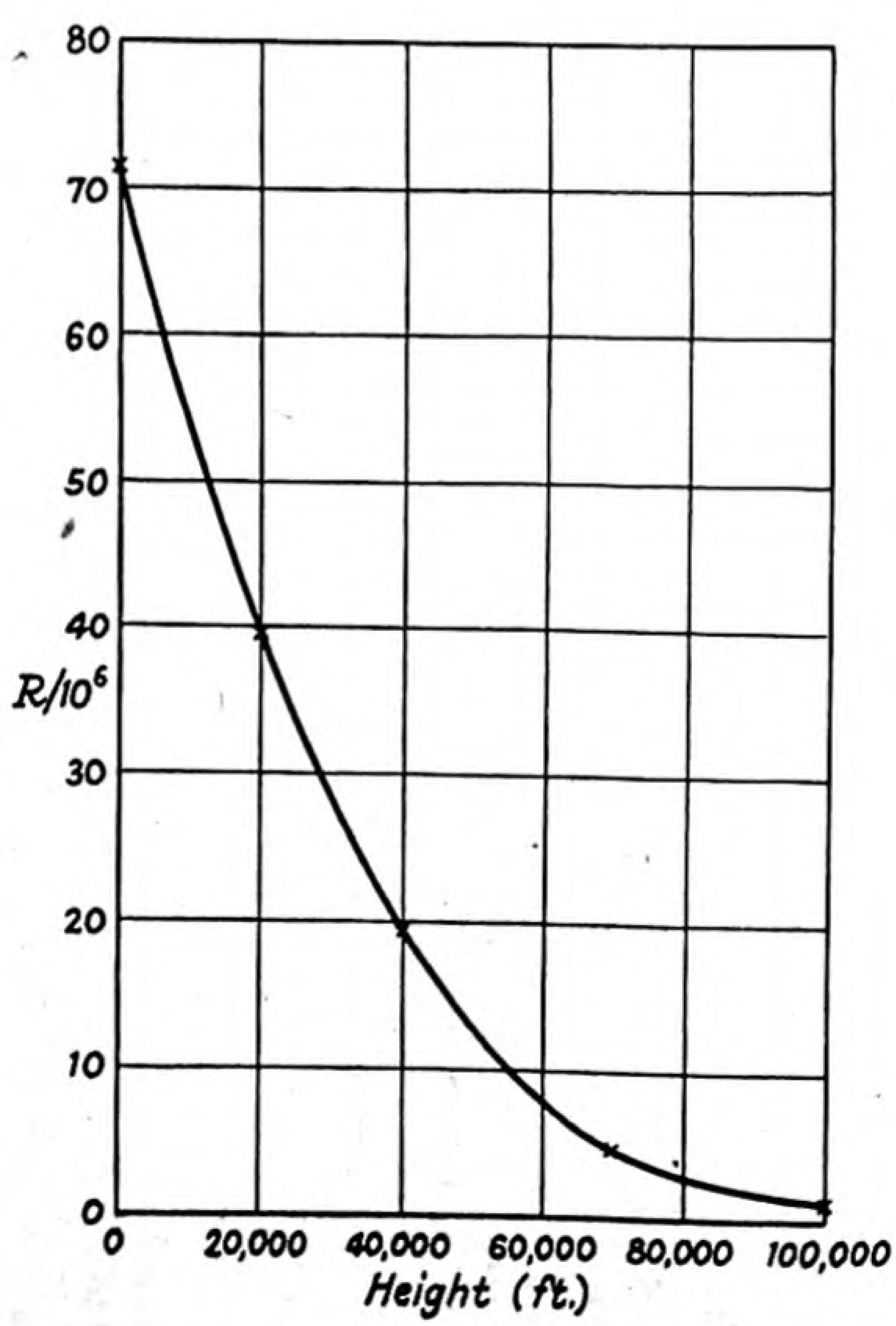

Fig. 1.—Reynolds' number for a Mach number of 1·0 for various heights. Based on a standard length of 10 ft.

There are other special features of compressible flow which must be emphasised in this context. As already remarked viscous and heat conduction effects become important not only inside boundary layers and wakes but also inside shock waves, and they may even be of some significance in extreme cases behind shock waves. There are interesting parallels and distinctions to be drawn between shock waves and boundary layers. Thus, in a shock wave the rates of ch nge of velocity and temperature normal to the wave are large compared with the rates of change parallel to the wave, and the wave is taken to be of very small thickness. These conditions are incompatible with those assumed to be characteristic of the flow in a boundary layer. Hence, in region of interaction of a shock wave and a boundary layer the assumptions and therefore the conclusions of boundary layer theory cannot be expected to be valid. The subject of shock wave-boundary layer interaction is discussed more fully in §6, otherwise the discussion of boundary layer flow that follows will be confined to those regions considered well away from the zone of interference of shock waves.

We must note that in incompressible flow the effect of the boundary layer on the main stream flow is, in general, small compared with the effect of the body on which the boundary layer develops, and it can usually be estimated with fair accuracy. It may be regarded as equivalent to the effect of a small displacement of the boundary, which effect is small in incompressible flow because it spreads without bias in all directions throughout the medium. In compressible flow, however, sensitivity of the flow to small changes of the boundary becomes increasingly marked as the flow speeds approach that of sound. When the flow is supersonic the disturbances due to such effective boundary displacements are propagated along definite wave fronts, they may be intense and may be amplified on reflexion from other boundaries.

Thus, for compressible flow we must be critical of adopting the usual approach of regarding the flow outside the boundary layer as largely independent of the boundary layer or vice versa, in certain cases the two must be considered together.

2. BOUNDARY LAYER EQUATIONS. IMPORTANT PARAMETERS

With the assumptions discussed in the previous paragraph and the assumption that the effect of gravity can be neglected, the equations of motion and energy of a two dimensional or axi-symmetric viscous compressible fluid in laminar motion reduce to the following :—

Equation of motion

$$\rho\left[\frac{\partial u}{\partial t}+u\,\frac{\partial u}{\partial x}+v\,\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu\,\frac{\partial u}{\partial y}\right) \tag{1}$$

Equation of energy

$$\rho J\left[\frac{\partial}{\partial t}(c_pT)+u\,\frac{\partial}{\partial x}(c_pT)+v\,\frac{\partial}{\partial y}(c_pT)\right]=\frac{\partial p}{\partial t}+u\,\frac{\partial p}{\partial x} + J\,\frac{\partial}{\partial y}\left(k\,\frac{\partial T}{\partial y}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2. \tag{2}$$

The equation of motion in the y direction leads to the conclusion that the pressure is constant across the boundary layer provided the curvature and rate of change of curvature of the surface are small.

In addition we have the equations of continuity and of state. The former is

$$\left.\begin{aligned} &\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0, \text{ for two dimensional flow,}\\ &\quad\text{and}\\ &\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\rho u}{r}\cdot\frac{\partial r}{\partial x}=0, \text{ for axi-symmetrical flow,}\end{aligned}\right\} \tag{3}$$

the latter for a perfect gas is

$$\frac{p}{\rho}=\bar{R}T. \tag{4}$$

The coefficient of viscosity, μ, is a function of the temperature only and is most adequately described by Sutherland's formula

$$\mu = \text{const. } T^{1\cdot 5}/(T + C) \tag{5}$$

where C is a constant, which for air is generally taken to be 114. It is found that little accuracy is lost, however, if we use a formula of the type

$$\mu = \text{const. } T^{\omega}. \tag{6}$$

For air and a temperature range between about 90°K, and 300°K, the best value of ω to take is about 8/9, for higher values of the temperature a smaller value of ω is required.(2)

It may be assumed for most applications that the specific heat, c_p, is constant and that the Prandtl number, $\sigma = \mu c_p/k$, is also constant. It follows that the coefficient of heat conductivity, k, then varies with temperature as does the coefficient of viscosity, μ.

At the wall, where $y = 0$, u and v must both be zero. The temperature at the wall, T_w, may either be given or may be determined by a condition of zero heat transfer there, in which case $\left(\frac{\partial T}{\partial y}\right)_w = 0$. At the outer edge of the boundary layer the values of the velocity and temperature are determined by those of the free stream outside, and by the condition that the boundary layer and free stream flow must merge smoothly into each other.

A dimensional analysis of these equations readily reveals that the important parameters determining the boundary layer flow are the Mach number of the flow in some standard condition (e.g. that of the undisturbed main stream), the Reynolds' number based on a representative length of the body, the Prandtl number, the ratio of the specific heats (γ), the exponent ω in the viscosity-temperature relation (equation 6) and the ratio of the temperature of the wall to the temperature of the free stream. A complete theory would have to take account of and explore fully the possible variations of all these parameters, and consequently it is believed that the full development of such a theory will require the aid of the most modern computing devices. Nevertheless considerable progress has been made by various workers in special but important cases along simple analytical lines. It would be impossible in a paper of this kind to discuss the mathematical

details of the work that has been done, all that will be attempted will be a review of the more important results.

3. LAMINAR BOUNDARY LAYER

3.1 *Preliminary remarks on the case when* $\sigma = 1$. At an early stage it was noted that if the Prandtl number, σ, is equal to unity an important simplification in the analysis follows in certain cases. Thus with no heat transfer at the wall, the energy equation is immediately soluble to give the result that the total energy (or stagnation temperature) is constant across the boundary layer, and in particular the temperature at the wall is given by

$$T_w = T_1 + U^2{}_1/2Jc_p, \tag{7}$$

where suffix 1 refers to quantities in the free stream just outside the boundary layer. As already remarked the value of σ for air is not very different from unity, being about 0·72, so that results obtained for $\sigma = 1$ are of considerable practical significance. Equation (7) leads to the result that the increase in the wall temperature over the local ambient temperature of the free stream is very closely given by $(U_1/100)^2$ in degrees Centigrade where U_1 is in m.p.h. Thus, as the speeds of aircraft increase surface

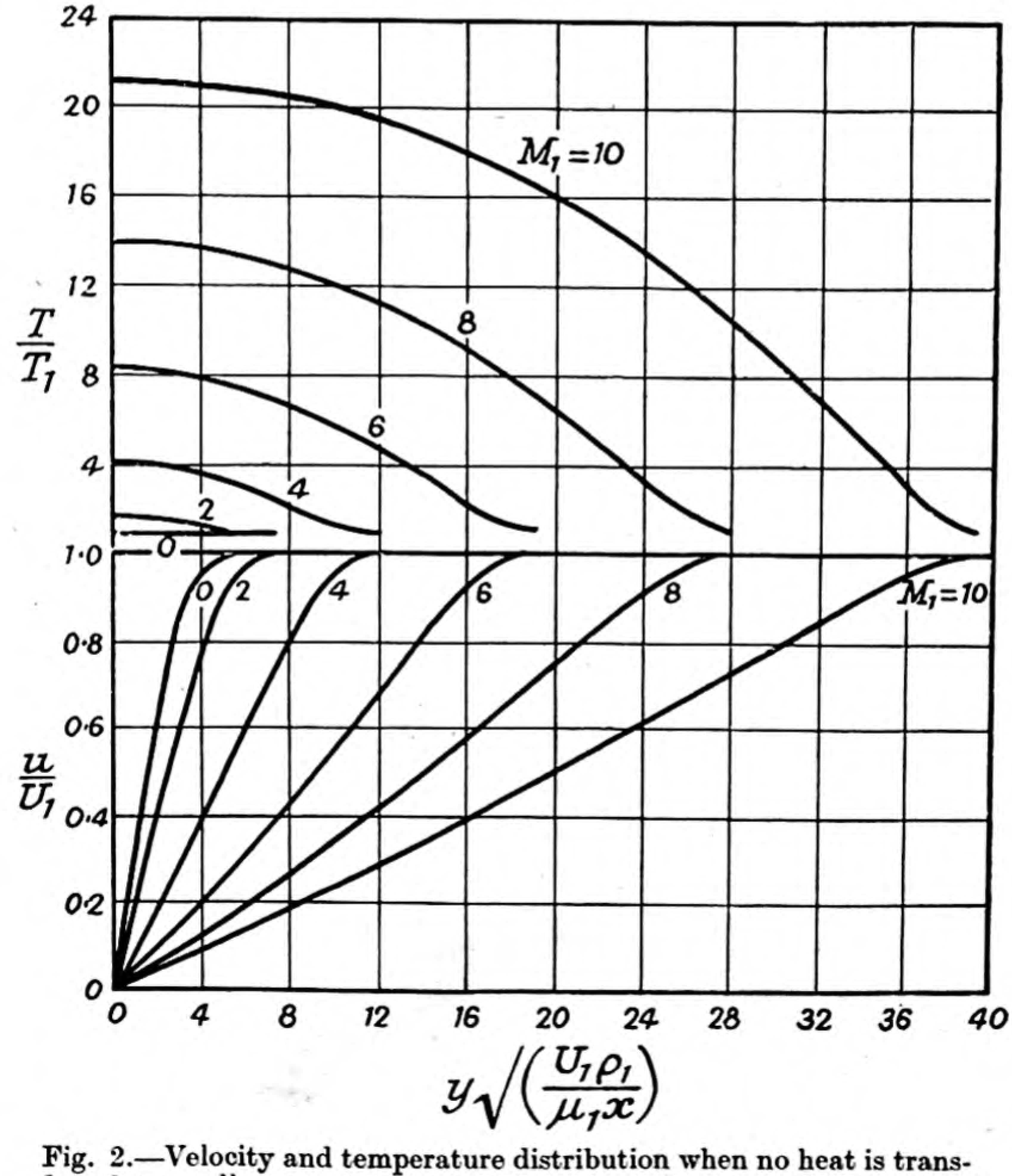

Fig. 2.—Velocity and temperature distribution when no heat is transferred to wall. $\sigma = 1{\cdot}0$, $\omega = 0{\cdot}76$. According to calculations of Karman and Tsien.

temperatures may reach uncomfortable or undesirable values, in spite of the relatively low ambient temperatures attainable at high altitudes, and designers of supersonic aircraft may ultimately be faced with the need to cool the air inside the aircraft.

For a flat plate at zero incidence and $\sigma = 1{\cdot}0$, with or without heat transfer, the equations of motion and energy become similar and with analogous boundary conditions, with the result that they lead to the solution that the total energy in the boundary layer is a linear function of the velocity. Following from this result Karman and Tsien[3] then showed that the energy equation could be reduced to an ordinary differential equation which was soluble by an iterative process. Results obtained by them for the velocity and temperature profiles in the boundary layer for various Mach numbers of the free stream and zero heat transfer are shown in Fig. 2. Corresponding results when the wall temperature is one quarter of the free stream temperature are shown in Fig. 3. It will be noted that with increase of Mach number

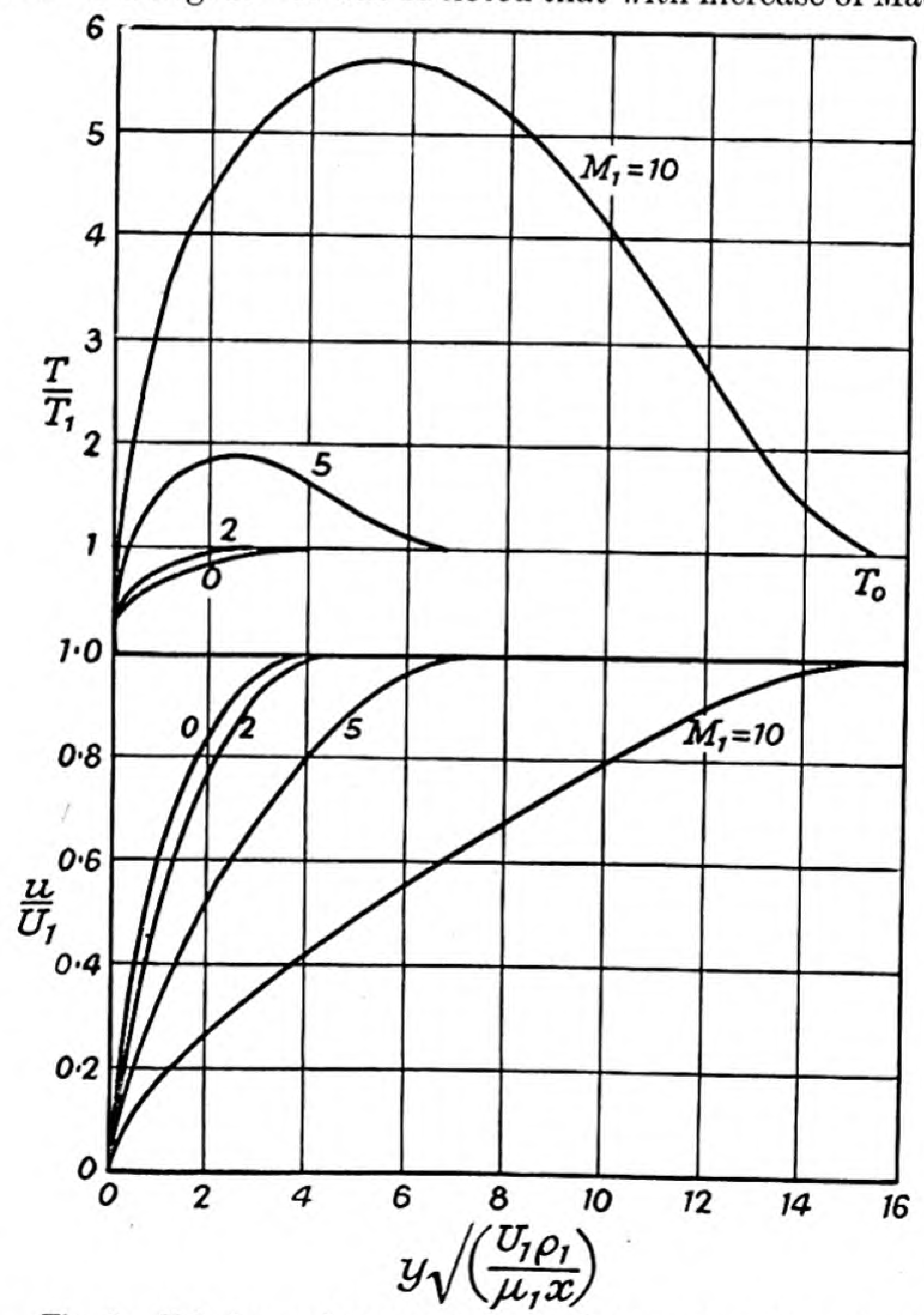

Fig. 3.—Velocity and temperature distribution when the wall temperature is one-quarter of the free stream temperature. $\sigma = 1{\cdot}0$, $\omega = 0{\cdot}76$. According to calculations of Karman and Tsien.

the boundary layer thickens. An approximate formula for the relative thickness of the boundary layer on a flat plate in compressible flow, due to Howarth[4], is

$$\delta/\delta_i \approx 1 + 0{\cdot}08\, M_1^{\,2}, \tag{8}$$

when δ is the boundary layer thickness and suffix i refers to incompressible flow. Thus at a free stream Mach number of about 3·5 the boundary layer thickness is roughly twice what it would be in incompressible flow. Another interesting point to note from Figs. 2 and 3 is that the velocity profile of the boundary layer with zero external pressure gradient tends to lose its characteristic convexity and becomes more linear with increase of Mach number.

3.2 *Further remarks on the laminar boundary layer on a flat plate at zero incidence.* The laminar boundary layer on a flat plate at zero incidence in uniform flow is clearly a problem that would attract attention from the first. The most important theoretical developments have been made by Busemann[5], Karman and Tsien[3], Hantzsche and Wendt[6], Brainerd and Emmons[7], Crocco[8].

It is probably fair to say that Crocco has taken the analytical development furthest. He showed that when $\omega = 1{\cdot}0$ the equation of motion of the boundary layer can be readily transformed to that for incompressible flow. As a result the velocity distribution expressed as a function of the distance $Y^* = \int_0^y (\mu_1/\mu)dy$, where μ_1 is the coefficient of viscosity of the free stream, is independent of Mach number. Further the local skin friction coefficient expressed as a function of distance along the plate is independent of Mach number, i.e.,

$$c_f\sqrt{R_x} = 0{\cdot}66411 \tag{9}$$

as in incompressible flow. These results remain true whatever the thermal conditions of the flow. We may note the fortuitous fact that for a large range of temperature of practical interest ω is of the order of 0·9 and is therefore very near to unity, and hence these results are of considerable practical significance.

Crocco then showed that the distribution of enthalpy ($\int Jc_p dT = Jc_p T$ for a perfect gas with constant specific heats) considered as a function of velocity in the boundary layer is practically independent of the value of ω, provided that σ is near to unity. This enabled him to develop a comparatively rapid method of solving the boundary layer equations for general values of ω and σ not far off unity. In particular, he deduced an important generaltion of equation (7) for the case of zero heat transfer, viz.:

$$T_w = T_1\left[1 + \frac{(\gamma-1)}{2} M^2{}_1\, \sigma^{\frac{1}{2}}\right]. \tag{10}$$

The numerical work of others (see e.g. Brainerd and Emmons) has confirmed this formula.

By means of an extension of Crocco's analysis and guided by the numerical results obtained by other workers for particular cases, the author[9] obtained the following formula for the local skin friction

$$c_f\sqrt{R_x} = 0{\cdot}664\left[0{\cdot}45 + 0{\cdot}55\,i\,(0) + 0{\cdot}09\,(\gamma-1)\,M_1^{\,2}\,\sigma^{\frac{1}{3}}\right]^{\frac{-(1-\omega)}{2}} \tag{11}$$

where $i\,(0) = T_w/T_1$. With zero heat transfer at the wall this formula becomes

$$c_f\sqrt{R_x} = 0{\cdot}664\left[1 + 0{\cdot}356(\gamma-1)M_1^{\,2}\,\sigma^{\frac{1}{3}}\right]^{\frac{-(1-\omega)}{2}} \tag{12}$$

These formulae are estimated to be reliable to within 1% error for values of the Mach number up to 10 and for values of ω and σ likely to be of practical interest when air is the working fluid. A comparison of the results given by equation (12) and the values calculated by various workers is shown in Fig. 4.

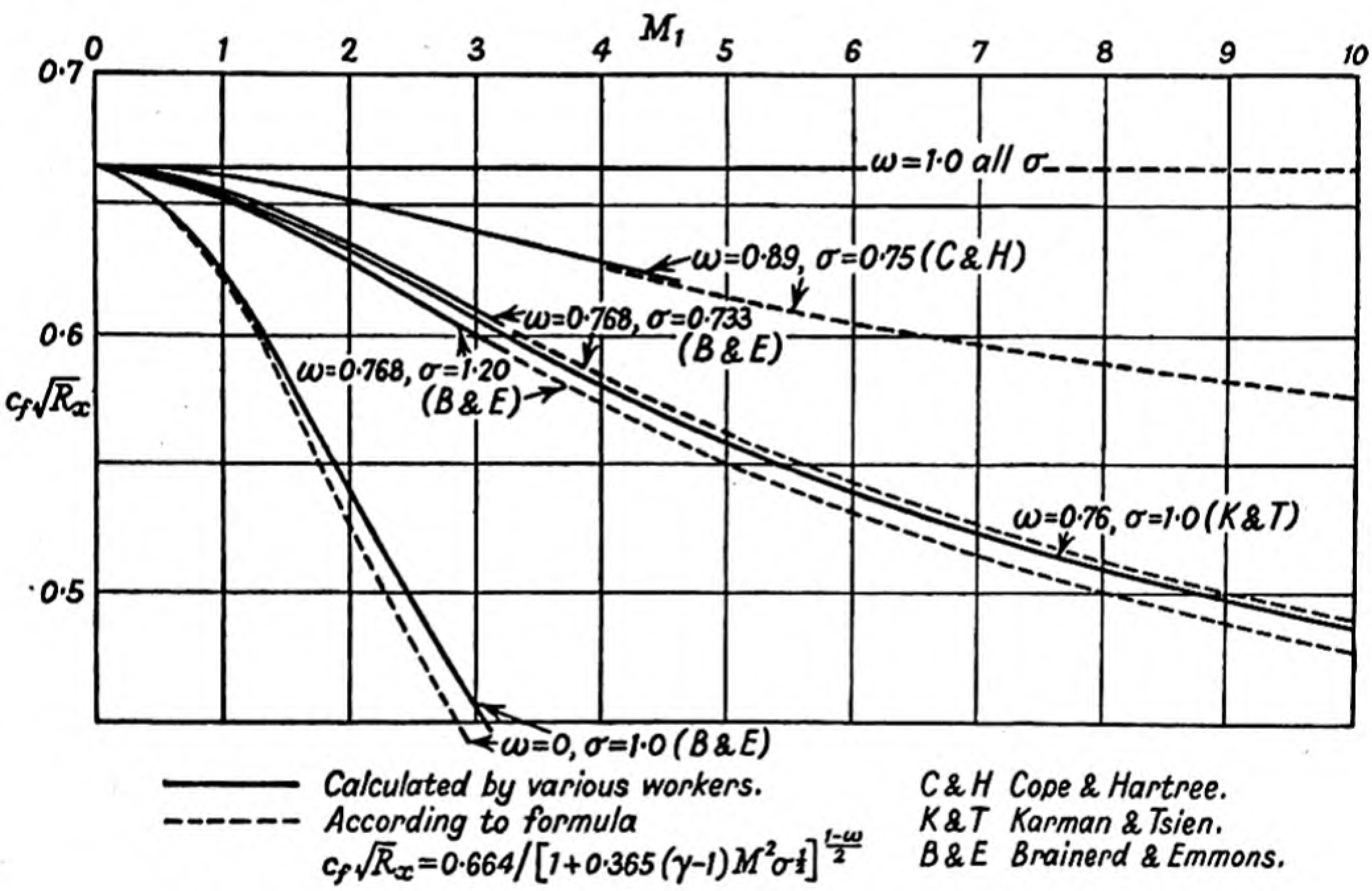

Fig. 4.—Skin friction on a flat plate at zero incidence. No heat transfer.

One other important result of Crocco's analysis must be mentioned here. Denoting the local rate of heat transfer from the wall to the fluid per unit area by q, and the local frictional force per unit area by τ_w, Crocco finds

$$\frac{q}{\tau_w} = \frac{c_p}{\sigma^{2/3}}\left[\frac{T_w - T_{th}}{U_1}\right] \tag{13}$$

where T_{th} stands for the temperature measured by a thermometer in the stream, and hence from (10)

$$T_{th} = T_1\left[1 + \frac{(\gamma-1)}{2}\,M_1^{\,2}\,\sigma^{\frac{1}{3}}\right].$$

Comparing (13) with the corresponding result obtained by Pohlhausen[10] for the case of incompressible flow we can conclude that the form of the law relating heat transfer and skin friction is the same at high speeds as for low if in the latter we substitute for the temperature of the main stream the thermometer temperature.

3.3. *Laminar boundary layer on a cylinder in steady flow.* As already remarked the general problem of the laminar boundary layer on a cylinder has not been solved although the possibilities of an attack on numerical lines using the ENIAC have been demonstrated by Cope and Hartree[2]. Of considerable analytic and practical interest, however, is the case when both σ and ω assumed equal to unity and there is zero heat transfer at the wall. In this case Stewartson[11] and Illingworth[12] have shown independently that it is possible to transform the boundary layer equations for a given main stream velocity distribution to the equations for an incompressible fluid but with a different main stream velocity distribution. It follows that to any incompressible boundary layer problem and its solution there is an associated compressible boundary layer problem and its solution, and the existing literature of incompressible flow boundary layer theory can be readily exploited to yield answers to interesting problems of boundary layers in compressible flow. For example, considering the problem of a constant retarding velocity gradient ($u_1 = u_0 - \beta_1 x$) Stewartson has deduced a rapid forward movement of separation with increase of M_0[11]. His results for the separation distance, x_s, based on Howarth's results for the corresponding incompressible flow problem[13], are as follows :—

M_o	0	2	4	6	10
$\dfrac{\beta_1 x_s}{u_o}$	0·120	0·096	0·062	0·044	0·024

The problem is to some extent an academic one since in practice in a region of compression a shock wave would develop and might have a profound effect on the boundary layer. However, the increased tendency to separation with increase of Mach number is noteworthy.

For the more general problem where ω and σ are not necessarily unity the methods that have been developed are approximate and in greater or less measure owe something to the Karman-Pohlhausen approach. For example, the author[14] has used a relation between skin friction and boundary layer momentum thickness, derived on analogous lines to the Karman-Pohlhausen method, to solve the momentum integral equation of the boundary layer (see Appendix I). The solution was arranged to satisfy the condition that the resulting skin friction on a flat plate with zero velocity gradient and no heat transfer should agree with equation (12). The final solution gives the momentum thickness (or skin friction) at any point on a cylinder expressed as an integral which involves the external velocity distribution up to that point and which can be readily evaluated graphically or numerically. When applied to the problem considered by Stewartson referred to in the previous paragraph, the method gave reasonable agreement with Stewartson's results. This severe test suggests that the

method should prove satisfactory for most practical cases where the skin friction or overall characteristics of the boundary layer (e.g. momentum and displacement thicknesses) are required.

4. STABILITY OF THE LAMINAR BOUNDARY LAYER

It is now accepted that the theory of the stability of the laminar boundary layer to small disturbances provides a useful guide to some of the important factors that control transition to turbulent flow. The physical causes of turbulence in the boundary layer are far from understood at present, but they appear to be associated with the presence of finite disturbances which may be introduced from outside the boundary layer (e.g. external turbulence) or be caused by surface imperfections (e.g. roughness, waviness, etc.). In the absence of such sources of disturbance it is inferred that transition to turbulence may be provided by sufficiently amplified disturbances initially of the type shown by the linearised theory to cause instability of the laminar boundary layer.

The theory of the stability of the laminar boundary layer in incompressible flow has been developed by a number of workers over a considerable period, but only recently has the theory been extended, by Lees and Lin[15,16], to the problem of the two dimensional boundary layer in compressible flow. It is impossible here to deal in any detail with the theory and all that will be noted are its essential features and the more important results. Each mode of the perturbation is assumed to be of the form $e^{i\alpha(x-ct)}$ times a function of y, the distance from the surface, where the wave length of the perturbation$=2\pi/\alpha$, and c is the phase velocity and is in general complex. Thus $c=c_r+ic_i$, where c_r and c_i are real, $c_i>0$ implies instability, $c_i<0$ implies stability and $c_i=0$ implies neutral stability. The equations of motion, continuity, energy and state can be made to yield six independent linear homogeneous equations in functions of the disturbance velocity components, pressure, density and temperature, and in general there are six homogeneous boundary conditions to be satisfied. In consequence, there is a characteristic relation between the parameters M_1, R_θ (the Reynolds' number based on the momentum thickness θ), α and c. This relation can be written $c_r=c_r\,(\alpha, R_\theta, M_1)$ and $c_i=c_i\,(\alpha, R_\theta, M_1)$, and hence $c_i=0$ yields a relation between α, R_θ and M_1 for which there is neutral stability. This relation can be plotted as a series of so-called neutral curves of α against R_θ for different values of M_1. Specimen curves are shown in Fig. 5 where, however, the wavelength parameter is not α, but the non-dimensional quantity $\alpha\,\theta$, written as α_θ. The region interior to a curve at a given Mach number corresponds to instability in the boundary layer, whilst the exterior region corresponds to stability.

The curves of Fig. 5 (*a*) refer to an insulated flat plate with mainstream Mach numbers of 0, 0·5, 0·7, 0·9, 1.1 and 1·3, whilst the curves of Fig. 5 (b) refer to a flat plate at a main stream Mach number of 0·7 and surface temperature ratios (T_w/T_1) of 0·7, 0·8, 0·9, and 1·25). For the latter case no heat transfer would occur for $T_w/T_1 \approx 1{\cdot}1$, so that for $T_w/T_1<1{\cdot}1$ heat is

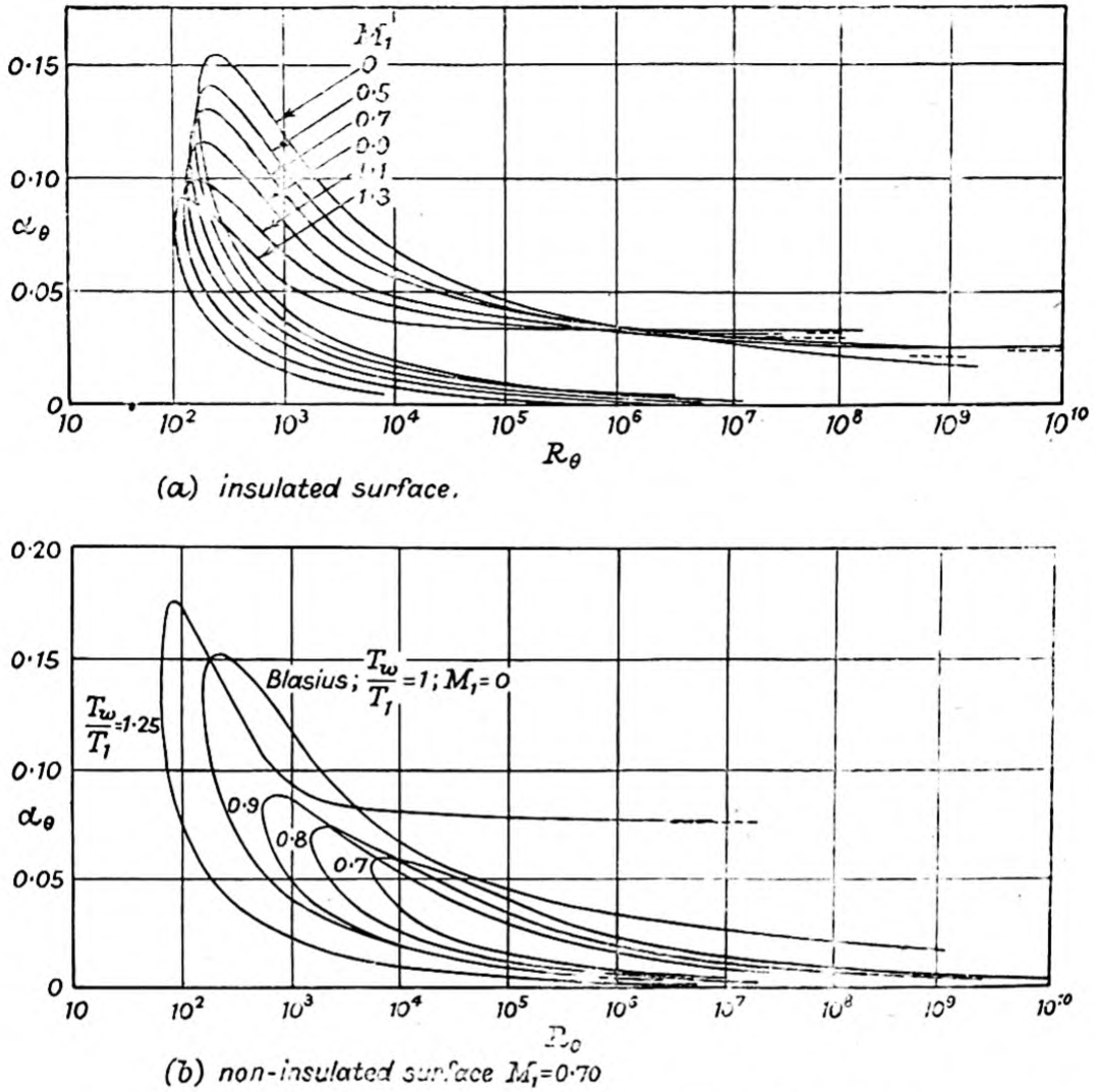

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Fig. 5.—Wave number α_θ against Reynolds' number R_θ for neutral stability of laminar boundary layer.

transferred from the fluid to the surface, but for $T_w/T_1 > 1·1$ the reverse occurs. The curves illustrate several important points. First, it will be noted that in each case there is a critical value of the Reynolds' number, R_θ, below which there is complete stability to all disturbances. Secondly, it will be noted that this critical value falls with increase of Mach number or with increase of heat transferred to the fluid. On the other hand a withdrawal of heat from the fluid is found to be stabilising and becomes increasingly so with increase of Mach number. Lees estimates that for $M_1 > 3$ (approx.) at 50,000 ft., or for $M_1 > 2$ (approx.) at 100,000 ft., the heat radiated from a surface under conditions of thermal equilibrium is sufficient to ensure stability to all small disturbances in the absence of adverse pressure gradients.

The maximum rate of amplification of the self-excited disturbance varies as the inverse square root of the critical value of R_θ. Hence any circum-

stance tending to increase the stability by increasing the critical value of R_θ will also decrease the initial rate of amplification and therefore will presumably help to delay transition for a given level of free stream turbulence. Measurements by Liepmann and Fila[17] of the movement of the transition point on a flat plate at low speeds confirmed that an appreciable forward movement of transition occurred when heat was applied to the surface. This movement was particularly marked when the level of the free stream turbulence was low. Frick and McCullough[18] made similar measurements on a low drag aerofoil and also found a forward movement of transition with heating of the surface, except when they heated the nose section. The stabilising effect of the favourable pressure gradients there was presumably too great to be overcome by the effect of the heat transfer to the fluid.

5. THE TURBULENT BOUNDARY LAYER

When considering the turbulent boundary layer in compressible flow we must accept the existence of fluctuations in density, temperature, viscosity and conductivity as well as the fluctuations in velocity met with in incompressible flow. These additional terms coupled with an insufficient knowledge of the general nature of turbulence leave it impossible to proceed very far with any theoretical treatment beyond the formal statement of the boundary layer equations. An accepted approach on the semi-empirical lines that have given us working formulae for the characteristics of the turbulent boundary layer in incompressible flow must necessarily await the production of a considerably greater body of experimental data for compressible flow than we have at the moment.

However, an examination of the turbulent boundary layer equations reveals certain facts of some importance. Firstly, it can be deduced that when $\sigma = 1$, the relation between mean* total energy and mean velocity parallel to the surface in the boundary layer is precisely the same as in the laminar layer, viz. with zero heat transfer at the wall the mean total energy is constant, whilst on a flat plate at zero incidence with or without zero heat transfer, the mean total energy is a linear function of the mean velocity. Even when σ is not unity we may expect that the powerful mechanism of energy interchange of the eddying motion in the turbulent boundary layer will tend to ensure a more uniform distribution of total energy than in the laminar boundary layer. For the case of zero heat transfer at the wall we have noted that for a laminar boundary layer (equation 10)

$$T_w = T_1 + \frac{U_1^2}{2Jc_p} \cdot \sigma^{\frac{1}{2}}. \qquad (10)'$$

For the turbulent layer at low speeds Squire[19] has deduced the formula

$$T_w = T_1 + \frac{U_1^2}{2Jc_p} \cdot \sigma^{\frac{1}{3}}, \qquad (14)$$

showing in the change of the exponent of σ from $\frac{1}{2}$ to $\frac{1}{3}$ the increased tendency to uniformity of total energy across the turbulent boundary layer. Since equation (10) applies at all speeds it is unlikely that Squire's formula, equation (14), will be in serious error at high speeds.

*Here the term " mean " is used in the sense of a time mean taken at a point, and the value of a quantity at any instant is the sum of its mean and the turbulent fluctuation.

There are at present a few available experimental data on the mean velocity distributions in turbulent boundary layers at high speeds. They are all measurements made in boundary layers on wind tunnel walls and as such may be regarded as a guide to the ideal case of a boundary layer on an insulated flat plate at zero incidence in a uniform stream, if we can disregard the prior influence of the nozzle in the case of supersonic flow, the random but relatively large pressure gradients that are liable to be present in high speed tunnels and the effects of possible shock waves both upstream and downstream of the measuring positions. The scatter of the available data is large and indicates that these factors are not always negligible, it also shows the formidable difficulties associated with attempting accurate measurements in boundary layers at high speeds. One can only conclude from the data that any variation with main stream Mach number of the velocity profile of the turbulent boundary layer, at least for Mach numbers up to about 2·5, is readily masked by experimental scatter. For the present, therefore, there is no clear cut evidence to suggest that we should represent the velocity profile by other than one of the formulae that are normally accepted for the boundary layer in incompressible flow, viz. a power law or the " log " law. Even so we must note that the choice of the quantities of normally used to present these laws in their most general non-dimensional forms, in which viscosity and density are parameters, is to a considerable degree arbitrary for compressible flows in the absence of more experimental data. Since these forms are required if any deductions as to skin friction are to be made, it follows that widely differing results for the skin friction may follow on equally plausible assumptions. Thus, one can generalise the relation deduced by Squire and Young[22] for incompressible flow, viz. $\rho_1 U_1 \theta/\mu_1 = Ae^{B\zeta}$, where $\zeta^2 = \rho_1 U_1^2/\tau_w$ and A and B are constants, to several different forms as, for example, $\rho_1 U_1 \theta/\mu_w = Ae^{B\zeta}$, with $\zeta^2 = \rho_1 U_1^2/\tau_w$, or $\zeta^2 = \rho_w U_1^2/\tau_w$. If we assume $\zeta^2 = \rho_1 U_1^2/\tau_w$ and assume also that the values of A and B are unchanged by compressibility then the relation $\rho_1 U_1 \theta/\mu_w = Ae^{B\zeta}$ is the same as that for incompressible flow with the Reynolds' number reduced by the factor μ_1/μ_w. Hence a slow increase of skin friction with increase of Mach number would follow. However, such experimental data as are available indicate a fall of skin friction with Mach number. If, however, we assume $\zeta^2 = \rho_w U_1^2/\tau_w, \rho_1 U_1 \theta/\mu_w = Ae^{B\zeta}$, and take A and B to be the same as for incompressible flow, then one obtains the relation between overall skin friction coefficient, Mach number and Reynolds number illustrated in Fig. 6. Here a reduction of skin friction with Mach number results of an order that is consistent with the existing experimental data. Further experimental results are required to check conclusively whether the curves of Fig. 6 describe the relation between skin friction and Mach number with reasonable accuracy. Almost exactly the same results are obtained by a generalisation of the power law for the velocity distribution in the form $u/U_\tau = 8{\cdot}7\eta^{1/7}$, where $U_\tau^2 = \tau_w/\rho_w$, and $\eta = \rho_w U_\tau y/\mu_w$*.

*Recently available analyses of experimental data[29],[30] suggest that the fall of skin friction with Mach number is about half that indicated in Fig. 6.

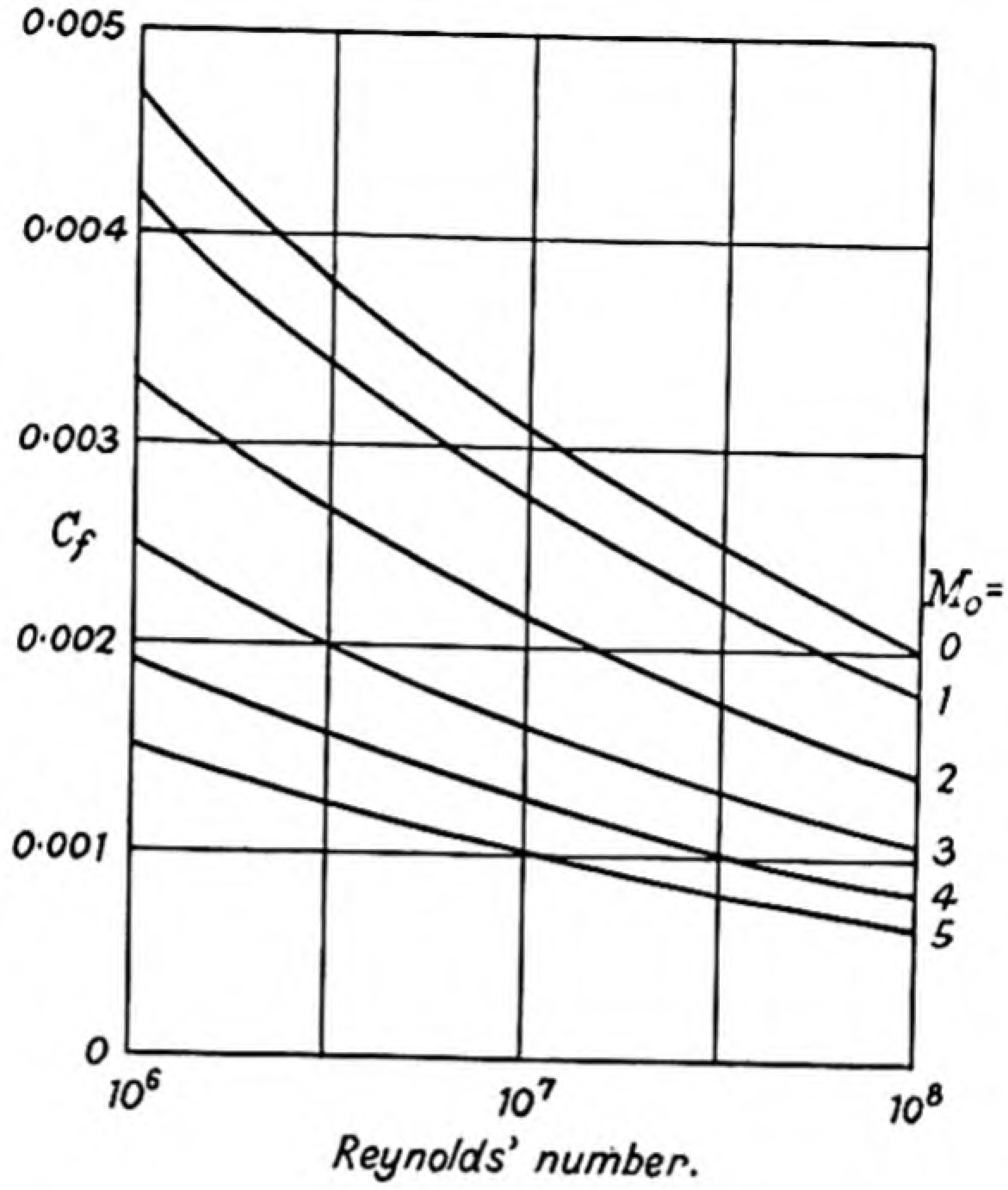


Fig. 6.—Variation of overall skin friction coefficient of flat plate at zero incidence with Reynolds' number and Mach number. Boundary layer completely turbulent. Calculated according to relation $\rho_1 U_1 \theta/\mu_w = A\exp.(B.\zeta)$, where $\zeta^2 = \rho_w U_1{}^2/\tau_w$, $A = 0{\cdot}2454$, $B = 0{\cdot}3914$.

Once the skin friction on a flat plate at zero incidence can be conclusively established it will not be difficult to deal with the more general case of the turbulent boundary in the presence of a non-uniform external pressure distribution. This can be done on lines analogous to those successfully used for incompressible flow(21), since it should be possible to extract from the flat plate results a local relation between skin friction intensity and momentum thickness which when combined with the momentum equation of the boundary layer (see Appendix 1) should yield an equation readily soluble for the skin friction and momentum thickness distributions (for an early attempt on these lines see Young and Winterbottom(22)). Finally it should be possible to calculate the profile drag of a wing for any Reynolds' number and transition position by following the development of the boundary layer in its laminar and turbulent stages to the trailing edge and thence solving the momentum equation of the wake to yield the value of the wake momentum thickness far downstream and hence the profile drag. The effect of the boundary layer in modifying the effective shape of the wing and hence the wave drag can also be estimated, and this can be regarded legitimately as a contribution to the profile drag. The results of some preliminary calculations of the profile drag of a 6% thick biconvex section are shown in Fig. 7. These calculations are based on the results of a hypothesis that gives practically constant skin friction with varying Mach numbers for the flat plate at zero incidence. It will be seen that even

at a Mach number of 1·5 the profile drag is by no means negligible in comparison with the wave drag and with increase of Mach number the profile drag becomes an increasingly important proportion of the total drag.

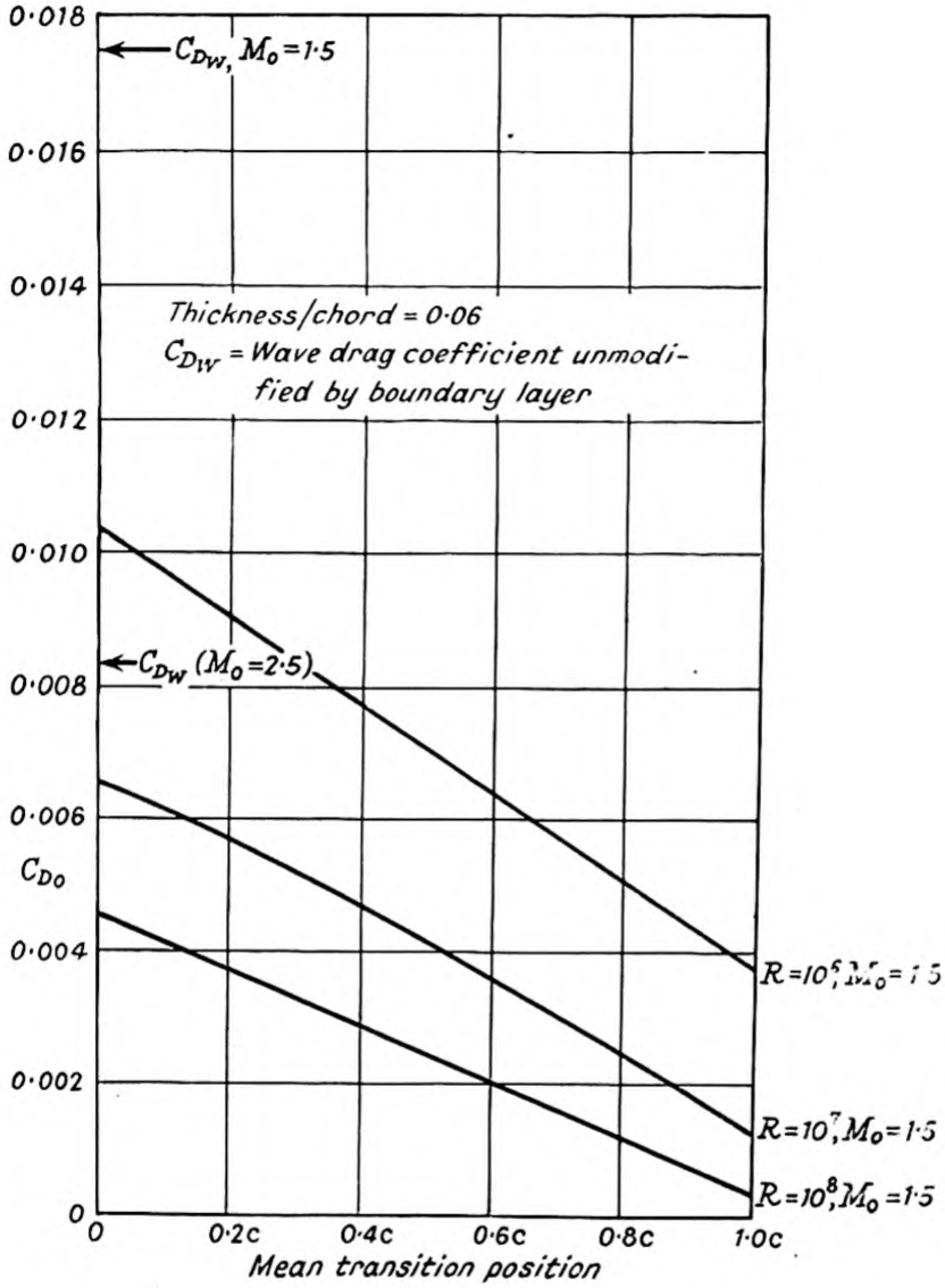


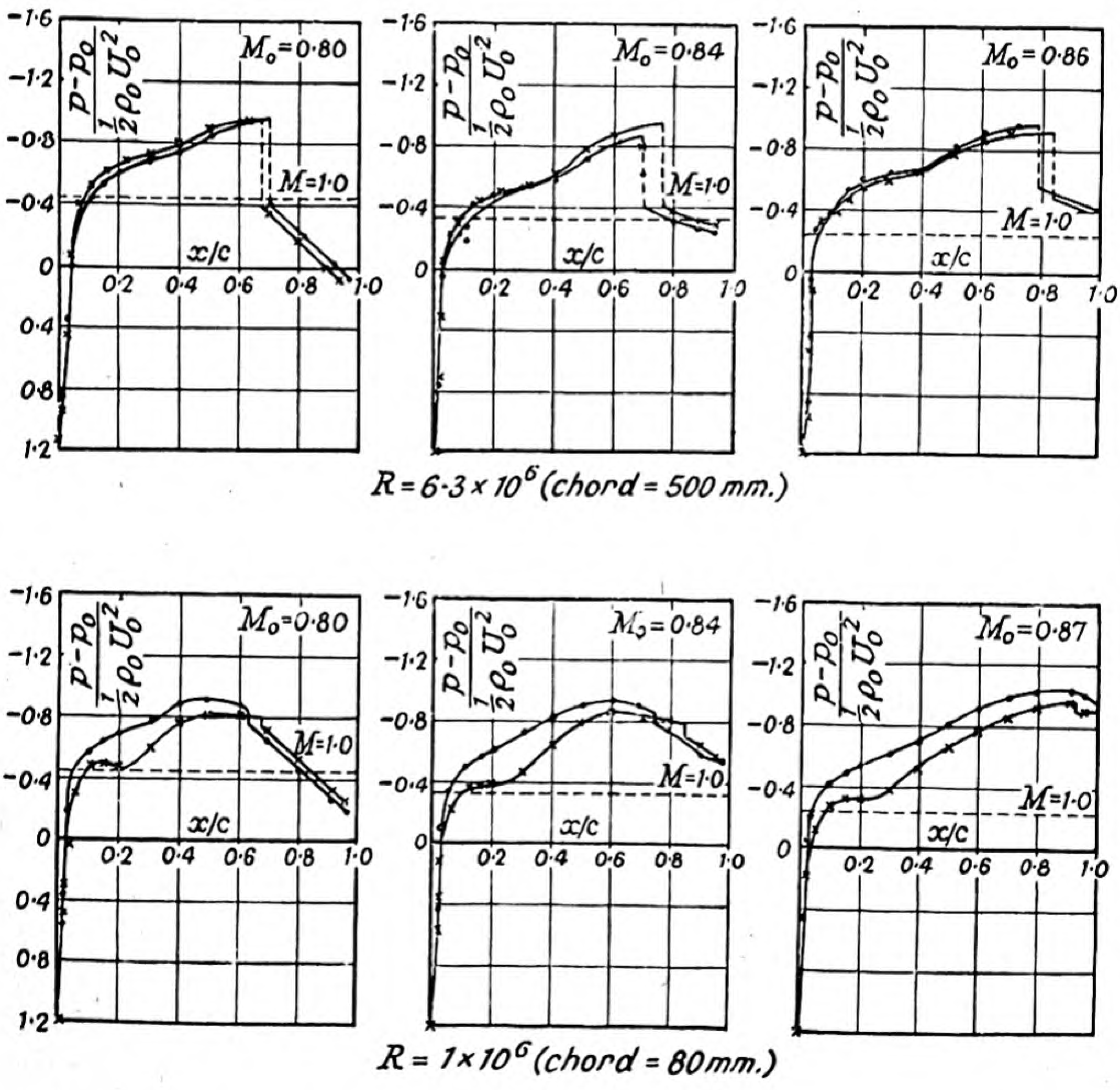
Fig. 7.—Calculated profile drag of a 6% biconvex section at supersonic speeds.
$C_D = \text{DRAG}/\frac{1}{2}\rho_0 U_0^2$

6. SCALE EFFECT AND SHOCK WAVE-BOUNDARY LAYER INTERACTION

There is already enough experimental data to indicate that changes of Reynolds' number are not likely to be less important in compressible flow at high speeds than they are in incompressible flow. Fig. 8 shows the results of pressure distribution measurements made on two geometrically similar aerofoils, one of 500 mm. chord and one of 80 mm. chord, under identical conditions at high subsonic speeds. It will be seen that there are marked

changes in the pressure distributions with the change of scale. It seems probable from evidence to be discussed that these changes derive mainly from the fact that for the larger wing the boundary layers were turbulent ahead of the shock waves springing from the surface, but for the smaller wing they were laminar. Again, towards the rear of the upper surface of an aerofoil at incidence at supersonic speed and at low Reynolds' numbers separation of flow has sometimes been observed, where inviscid flow theory would predict a favourable pressure gradient in which such separation would not be expected. Hence it appears that the pressure rise across the trailing edge shock wave is diffused upstream in the subsonic part of the boundary layer and may provoke separation.

Other examples can also be presented of viscous effects modifying in greater or less measure the characteristics of the flow about a body from those expected in inviscid flow, and they all serve to support the view



Section :—N.A.C.A. 0 0015—1·1·40, $\alpha = 0°$.

Fig. 8.—Measured pressure distributions on two aerofoils of same section but different chords (Goethert).

that the assessing of scale effects at high Mach numbers will require the prior understanding of the nature of the interaction of shock-waves and boundary layers. Hence this subject is attracting the attention of a number of research workers. It presents by no means an easy field to explore, and there are serious difficulties in the way of both theoretical and experimental investigations, nevertheless important and illuminating results are already available and these will be briefly discussed.

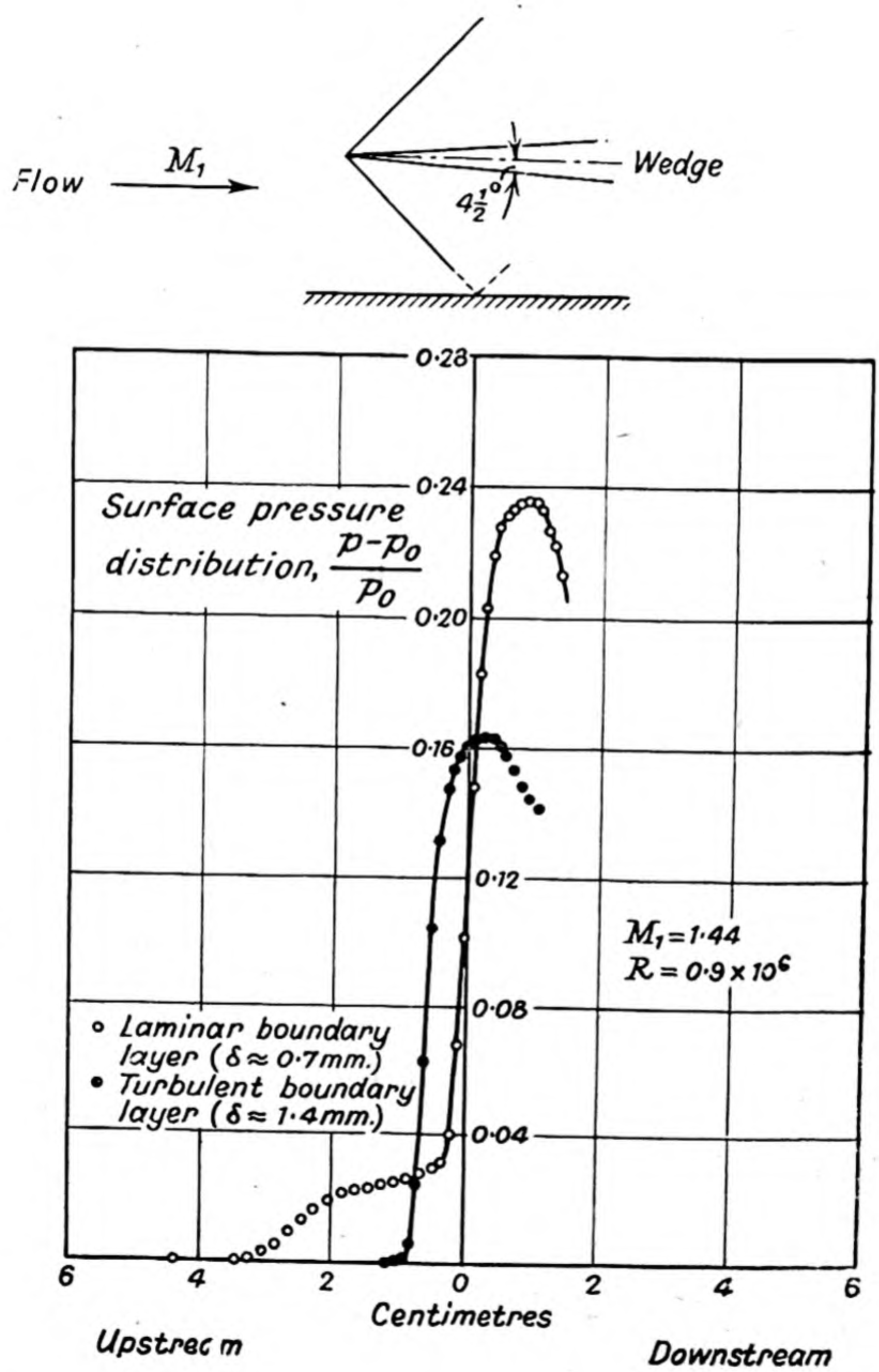


Fig. 9.—Reflection of a shock wave from a flat surface.
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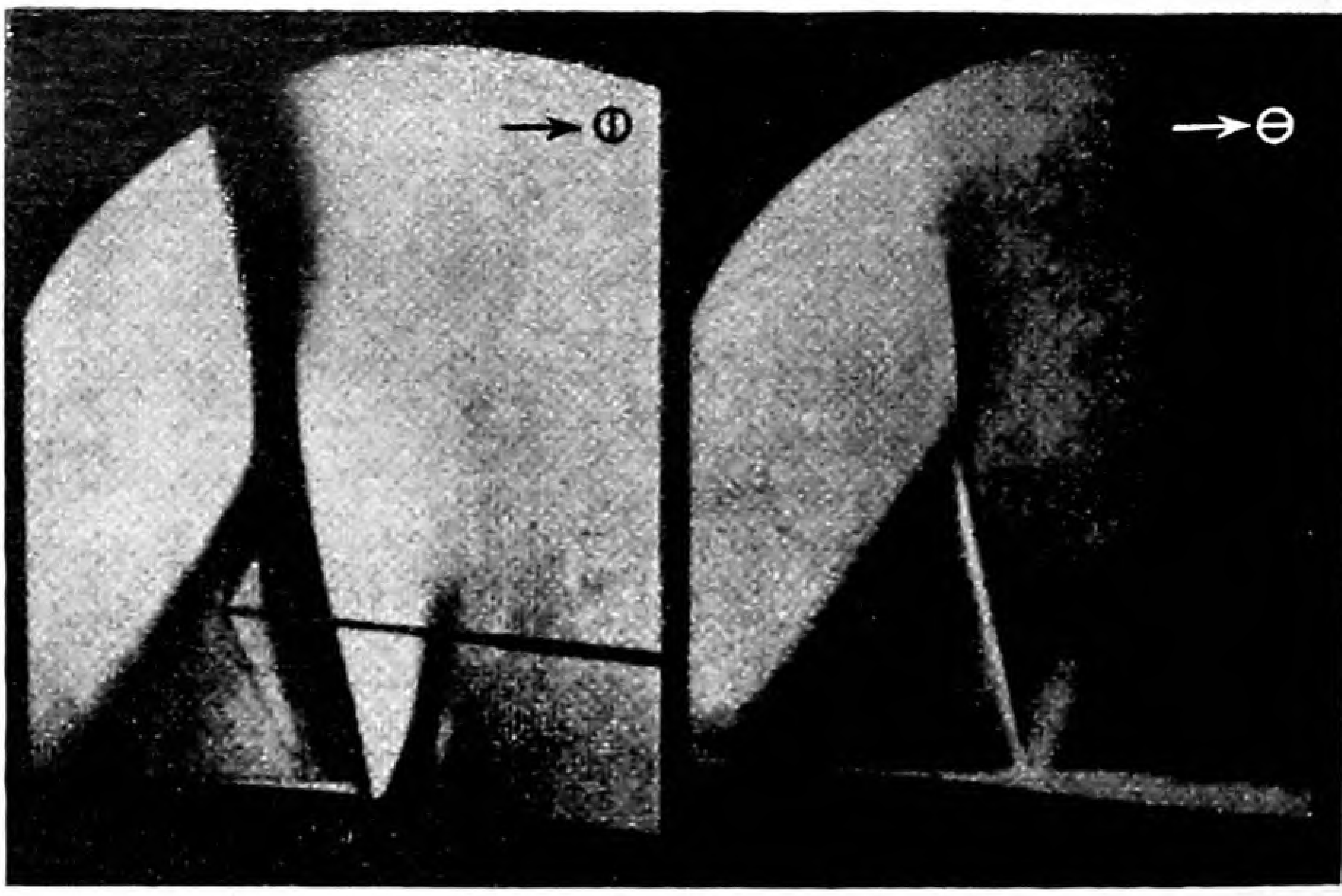
When a disturbance approaches a boundary layer it will be partly reflected and partly refracted in the outer part of the boundary layer if the flow there is supersonic. Theory indicates that when the local Mach number is greater than $\sqrt{2}$ the reflected disturbance is of opposite kind to the incident disturbance, but where the local Mach number in less than $\sqrt{2}$ the reflection is of the same kind[23]. When it reaches the inner region of subsonic flow, however, where pressure changes can be propogated upstream, the pressure changes due to the disturbance are diffused both upstream and downstream. We find in consequence that the pressure distribution at the surface near the foot of an incident shock wave shows a rising pressure distribution ahead of the wave foot which continues beyond the wave foot; thus the almost abrupt rise in pressure at the wave front found in the main stream is smoothed and "softened" at the surface[24,25,26,27]. It is here that the difference between the laminar and turbulent boundary layer manifests itself, because experiments indicate that this diffusion of pressure at the surface is much more marked when the boundary layer is laminar than when it is turbulent[24,26] (see Fig. 9). The experimental data indicate that the distance for upstream diffusion of the pressure rise across the shock can be as much as a hundred times the boundary layer thickness when it is laminar, but the corresponding distance when the boundary layer is turbulent is no greater than ten times the boundary layer thickness. This result probably follows from the fact that when the boundary layer is laminar there is a relatively thicker region of subsonic flow than when it is turbulent. Further, the rising pressure ahead of the shock distorts the velocity distribution and thickens the subsonic region rather more effectively for the laminar boundary layer than for the turbulent boundary layer.

Under the influence of the pressure rise the boundary layer may separate. It it does so a strong compression wave springs from the point of separation to join up with the main shock wave to form a bifurcated wave (see Figs. 10 and 12). The effect of the boundary layer separation will be in turn to modify the overall pressure distribution and hence the shock wave. The tendency to separation will be a function of scale, the nature of the flow in the boundary layer as well as of the strength of the shock wave. As might be expected, if the boundary layer is laminar it is more likely to separate than when it is turbulent.

In general, therefore, various possibilities present themselves when we consider a region of shock wave-boundary layer interaction. The approaching boundary layer may be laminar or turbulent, it may or may not separate, if it is laminar and separates it may re-attach itself to the surface in a laminar or turbulent state. To each case we may expect a characteristic shock wave pattern and a modification of the surface pressure distribution. Examples of all such cases can be readily culled from the available experimental literature (see Figs. 10, 11, 12 and 13) [23,24,25], but much research remains to be done before the nature of the interaction and its full effects are completely understood and predictable in every case.

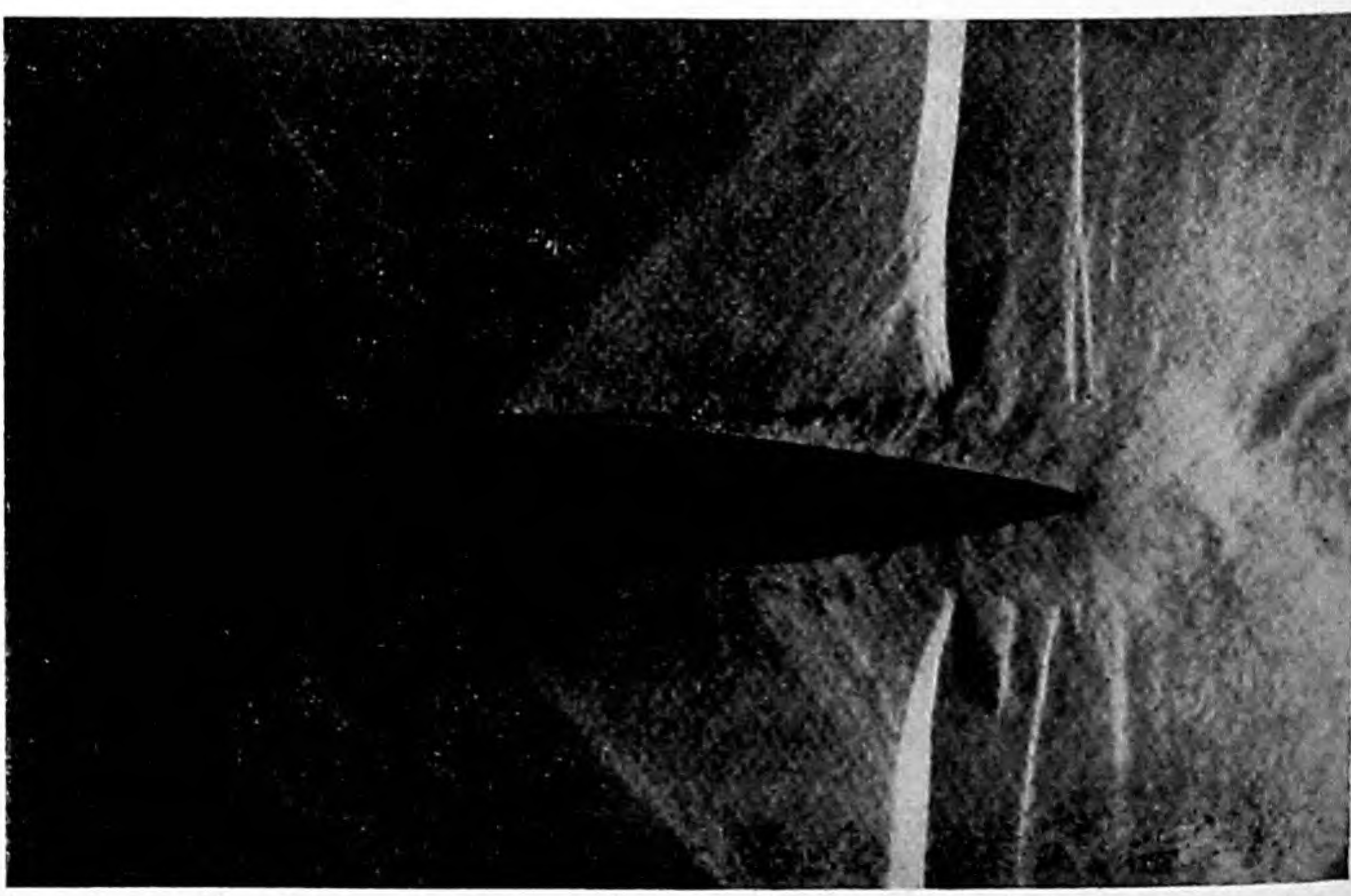
To illustrate the complexity of the problem the case of the laminar boundary layer that has not separated or has suffered only a local temporary



(*a*)

M_0 = Mach number ahead of shock = 1·225. R = $1{\cdot}325 \times 10^6$.

Φ knife edge normal to flow. ⊖ knife edge parallel to flow.



(*b*)

M_0 = 0·895. R = $8{\cdot}77 \times 10^5$.

Fig. 10.—Schlieren pictures illustrating interaction of shock waves and boundary layers. Boundary layer is laminar ahead of shock and separates.

(*a*) Flow past a curved plate.

(*Reproduced from Mitteilungen aus dem Institut für Aerodynamik.*)

(*b*) **Flow past aerofoil.**

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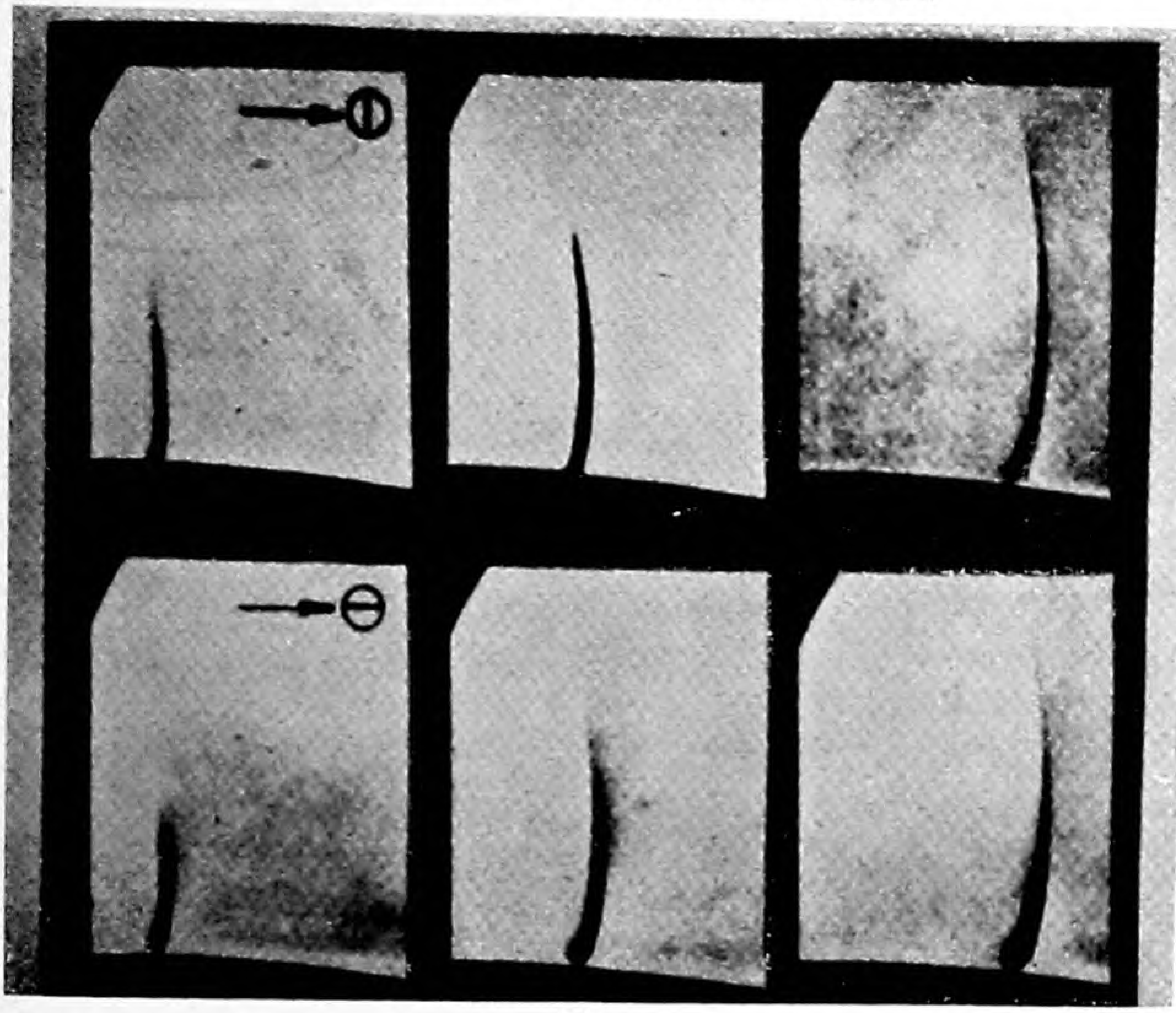
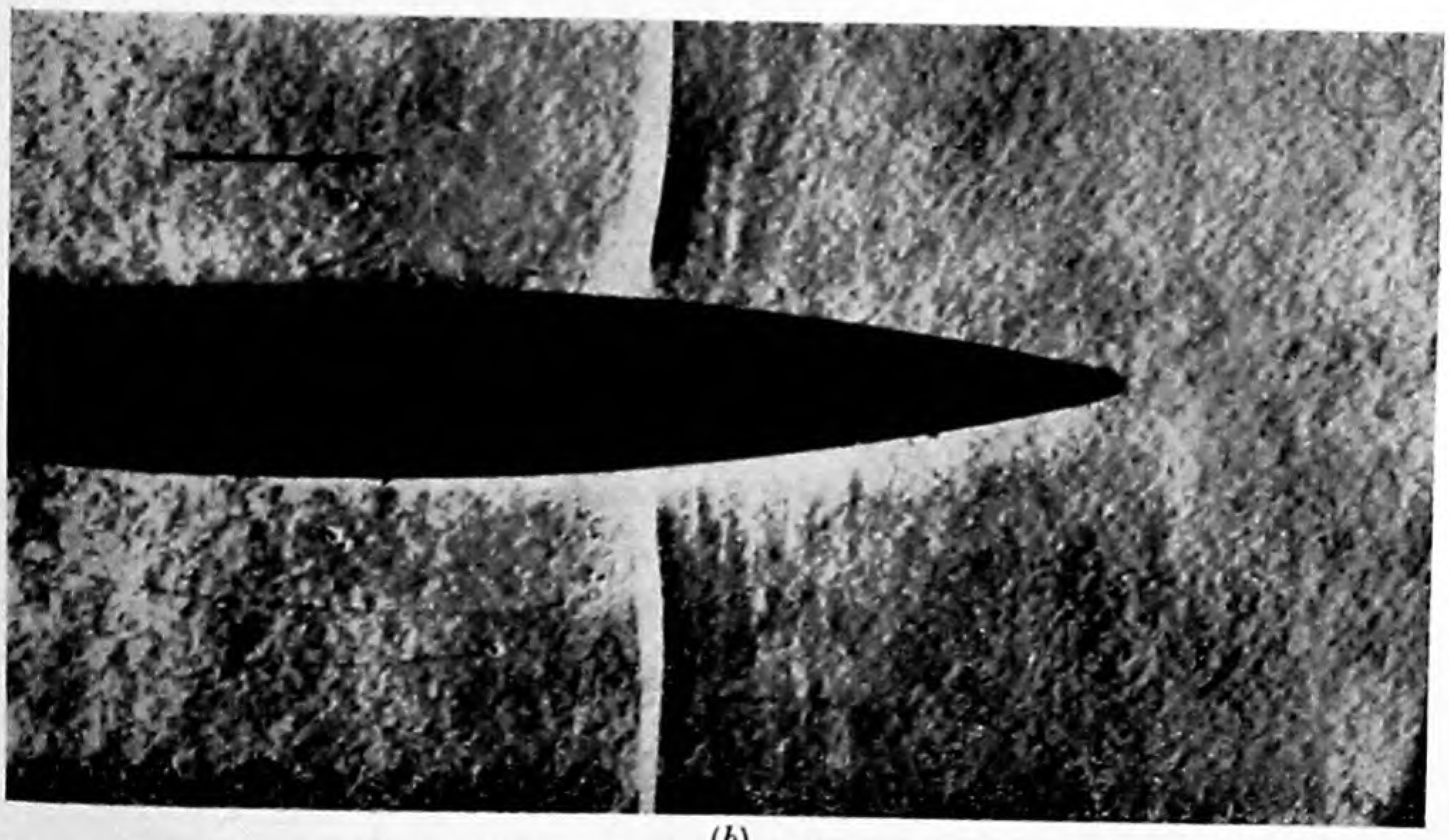


(a)

$M_0 \equiv$ Mach number ahead of shock

1·12 1·26 1·31

ⴲ knife edge normal to flow ⊖ knife edge parallel to flow.



(b)

$M_0 = 0{\cdot}843$. $R = 1{\cdot}69 \times 10^6$.

Fig. 11.—Schlieren pictures illustrating interaction of shock waves and boundary layers. Boundary layer is turbulent ahead of the shock and does not separate.

(a) Flow past curved plate.

(Reproduced from Mitteilungen aus dem Institut für Aerodynamik.)

(b) Flow past aerofoil.

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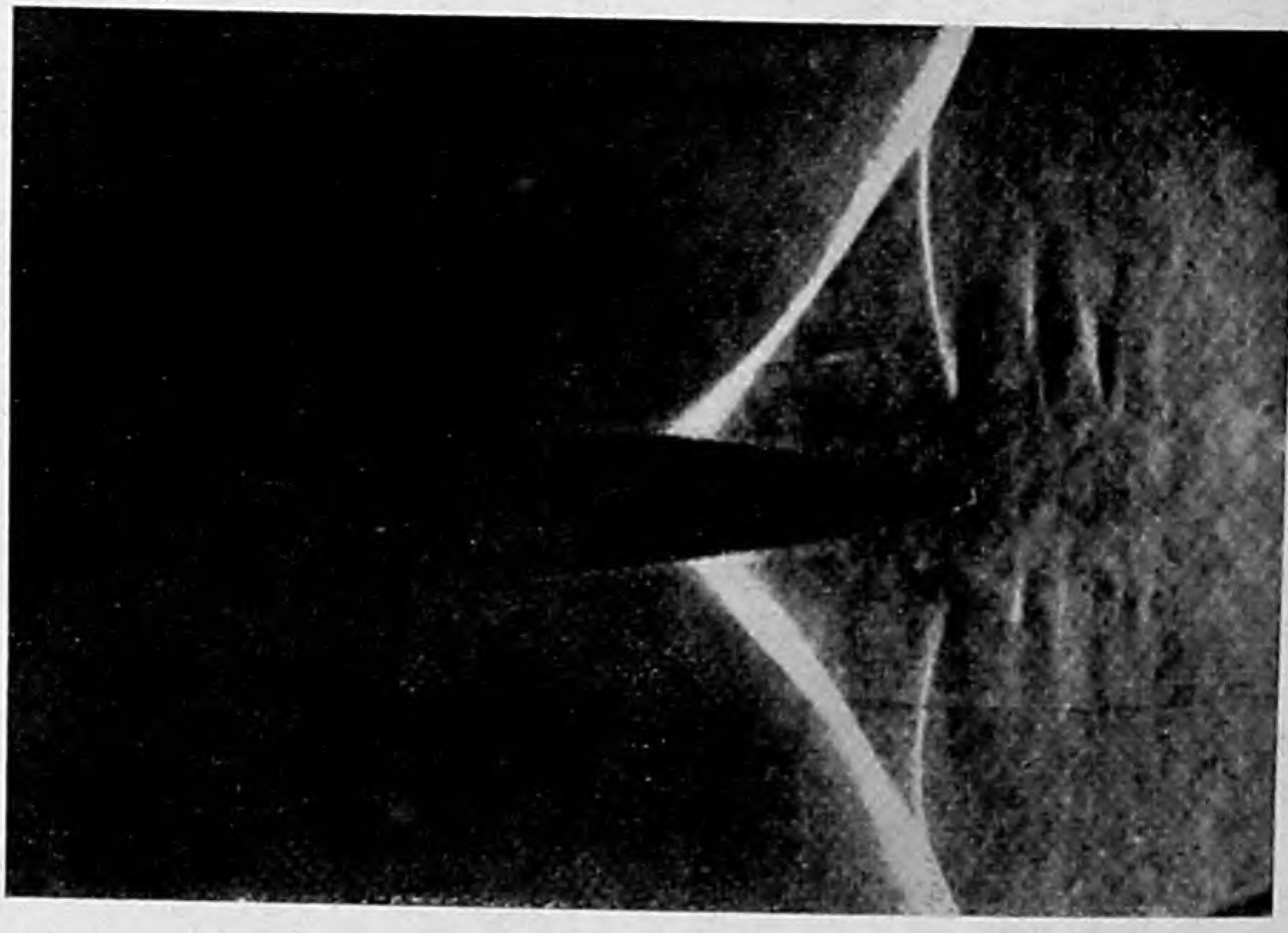
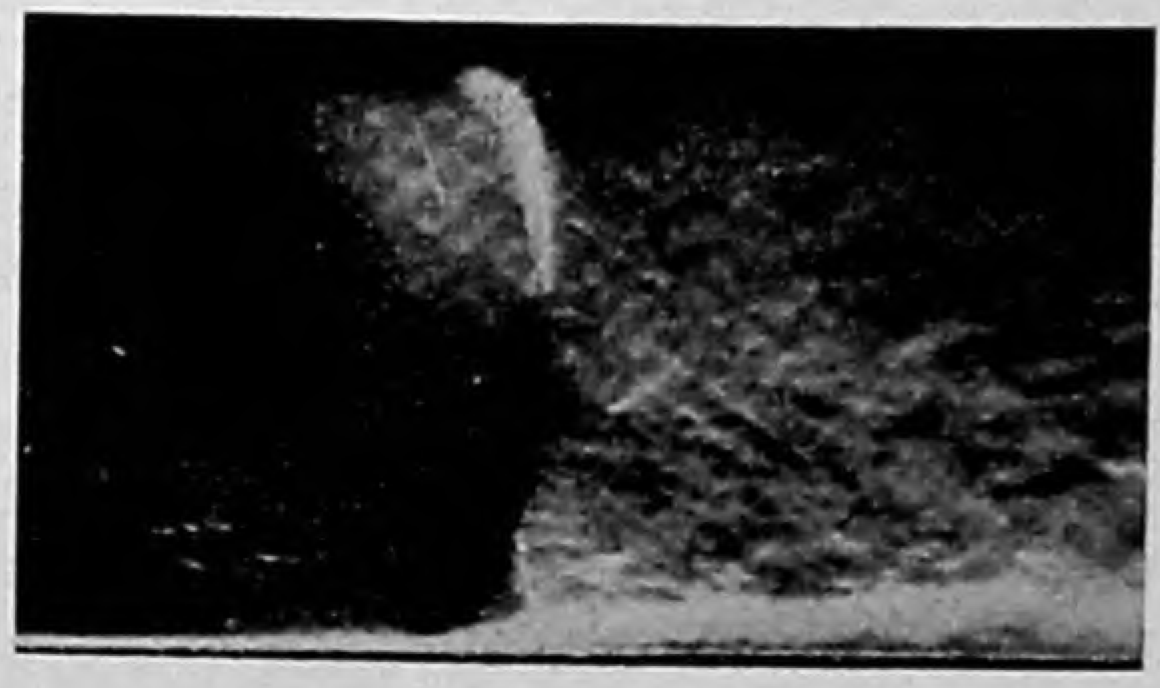


(*a*)

$M_0 = 0{\cdot}895$. $R = 1{\cdot}75 \times 10^6$.



(*b*)

$M_0 = 1{\cdot}443$.

Fig. 12.—Schlieren pictures illustrating interaction of shock waves and boundary layers. Boundary layer is turbulent ahead of the shock and separates.

(*a*) Flow past aerofoil.

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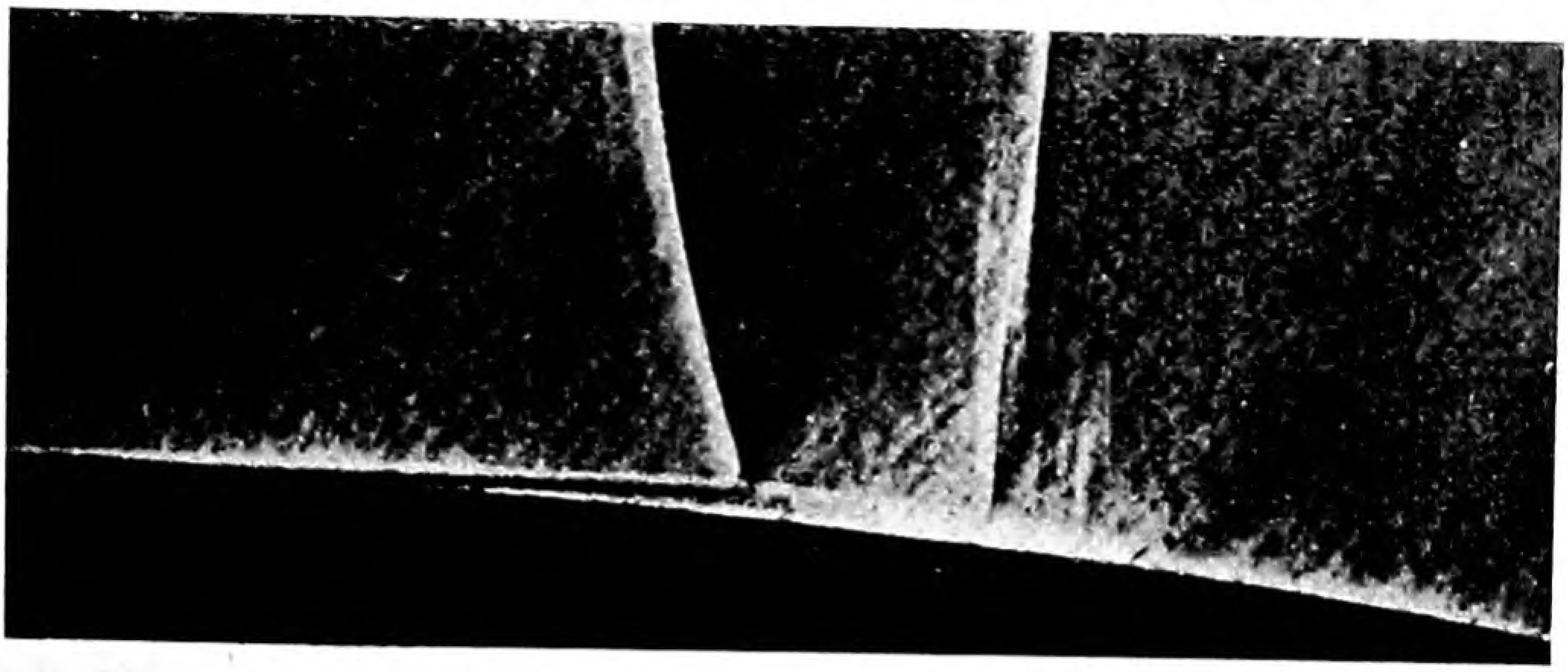
(*b*) Flow past flat surface.

(*Reproduced from the Proceedings of the Royal Society.*)



$M_0 = 0{\cdot}843$. $R = 8{\cdot}45 \times 10^5$.

Fig. 13.—Schlieren picture illustrating interaction of shock waves and boundary layers. Flow past aerofoil. Boundary layer is laminar ahead of the shock and does not separate completely.

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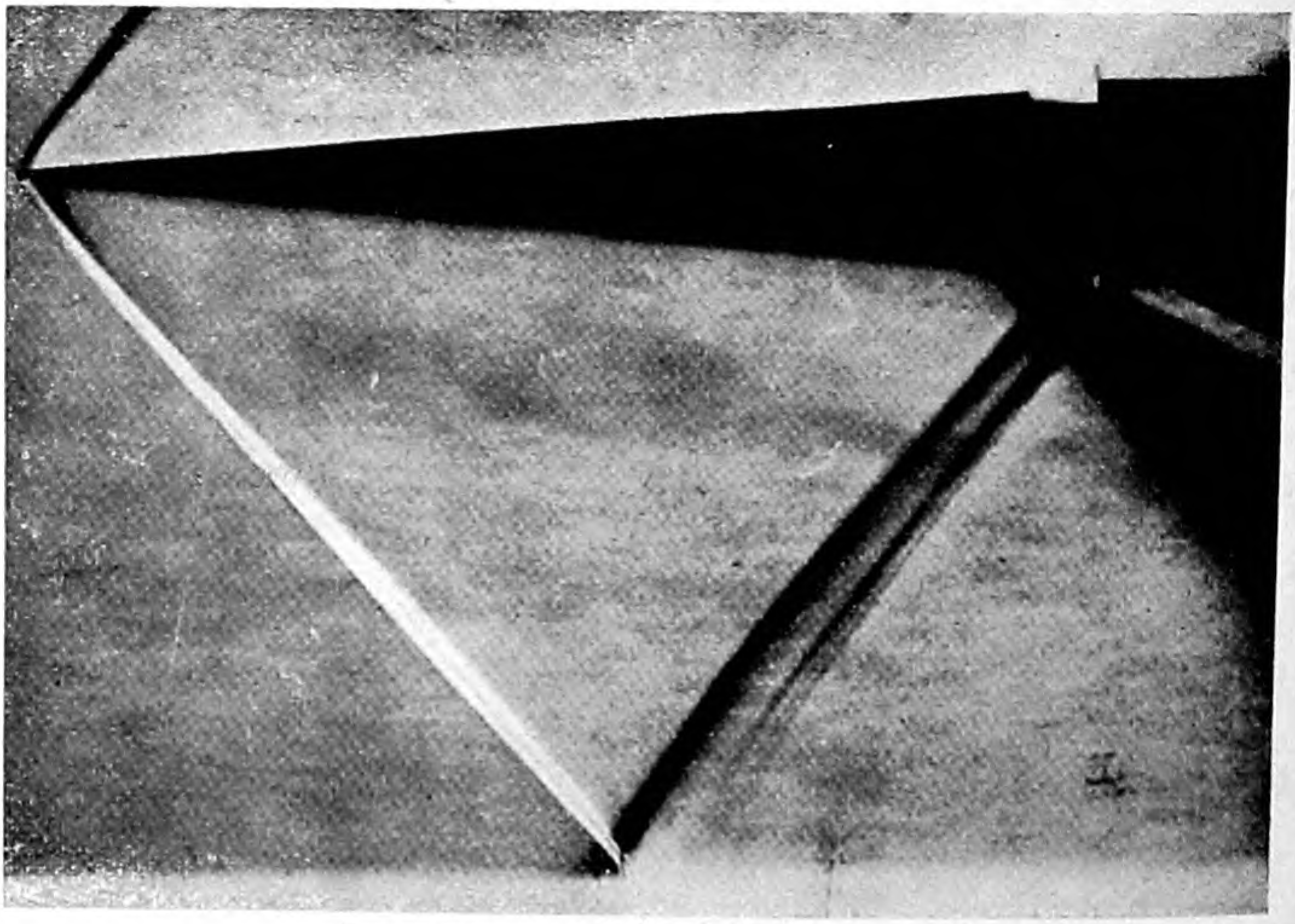
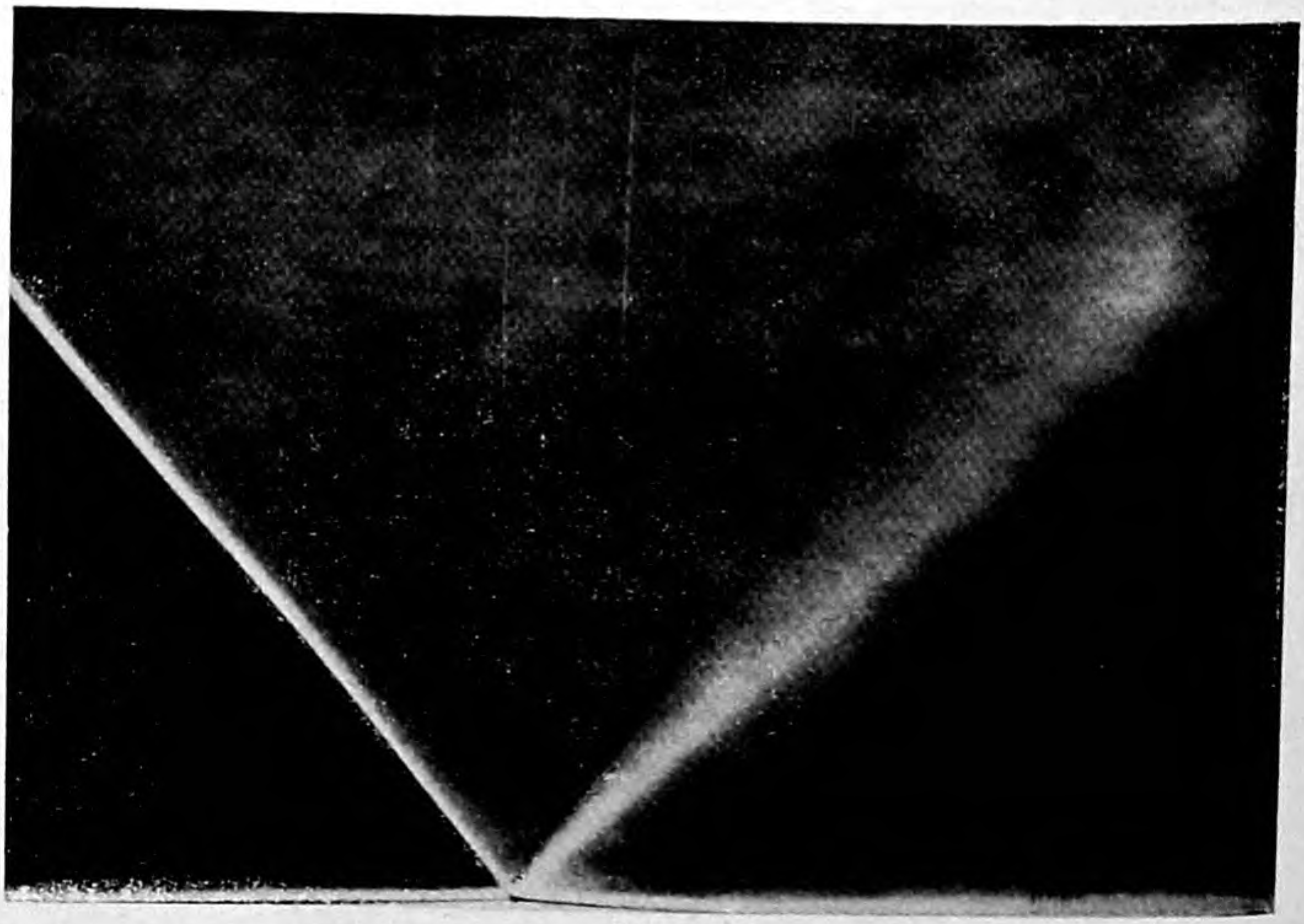
separation is worth some discussion (see Fig. 13). Here we see that the shock approaches the boundary layer at an angle somewhat less than 90° to the main flow direction and is reflected from the boundary layer by a fan of expansion waves. The boundary layer thins down slightly behind the shock. A similar pattern has been noted when an oblique shock has been reflected from a surface on which the flow in the boundary layer was laminar (see Fig. 14(*b*))[28]. Thus, it appears that with a laminar boundary layer that does not separate completely the surface behaves very much as if it were that of a free jet, as far as the reflection of incident disturbances is concerned. On the other hand, when the boundary layer is turbulent a shock is reflected as a shock (see Fig. 14(*a*)) as would normally be expected with a solid surface. A simplified picture of what has been suggested to occur with the laminar boundary layer is shown in Fig. 15(*b*). The relatively slow rate of thickening of the boundary layer ahead of the shock induces a family of relatively weak compression waves in the main flow converging to a region well away from the surface. The laminar boundary layer cannot sustain large adverse pressure gradients, particularly if it has already suffered temporary separation, and the expansion fan reflecting from the foot of the shock ensures that the pressure gradient along the surface remains small. In addition, it deflects the thickening or separating boundary layer back to the surface, and the displacement thickness is reduced aft of it. Consequently a second family of convergent compression waves is induced in the main flow. Well away from the surface the two families of compression waves and the expansion fan result in a net compression, the reflexion of the incident shock wave then approximates to that for a solid surface in inviscid flow. When the boundary layer is turbulent its initial thickening ahead of the shock takes place over a much smaller distance and is consequently more intense. The resulting compression waves induced in the main flow converge more rapidly, and with an incident oblique shock they tend alone to provide the compression shock required for "solid surface" reflexion (Fig. 15(*b*)). However, something of the modifying influence



(*a*)



(*b*)

Fig. 14.—Reflection of a shock wave from a wall with (*a*) turbulent boundary layer, (*b*) laminar boundary layer. No complete separation occurs in either case.

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characteristic of a laminar boundary layer may still be present with a turbulent boundary layer, and a small localised expansion fan just aft of the incident shock is sometimes noted.

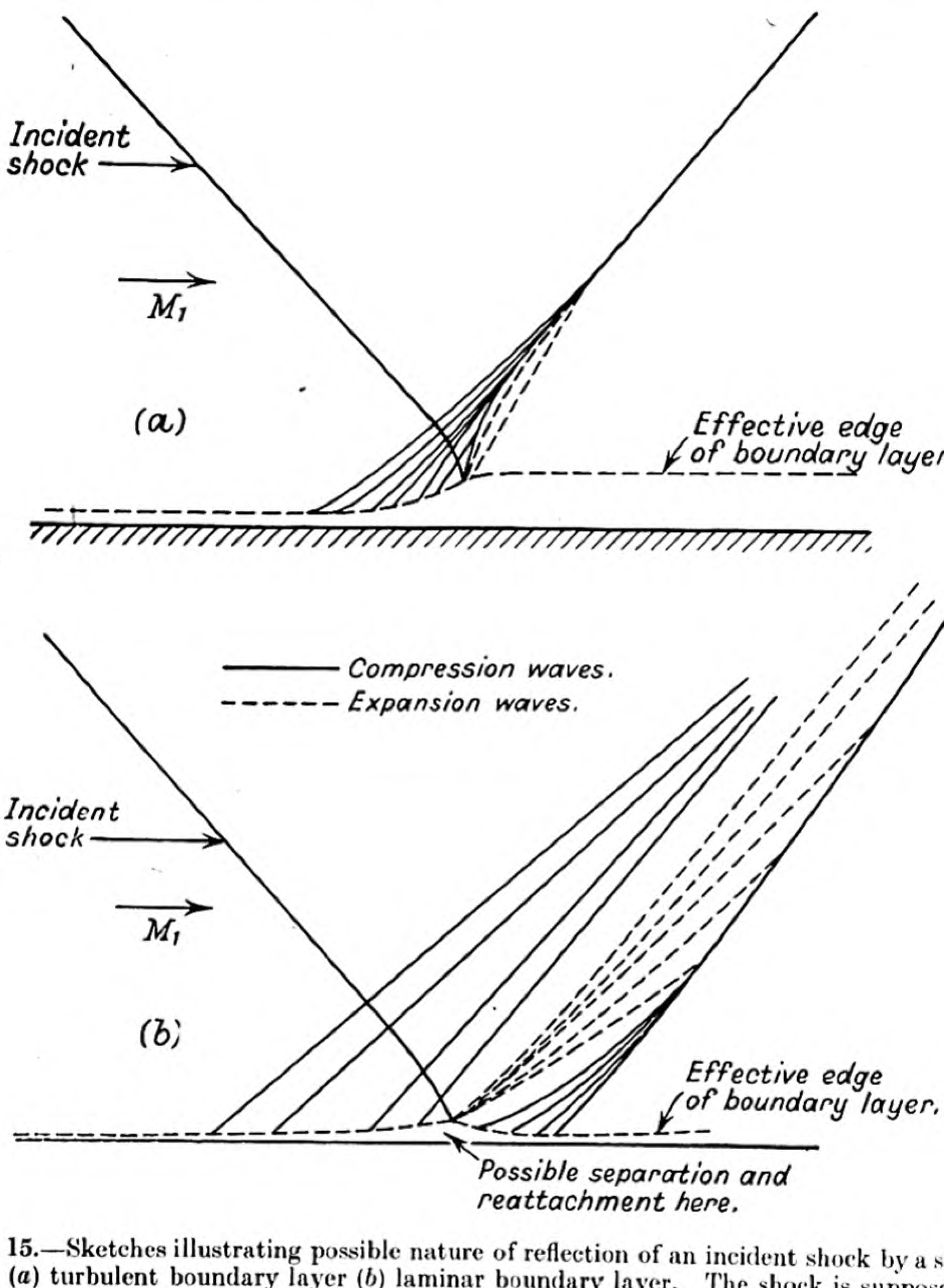

Fig. 15.—Sketches illustrating possible nature of reflection of an incident shock by a surface with (*a*) turbulent boundary layer (*b*) laminar boundary layer. The shock is supposed not to induce complete separation of the boundary layer.

This brief discussion will indicate that here we are on the threshold of a subject of considerable importance to those interested in the flow of real fluids high at speeds, and it will undoubtedly attract the attention of research workers for many years to come.

7. ACKNOWLEDGEMENTS

The author wishes to thank Professor Liepmann of the California Institute of Technology, Professor Lees of Princeton University, Professor Ackeret of E.T.H. Zurich, Mr. Fage of the N.P.L., and the Director of the

N.A.C.A., for permission to refer to their works and to reproduce diagrams taken from their publications.

Some of the material in this paper was used originally in a Section Lecture of the Royal Aeronautical Society on 21 March, 1950.

APPENDIX 1.

MOMENTUM—INTEGRAL EQUATION OF THE BOUNDARY LAYER

Two-dimensional steady flow.

If equation (1) is integrated with respect to y through the boundary layer, or alternatively the balance of rate of change of momentum and frictional and pressure forces for an elementary section of the boundary layer is considered, the following equation can be derived :—

$$\frac{d\theta}{dx} + \theta\left[\frac{(H+2)}{U_1}\frac{dU_1}{dx} + \frac{1}{\rho_1}\frac{d\rho_1}{dx}\right] = \frac{\tau_\omega}{\rho_1 U_1^2} \qquad (15)$$

where θ is the momentum thickness of the boundary layer

$$= \int_0^\delta \frac{\rho u}{\rho_1 U_1}\left(1 - \frac{u}{U_1}\right) dy,$$

and $H = \delta^*/\theta$, where δ^* is the displacement thickness of the boundary layer

$$= \int_0^\delta \left(1 - \frac{\rho u}{\rho_1 U_1}\right) dy.$$

As written above the equation is applicable to both laminar and turbulent motion.

Axi-symmetric steady flow.

The momentum integral equation for axi-symmetric flow is :—

$$\frac{d\theta}{dx} + \theta\left[\frac{(H+2)}{U_1}\frac{dU_1}{dx} + \frac{1 d\rho_1}{\rho_1 dx} + \frac{1 dr_0}{r_0 dx}\right] = \frac{\tau_\omega}{\rho_1 U_1^2} \qquad (16)$$

where r_0 is the local radius of cross-section of the body ; if ϕ is the angle between the local tangent plane and the axis, then the momentum (θ) and displacement thicknesses (δ^*) are

$$\theta = \int_0^\delta \left(1 + \frac{y}{r_0}\cos\phi\right)\frac{\rho u}{\rho_1 U_1}\left(1 - \frac{u}{U_1}\right) dy,$$

and

$$\delta^* = \int_0^\delta \left(1 + \frac{y}{r_0}\cos\phi\right)\left(1 - \frac{\rho u}{\rho_1 U_1}\right) dy.$$

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7

The Effect of Concentration on the Settling of Suspensions and Flow through Porous Media

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ABSTRACT. An expression is obtained for the rate of settling of suspensions, applicable particularly to concentrated suspensions. Experimental evidence in support of the theory is found in published data; for a suspension of spheres there is good agreement from at least 7 up to 50% volume concentration. The influence of particle shape is discussed in a preliminary way. The behaviour of flocculated systems and non-rigid particles (blood cells) is also examined.

The flow of fluids through porous media is an allied process. The expression derived for sedimentation modified by a factor allowing for the difference in the arrangement of particles in a packed bed compared with a settling suspension, is found to agree quantitatively with experimental and theoretical relations for viscous flow through beds of particles.

SYMBOLS

c	=	concentration, volume of solids to total volume of system.
k	=	Einstein's shape factor (=5/2 for spheres).
Q	=	Vand's interaction constant (=39/64 for spheres).
U_o	=	Stokes' rate of settling of single particle in infinite fluid.
U_c	=	rate of settling of a suspension at concentration c.
	=	rate of fluid flow above a packed bed at concentration c. (Volume flow for unit area of empty tube).
U_r	=	relative velocity $= U_c/U_o$.
ζ	=	Fowler and Hertel's orientation factor.
η_o	=	absolute viscosity of pure fluid.
η_c	=	effective local fluid viscosity in a suspension. (not to be identified with bulk viscosity of suspension).
ρ	=	density of pure fluid.
σ	=	density of solid particles.
ψ	=	Wadell's particle shape factor, the sphericity, defined as ratio of surface area of sphere having same volume as particles to actual surface area ($=d/\delta$).
$\Delta P/L$	=	mean pressure gradient across porous media.
d	=	specific surface diameter (diameter of sphere having same ratio of surface area to volume as particle).
d_s	=	diameter of sphere or Stokes' diameter (diameter of sphere having same free falling speed as particle under same conditions)
δ	=	volume diameter (diameter of sphere having same volume as particle).
Δ	=	surface diameter (diameter of sphere having same surface area as particle).

1. INTRODUCTION

The rate of settling U_o of a single sphere of diameter d_s in an infinite fluid of viscosity η_o, is obtained by equating the viscous drag:

$$D = 3\pi\eta_o U_o d_s \tag{1}$$

derived by Stokes from the fundamental equations of viscous flow, with the effective gravitational force :

$$F = (\sigma-\rho)g\ \pi\ d_s^3/6 \tag{2}$$

where $(\sigma-\rho)$ is the density difference between solid and fluid. The terminal velocity is then given by Stokes' equation :

$$U_o = (\sigma-\rho)g\ d_s^2/18\eta_o \tag{3}$$

The limitations of Stokes' law are well known.(1) In particular the rate of settling is reduced near the walls of the containing vessel or in the neighbourhood of other spheres. It is with this latter aspect that we are concerned here and in the following discussion the other requirements for Stokes' settling (such as, smooth rigid spheres, low Reynolds' number, no solvation or electro-viscous effects, no 'slipping' of fluid at surface, etc.) are assumed to be satisfied.

A number of attempts(2-8) have been made at a hydrodynamical treatment of the mutual influence of particles in a suspension upon their rate of settling. The difficulties of the problem have, however compelled recourse to somewhat artificial models giving approximate solutions, none of which has shown satisfactory agreement with experimental observations. Furthermore, the suspensions are assumed to be extremely dilute.

In an extensive discussion, Burgers(3) has given the following picture of the settling of a suspension. The sedimentation of any given particle in the suspension is subject to two influences, one arising from the *motion* of the other particles and the other from their *presence*. (i) If all but a single particle are imagined to have the same density as the pure fluid and not to settle on their own account, they will nevertheless tend to be carried down by the velocity imparted to the fluid in consequence of the motion of the single sedimenting particle. When all the particles have their normal density, there is a mutual downward drag. This tendency is offset however, by the upward return flow that must occur in a closed vessel. The net effect on any given particle may be either an accelerating or retarding force depending upon the spatial distribution of the neighbouring particles. (ii) There is a retarding influence on any given particle arising from the mere presence of other particles. If as before, these are imagined to have the same density as the fluid, the field of flow of the single sedimenting particle will be distorted because the other particles, owing to their rigidity, cannot take part fully in the deformation of the fluid. In consequence the sedimenting particle experiences a retardation, which appears to it to arise from an increase in the viscosity of the fluid. The effect is of exactly the same nature as the increase in apparent bulk viscosity of a fluid on adding rigid particles ; it is not however of the same magnitude as will be seen later.

The terminal velocity of a particle in a suspension is obtained by equating the viscous drag, modified by the influences described above, with the effective gravitational force. It is usually assumed that in a suspension the effective gravitational force is determined by the buoyancy of the suspension and not that of the pure fluid. However, any additional buoyancy is a consequence of the motion of the particles ; for instance, the apparent weight of an object suspended in a liquid is not affected by the

presence of other objects in the fluid. A full analytical treatment of the influences described above would therefore automatically include any extra buoyancy effects and the effective gravitational force for a suspension, as for a single particle, is given by equation (2). This is the point of view adopted by Burgers.

Burger's analytical treatment of the processes described above, which is outlined in the next section, is not complete. Although it is established that the rate of settling of any given particle is dependent on the distances and arrangement of all other particles, no consideration is given to the question whether the settling process may not itself establish an *equilibrium* arrangement of particles. It is fairly obvious that this must be the case. If it were not, then the rate of settling of individual particles would vary with the local particle distribution and the characteristic feature of suspensions—that they settle as a whole—would not be observed.

The assumption of an equilibrium particle arrangement is the central point of the present discussion. On this basis it is possible to obtain (Section 3) an expression for an equilibrium rate of settling, applicable at least to the more concentrated systems. Some confirmatory experimental evidence is analysed in Section 4. The treatment of Section 3 is then applied to the flow of fluids through packed beds (Section 5).

2. DILUTE SUSPENSIONS

After an unsuccessful attempt[3] to obtain a solution starting with the Stokes' field of flow for a single sphere in an infinite fluid, Burgers[7] assumed that the return flow due to each sphere could be represented by a " diffuse " force field acting on the fluid in equilibrium with the gravitational pull considered to act as a point force on the fluid, the effective gravitational force in this model being the difference between weight of particle and pure liquid displaced. A further field is added, so chosen as to annul the residual motion at the surface of a sphere that arises when it is introduced into the combined field resulting from the diffuse and point forces. The fields are assumed to be infinitely extensive. The diffuse force field is chosen so that Stokes' solution is obtained when the interparticle distance is very great and includes the boundary condition that there shall be no motion at the base of a containing vessel. It is shown[8] that the solution obtained will satisfy also the boundary conditions at the vertical walls for the case of a suspension enclosed between two infinite parallel plane walls.

The total effect of a number of spheres is obtained additively, the suspension being assumed so dilute that no interactions occur between the fields appropriate to each sphere. The final result gives the relative velocity in a dilute suspension :

$$U_r = \frac{U_c}{U_o} = \frac{1}{1 + (\lambda_{\mathrm{I}} + \lambda_{\mathrm{II}})c} \tag{4}$$

where U_c is the mean settling rate of a suspension of equal spheres at volume concentration c and U_o is the Stokes' velocity of an individual sphere. The coefficient λ_{I} represents, in a general way, the effect of the presence and the coefficient λ_{II} the effect of the motion of other spheres on

the settling rate of a given sphere. Their values depend on the spatial arrangement of the spheres. For a random distribution, Burgers finds $\lambda_{I} = 15/8$ and $\lambda_{II} = 5$, and thus for very dilute suspensions of randomly distributed spheres

$$U_r \simeq 1 - 6\tfrac{7}{8}\,c \qquad c \ll 1 \tag{5}$$

The same result but with a numerical coefficient of 7·1 instead of 6·9 was obtained earlier by Kermack, M'Kendrick and Ponder(5). In the case of regular arrangements, the coefficients λ_{I} and λ_{II} are themselves functions of the concentration but are only slightly different for cubic and rhombohedral arrangements. Burgers emphasises the preliminary nature of the calculations, particularly in the evaluation of λ_{II}, which, although probably of the right order, might be different(8) for a suspension enclosed between parallel walls of finite extent.

Burgers does not discuss what kind of spatial distribution will actually occur in a settling suspension. In his analysis the motion of either of two spheres is unaffected by interchanging them and they consequently acquire the same terminal velocity. Furthermore, although lateral forces perpendicular to the direction of motion operate on each sphere, they are equal and there is no relative displacement. But the case will be otherwise with a number of spheres. The magnitude of the interaction between pairs of spheres depends on the distance between them. Consequently the local rate of settling will vary with the local distribution of spheres. It must be assumed however, since suspensions are observed to settle as a whole that the lateral components will produce a more or less uniform spacing in any horizontal layer. Furthermore, Burgers' analysis takes no account of the effect of the distribution in one layer on the drag in the layer above. (It is for this reason that Burgers finds little difference in the rate of settling of cubic and rhombohedral distributions, whereas the cubic arrangement would be expected to settle faster.) In actual fact it must be supposed that the upward flowing liquid will arrange the successive layers to give a minimum overall resistance or maximum rate of settling. Thus, the spheres will tend to become marshalled into vertical columns. Burgers' calculations will give then the settling rate of systems in which the initial spatial distribution is maintained by some extraneous mechanism, for example long-range forces. When the spheres are free to move, the rate of settling will increase steadily until an equilibrium arrangement has been produced. In the case of very dilute suspensions the initial distribution might persist for an appreciable time, and for a random arrangement equation (5) might then be adequate. For more concentrated systems on the other hand, only a short initial accelerating period would be expected; the final equilibrium settling rate will then exceed that indicated by equation (5).

3. CONCENTRATED SUSPENSIONS

The problem of determining the settling rate of a suspension is much simplified if we assume that an equilibrium particle arrangement on the lines considered above, has been established. We are then concerned with

the general motion of a large number of particles which can be examined in terms of the behaviour of a single *representative* particle. Instead therefore of a statistical treatment, attention can be restricted to average effects. A reasonable assumption is that the average rate of settling can be obtained by considering the motion of a single particle in a homogeneous suspension having the same concentration at every point. This assumption is likely to be satisfied when the interparticle spacing is comparable with the particle diameter. In this connexion it should be remembered that even at concentrations as low as 5% by volume, the mean distance between the surfaces of adjacent particles is only about twice their diameter; and at 10% concentration the spacing is less than a diameter.

The presence of the particles can be regarded as increasing the effective local viscosity of the fluid, whilst their motion is considered to produce two effects, a uniform upward fluid velocity and an increased buoyancy. The motion of the representative particle can then be treated as the Stokes' motion of a particle in a fluid of increased viscosity and density on which a uniform upward velocity has been impressed.

3·1. *Suspension of Spheres.*

(i) The effect of the *presence* of particles in a suspension is considered first. The introduction of rigid particles into a fluid in motion results in a distortion of the original field of flow since the fluid, in the absence of slipping, is reduced to rest at the surfaces of the particles. For example, the presence of a single sphere in a linear shearing field results in a retardation of the fluid in all planes parallel to the direction of motion, the integral of the retardation over any plane having a constant value irrespective of the distances of the sphere (at sufficiently large distances). The summation of the separate effects due to a number of spheres (second-order mutual interactions being neglected) results in a decrease in the *mean* rate of shear between two parallel planes. Einstein's[9] well-known expression for the viscosity of a suspension is then obtained:

$$\eta_c \simeq \eta_0(1 + kc) \qquad (6)$$

where η_c is the apparent viscosity of the suspension, η_0 is the viscosity of the pure liquid and k is a numerical constant, usually described as a shape factor, having the value 5/2 for rigid spheres. Similar results with different values for the shape factor have been obtained for non-spherical particles [10, 11, 12] and non-rigid spheres[13].

The assumption that each sphere makes its own independent contribution to the local retardation and that these can be combined additively, limits the application of expression (6) to dilute systems, the upper limit of concentration being experimentally[14, 15] about 2%. A number of attempts[16-20] have been made to extend the treatment to more concentrated suspensions by introducing additional fields to compensate the mutual disturbances at the surfaces of the particles. The difficulties are very great and no complete solution has been found. Furthermore, in a uniformly sheared suspension, relative motion between parallel layers of spheres results in collisions, which are frequent even at low concentrations.

Now for a settling suspension the problem is somewhat simpler. What is required is the effective local resistance to shearing that would arise if there were a very small relative displacement between a given particle and the rest of the suspension. There is, in the case of equilibrium settling, no actual relative motion between parts of the suspension nor rotation of individual particles and the major difficulties in extending expression (6) do not arise. The local resistance to shearing, a uniform concentration being assumed, can therefore be obtained by integrating Einstein's result for an elementary volume of suspension. It is however necessary for the more concentrated suspensions, to include the mutual interaction neglected in Einstein's analysis. The solution required has been given by Vand[20] in the course of a theoretical treatment of the more complex case of the apparent bulk viscosity of a concentrated suspension. This is :

$$\eta_c = \eta_o \, exp(kc/1—Qc) \qquad (7)$$

where, for the present case, η_c is the effective local viscosity. The shape factor k has the usual value 5/2 and the interaction constant Q is 39/64 for rigid spheres according to Vand's computations.

The viscous drag on a given representative sphere in a suspension may then be derived as if there were no other particles present (excluding for the moment the contributions arising from their motion) and as if the viscosity has everywhere the constant value η_c given by equation (7). That is the drag is Stokes' solution $3\pi\eta_c U d_s$ where η_c replaces η_o.

It must be emphasised that equation (7) does not give the apparent bulk viscosity that would be observed if, for example, the suspension were sheared in a viscometer, but only the effective resistance to a negligibly small displacement. It is impossible to test it experimentally. The measured viscosity would exceed that calculated from equation (7). For this reason, attempts[21, 22] to modify Stokes' law by substituting *measured* values of the bulk viscosity for the pure fluid viscosity cannot be expected to give satisfactory results.

(ii) The motion of the particles produces a number of effects. In Stokes' solution for a single sphere in an infinite fluid, the sphere drags the whole of the fluid along with it. When the fluid is enclosed in a vessel, this is of course impossible and there is a return flow producing a retarding effect which does not however invalidate Stokes' solution unless the vessel is insufficiently large compared with the diameter of the sphere[23]. (It has been shown[6,24,25] that two spheres settling in an *infinite* fluid (with no return flow) acquire velocities in excess of the Stokes' value ; and the case of a cloud of particles has been examined by Smoluchowski[6] and by Burgers[7].) Thus, in a suspension the motion of each particle tends to drag down all other particles and a compact cloud of particles in a very large vessel will settle at rates exceeding the Stokes' velocity of the individual particles. When the cloud completely fills the vessel the retarding effect of the return flow will however outweigh the mutual downward drag and the settling rate is reduced below the individual Stokes' velocity.

A number of attempts have been made to estimate this reduction in settling rate for very dilute suspensions. Cunningham(4) found a maximum value for the resulting extra drag in excess of the Stokes' resistance, by assuming each sphere to carry down an immobile liquid of diameter comparable with the distance between the sphere centres. Kermack, M'Kendrick and Ponder(5) assumed an immobile shell thickness equal to the sphere radius for reasons not stated, but probably because no sphere centre can enter within this region. The mean value of the return flow in the fluid outside the immobile envelope was then found by assuming it to be additive to the Stokes' components in the neighbourhood of the sphere. This gives $\lambda_{II} = 5{\cdot}5$ compared with Burgers' result $\lambda_{II} = 5$. Smoluchowski computed the local return flow velocity due to a large number of spheres (to be added to the local Stokes' component) by extending Lorenz's solution(23) for the motion of a single sphere approaching an infinite plane wall. The conclusion is reached that the return flow velocity will depend on the concentration and also on the local particle arrangement. Smoluchowski found however that the return flow component will have a constant value for a random arrangement of spheres.

This must also be the case for equilibrium settling otherwise there would be relative motion between parts of a suspension. The representative sphere may be supposed therefore to be settling in a pure fluid subject to a constant upward velocity (the presence of the spheres being accounted for by a viscosity increase, as discussed above). The magnitude of the return flow velocity u is determined by the continuity requirement that

$$(1-c)u + cU_c = O \tag{8}$$

where U_c is the actual rate of settling of the spheres and c is the cross-sectional area of the spheres cut by a horizontal plane as a fraction of the area of the plane. It is easily shown that the fractional solid cross-sectional area is equal, for an uniform suspension, to the volume concentration c. The value of u is therefore $-cU_c/1-c$ and the actual settling rate U_c will be equal to the Stokes' velocity U plus the return flow velocity u. That is

$$U_c = U + u$$

$$\text{or} \quad U = U_c/1-c \tag{9}$$

The viscous drag on the representative sphere is therefore from equations (7) and (9),

$$D_c = \frac{3\pi\eta_0 U_c d_s exp(kc/1-Qc)}{1-c} \tag{10}$$

A consequence of the return flow effect as treated above is that the introduction of an additional sphere into a suspension will require the displacement of an equal volume of suspension. The effective gravitational force is therefore determined by the density of the suspension, which is $c\sigma + (1-c)\rho$ and the effective gravitational force on the representative sphere is

$$F_c = (1-c)\ (\sigma-\rho)g\pi d_s^3/6 \tag{11}$$

(iii) The terminal velocity for equilibrium settling of a suspension of spheres is obtained on equating (10) with (11)

$$U_c = \frac{(\sigma-\rho)gd_s^2}{18\eta_0}.\ (1-c)^2\ exp(-kc/1-Qc) \tag{12}$$

or the relative rate of settling, compared with the terminal velocity U_o

$$U_r = U_c/U_o = (1-c)^2 \, exp(-kc/1-Qc) \tag{13}$$

In equations (12) and (13) the shape factor k and interaction constant Q have the values 5/2 and 39/64 respectively for suspensions of equal sized spheres.

This result may be compared with that obtained by Burgers. At very low concentrations, equation (13) becomes

$$U_c \simeq 1-4\tfrac{1}{2}c \tag{14}$$

Comparison with equation (5)

$$U_r \simeq 1-6\tfrac{7}{8}c$$

shows, as expected, a faster rate of settling for an equilibrium arrangement of spheres than for a fixed random arrangement. (Regular arrangements in Burgers' analysis settle even more slowly than a random arrangement of the same concentration.) No attempt has been made here to compare the expressions (14) or (5) with experiment; the experimental data examined in Section 4 in support of equation (13) does not extend to concentrations less than 5%.

When there are forces preventing the establishment of an equilibrium arrangement for examples in flocculated systems (Section 3.3) or in packed beds (Section 5), there will be according to the discussion in Section 2, a reduction in rate of settling below that indicated by equation (13), in deriving which the assumption has been made that the direction of the Stokes' field of flow of the representative sphere is parallel to the direction of settling. When there are interparticle forces maintaining an initial particle arrangement different from the equilibrium arrangement, the directions of the fields of flow of individual particles will depend on the arrangement. For the more concentrated systems, the reduction in relative velocity may be estimated by taking an average value of the directions of the individual fields (see Section 3.3).

The discussion has so far been restricted to suspensions of spherical particles all of the same size. The effect of particle shape is discussed below (Section 3.2). The question of the settling of a system containing a range of sizes presents a number of difficult problems. However, it is well known that there is in fact negligible differential settling when the range of sizes present is not too large, for example, less than 5 or 6 to 1. This suggests that the relation (13) which is based on the assumption of an equilibrium arrangement giving no differential settling, may quite well apply in the case of suspension having a moderate range of particle sizes. The major difficulty will then lie in the determination of the equivalent settling rate U_o at infinite dilution. It is suggested that the method proposed in the next section for defining the effective size of non-spherical particles could be employed to define that of unequal sized particles.

3.2 *Suspensions of non-spherical particles*

There are two aspects to consider in the case of suspensions on non-spherical particles: (i) the effect of the shape on the viscous drag for a single particle in an infinite fluid and (ii) the effect of the average shape on the

effective viscosity of the suspension. The disturbance produced in an infinite fluid by a non-spherical particle is approximately the same as for a sphere of suitably defined equivalent size and the interaction between the two effects will be small and is neglected in the following discussion.

(i) The viscous drag of a single particle of simple geometrical shape, (ellipsoids, cylinders, etc.) has been investigated rigorously (for example by Overbeck as reported by Davies[1]. However, the more important case is that of an irregular (rough) particle of more or less equal dimensions. Here the experiments of Eirich and co-workers [26] on the resistance to translation of porous or indented spheres show the liquid within the spheres to be carried along with them ; the same conclusion is reached from the calculations of Smoluchowski[6] and Burgers[7] on the motion of clouds of particles. It is thus reasonable to assume that the drag on a non-spherical particle can be estimated by replacing d_s in the Stokes' drag by an equivalent diameter. For this purpose it seems appropriate to choose the diameter Δ of a sphere having the same surface area as the particle. In the case of geometrical shapes also there is reasonable agreement between the drag estimated from the surface diameter and the more exact calculations. If the volume of the irregular particle is expressed as $\pi\delta^3/6$, the terminal velocity is then

$$U_o = (\sigma - \rho)g\ \delta^3/18\eta_o\Delta$$

This is more conveniently written in terms of Wadell's[27] particle shape factor, the sphericity ψ, defined as the ratio of surface area of a sphere having the same volume as the particle to the actual surface area :

$$U_o = \psi^{\frac{1}{2}}(\sigma - \rho)g\ \delta^2/18\eta_o \qquad (15)$$

An unpublished statistical examination by the author of data reported by Pettyjohn and Christiansen[28] on the settling of isometric particles of simple geometrical shapes, confirms that equation (15) is an adequate expression for estimating the terminal velocity of a single non-spherical particle.

(ii) The question of the effect of particle shape on the effective viscosity is a more difficult problem. The *bulk* viscosity is very sensitive to departures from spherical shape. At low concentrations, the behaviour is given by Einstein's relation with a value of k depending upon the shape and on their average orientation. The basic calculations are due to Jeffrey[12] and values of k can be computed if the orientation of the particles is separately known ; this latter question has been much discussed without any final conclusions[26]. In the case of a settling suspension, all orientations might be supposed equally probable and Burgers'[29] numerical evaluation of Jeffrey's results for this case might be used. On the other hand, the major part of the increase in the value of k is a consequence of rotation, which is assumed inappreciable in sedimentation, and the shape factor for smooth particles may well be only slightly larger than the value 5/2 for spheres, except for markedly assymetrical particles.

In the case of irregular (rough) particles of more or less equal dimensions, the shape factor is, according to Eirich[26], determined chiefly by the outer outline, the fluid contained in indentations at the surface being effectively part of the particle, as mentioned above. This amounts to an

increase in concentration (affecting however only the viscosity effect). The same assumptions as above on the amount of included fluid lead to the conclusions that the shape factor k for an irregular particle will be

$$k = 5/2\psi^{-3/2} \qquad (16)$$

(iii) To sum up : for irregular isometric particles the rate of settling of a suspension may be estimated from equation (13) with U_o and k given by equations (15) and (16) respectively. For smooth particles U_o may be obtained approximately from equation (15) or more exactly from, for example Davies'[1] evaluation of Overbeck's analysis ; k on the other hand is not likely to be much greater than 5/2 except for unusual shapes, an upper limit being set by the value of k computed for the viscosity case.

3.3 *Flocculated Suspensions*

Since aggregated systems are generally speaking more important industrially than dispersed systems, it is worth while enquiring whether the present treatment can be applied, even in an approximate way. Some flocculated systems consist of a continuous network that does not settle at all or only very slowly at a decreasing rate. These are not considered here although the compaction of a floc is accompanied by the exusion of liquid, for example " bleeding " of cement, the process being essentially the same as flow through a packed bed. In other flocculated systems such as glass powders aggregated by a flocculating agent, there is a constant rate of settling preceding the falling rate period and the following discussion refers to this constant rate period. The essential feature is that a constraint is imposed on the possible spatial distribution and this results in a reduced rate of settling when the floc structure extends throughout the containing vessel. At the lower concentrations for example under 15—20%, the bonding forces may not be able to maintain a continuous structure and large clumps separated by fine channels then appear ; and at still lower concentrations the clumps form discrete units settling individually.

For sufficiently great dilutions the rate of settling in the latter case, might perhaps be estimated from Smoluchowski's[6] or Burgers'[8] calculations on the motion of clouds in an infinite fluid. The channelling stage in the intermediate concentration range is an example of capillary flow.

The rate of settling of the continuous floc structure filling in entire vessel at the higher concentrations can be estimated by modifying equation (13). As remarked above, allowance must be made for the fact that the direction of the fields of flow of individual particles are not parallel to the direction of settling. In a very approximate way, this can be done by taking an average value for the direction. By the same arguments as have been used by Fowler and Hertel[30] for flow through short capillaries forming a porous bed, the rate of settling will be reduced approximately by the average value of $\sin^2\phi, \phi$ being the angle between the normal to an element of surface of the particle and the direction of settling. This assumes an uniform arrangement of particles in three perpendicular directions, which will be justifiable for moderate to high concentrations. At lower concentrations a more detailed analysis would be required. For spheres (or any convex equidimensional particle $\overline{\sin^2 \phi} = \zeta = 2/3$ and the relative rate of settling

of a flocculated suspension (providing it fills the containing vessel) at sufficiently high concentrations is then approximately :

$$U_r = U_c/U_o = \zeta(1-c)^2 \exp(-kc/1-Qc) \qquad (17)$$

where for spheres and equidimensional irregular particles $\zeta = 2/3$.

It should be remarked that flocculated suspensions settle according to equation (17) more slowly than dispersed suspensions at high concentrations. At low concentrations, when the clumps are well separated, the settling rate will exceed that for a dispersed system and even exceed the Stokes' velocity of the constituent particles. Thus to secure the most rapid sedimentation at high concentrations it is necessary to add a dispersing agent, whilst at low concentrations flocculation will be more effective.

4. EXPERIMENTAL EVIDENCE

The literature contains very few observations on the rate of settling of concentrated suspensions, the most extensive being due to Steinour[31]. The examination of the validity of equation (13) is based largely on Steinour's data although a few other measurements have been used. In analysing his data, Steinour argued by analogy from Stokes' law that $U_c = (1-c)^2 f(c)$ and determined empirically the form of $f(c)$ from data on the settling of dispersed suspensions of spheres.

In order to obtain a convenient form for analysing the data, expression (13) is re-arranged :

$$\log_e U_c/(1-c)^2 = -kc/1-Qc + \log_e U_o \qquad (18)$$

$$\text{or} \quad y = bx + a$$

$$\text{where } y = \log_{10} U_c/(1-c)^2 \quad x = c/1-Qc$$

A plot of y against x should give a straight line of slope $b = -k/\log_e 10$ and intercept $a = \log_{10} U_o$. (The value of the interaction constant $Q = 39/64$ is assumed correct ; it enters only into terms involving higher powers of the concentration than the first.) For suspensions of spheres (whether dispersed or flocculated), the slope should be $b = -2{\cdot}5/\log_e 10 = -1{\cdot}09$. In the case of suspensions of non-spherical particles, no measurements are available of the particle shape and no estimate can be made of the slope. All that can be said is that it should not be less than $-1{\cdot}09$. As a rough check however, we may compute from the observed slope the value of the sphericity ψ according to equation (16), and compare the order of the result with values of ψ obtained by direct measurement on similar materials.

For dispersed suspensions of spherical or non-spherical particles, the observed intercept should have the value $a = \log_{10} U_o$ or in the case of *concentrated* flocculated suspensions, $a = \log_{10} \zeta U_o$ with $\zeta = 2/3$. For one set of Steinour's data (tapiocas pheres) an independently measured value of U_o is available and this provides a reliable value of $\log_{10} U_o$ for comparison with the intercept. Unfortunately in all other cases, the terminal velocities have to be estimated from reported particle sizes. Except for one sample (Emery E) obtained by sieving, for which a reliable estimate of size can be made, the reported sizes are somewhat uncertain. They were obtained either by air permeability measurements or by the A.S.T.M. Wagner

turbidimeter. In the first method the size is deduced from the rate of flow of air through a packed bed of powder under a given pressure gradient. It is rejected for the present purposes on two grounds ; firstly, there is for fine powders appreciable diffusion as well as bulk flow of the air and the exact nature and magnitude[32] of the correction required is uncertain ; secondly, the usual interpretation of the measurements gives, not the Stokes' diameter but the specific surface diameter d, (the size of sphere having the same ratio of volume to surface as the particle) and a shape factor is therefore required to determine the terminal velocity. The second method (Wagner turbidimeter) is more reliable and measures the Stokes' diameter ; the sedimentation of a very dilute suspension is observed by recording the changes in intensity of light transmitted through the suspension at a known depth. The method assumes that the ratio of scattering cross-section of particles in a light beam to their geometrical cross section is independent of the particle size ; this assumption although not generally true is probably more or less justified for the range of sizes examined by Steinour. The values of U_o have therefore been computed from particle sizes as measured in the Wagner turbidimeter. In general, the method will tend to overestimate the size, particularly if large particles are present.

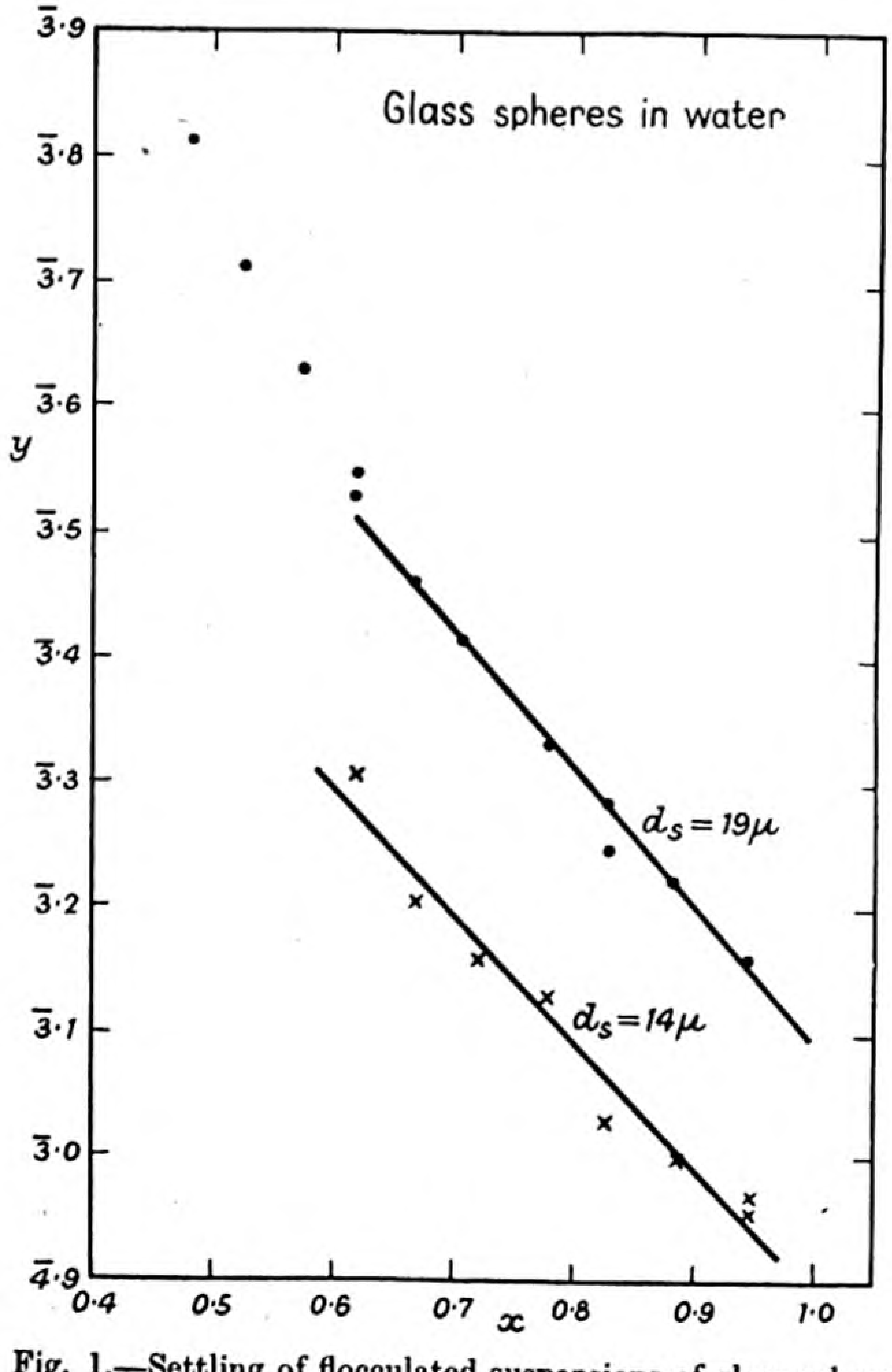

Fig. 1.—Settling of flocculated suspensions of glass spheres.

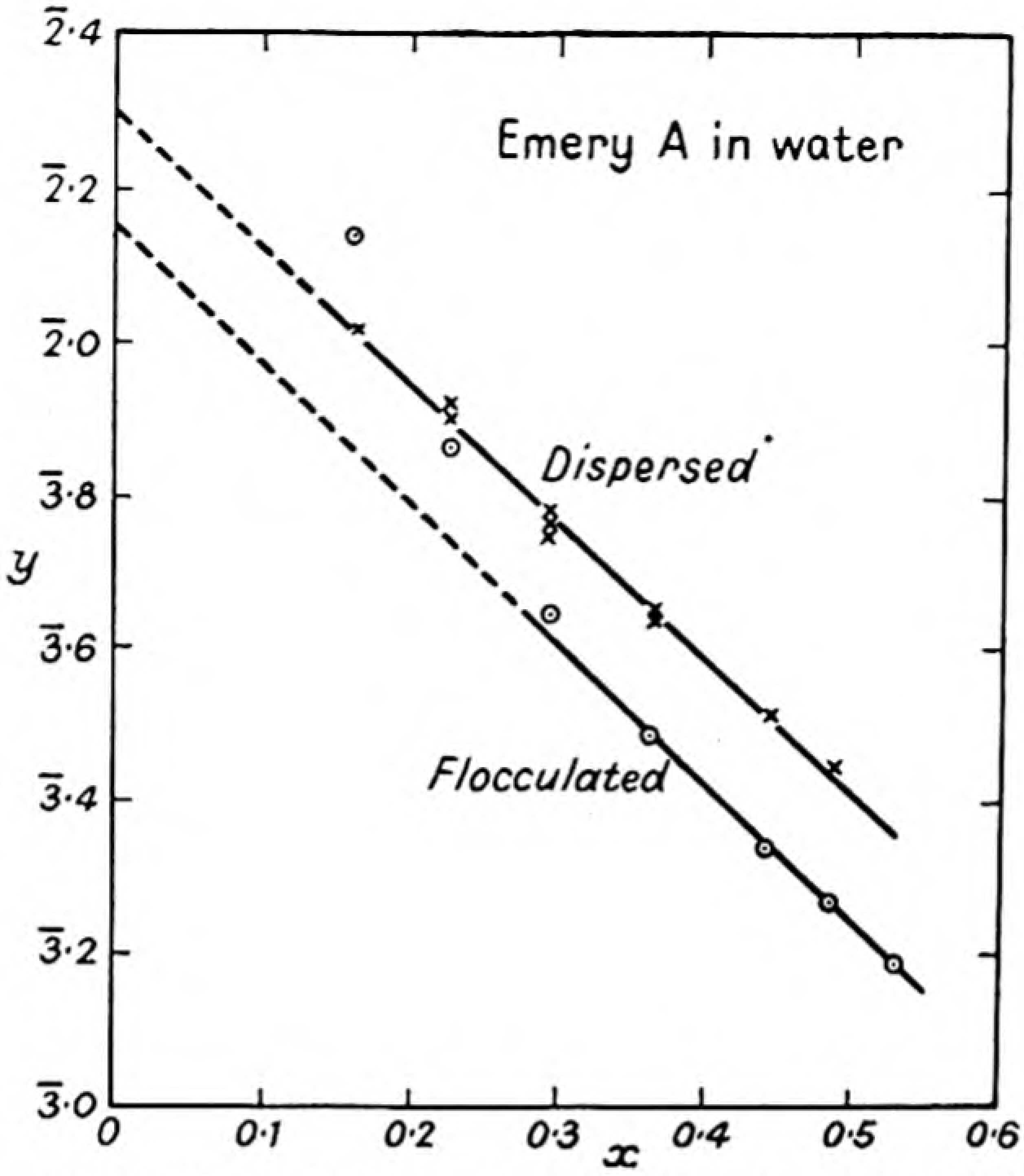

Fig. 2.—Settling of suspensions of non-spherical particles.

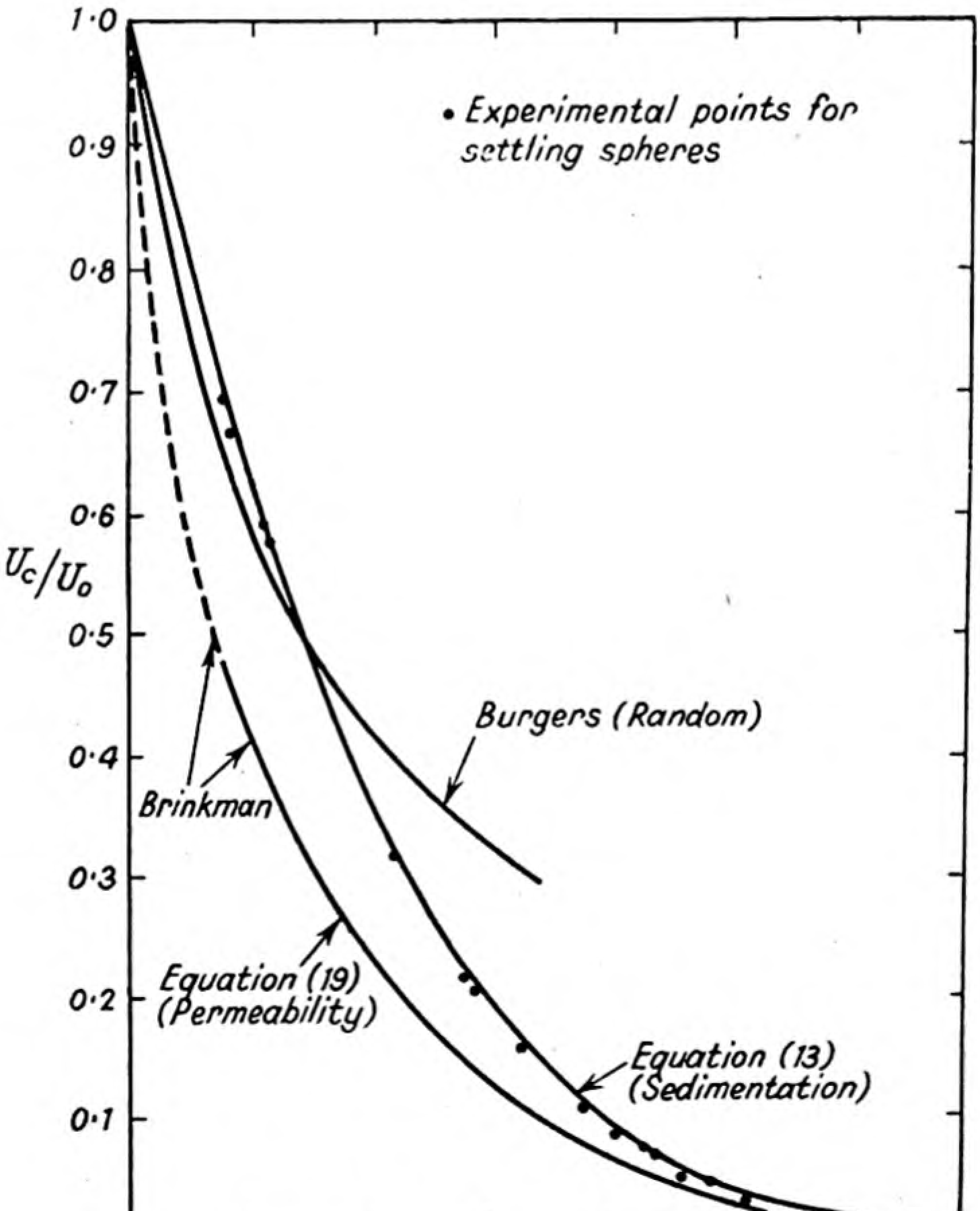

Fig. 3.—Relative velocity and concentration in sedimentation and permeability.

Repeated measurements of the rate of settling at the same concentration show some scatter and the data have been examined statistically. It is assumed that the concentration has been measured with a high order of accuracy and that the scatter arises from variations in the rate of settling (due either to experimental inaccuracies, or to an inherent lack of reproducibility in the process). The data have then been fitted by the linear regression of y on x to give the best estimates of the slope b and intercept a. A statistical test can be applied to determine whether the observed slope differs significantly from the expected value $-k/\log_e 10$. A similar test can be applied to the comparison of the observed intercept with $\log_{10} \zeta U_o$ in only one case, because the data are insufficiently extensive; in the other cases, nothing can be said either way. (See Note 2 to Table 1.)

4.1. *Results of analysis of Steinour's data :*

The results of the analysis of Steinour's data are given in Table 1 and some of the regression plots are shown in Figs. 1 and 2. The data (mean values only) for dispersed suspensions of spheres are plotted as relative velocity against concentration in Fig. 3, where the agreement with theory is seen to be satisfactory.

(*i*) *Values of slope b :* In all cases for suspensions of spheres (dispersed or flocculated—Table 1, Group A) the slope b is not significantly different from the theoretical value of $-1{\cdot}09$, corresponding to the shape factor $k = 2{\cdot}5$. The most extensive set of results (tapioca spheres) covers a very wide range of concentration from 7 to 50%. For the flocculated spheres, whilst the slope is also not different from $-1{\cdot}09$, the linear relation fails as expected, for the lower concentrations (Fig. 1).

In Group B of Table 1, the values of the sphericity ψ have been calculated from the observed slopes for suspensions of non-spherical particles. They agree with typical values for similar materials : Wadell[27] and Heywood[33] give $\psi = 0{\cdot}64$ to $0{\cdot}83$ for coal dust, crushed glass, sand, etc. This evidence cannot be regarded as more than suggestive. It is particularly satisfactory however, that the slopes for the dispersed and flocculated suspensions of Emery A should not differ significantly (Fig 2).

(*ii*) *Values of intercept a :* The free falling speed U_o is known reliably only for tapioca spheres and Emery E. In these cases and also for the flocculated suspensions of ground glass 4 and 5 the observed slope agrees very well with the expected value. In other cases the agreement between observed and calculated values is good on the whole. The largest discrepancy occurs for the flocculated suspensions of glass spheres 1 and 2 ; the values U_o are however very unreliable, being derived from air permeability measurements in these cases. The application of a correction for diffusion in the permeability measurement would reduce the discrepancy. A particular point is that the observed intercepts for the dispersed and flocculated states of Emery A are $\bar{2}{\cdot}31$ and $\bar{2}{\cdot}16$ respectively, giving a ratio U_o (flocculated) to U_o (dispersed) of $0{\cdot}71$, which is in good agreement with the expected value $\zeta = 2/3$. This result is independent of the estimation of particle size and free falling speed.

TABLE 1. *Settling of Suspensions.*

Data of Steinour[31] and Whittington[22]

	System	Observed Values (Regression equation)					Expected Values		Sphericity ψ (calculated from coefficient b)	Reported Stokes' Diameter, d_s (micron)	Estimated Terminal Velocity, U_o (calculated from d_s (cm/sec.)	Range of Volume Concentration
		Slope (coefficient) b	Intercept (constant) a	Standard Error of coefficient	Standard Error of Residuals	Number of Degrees of Freedom	Slope $k/\log_e 10$	Intercept $\log_{10} \zeta U_o$				
	Group A.—Suspensions of Spheres											
Dispersed	Tapioca spheres	—1·12	$\bar{1}\cdot012$	0·0198	0·0272	44		$\bar{1}\cdot049$			0·112	0·07—0·50
	Glass spheres A	—1·16	$\bar{2}\cdot068$	0·0420	0·0158	11	—1·09	$\bar{2}\cdot18$	Nominally	13·3	0·0153	0·15—0·35
Flocculated	Glass spheres 1	—1·10	$\bar{2}\cdot196$	0·0388	0·00899	4		$\bar{2}\cdot34$	ψ = 1	20·1	0·0331	0·47—0·60
	Glass spheres 2	—1·01	$\bar{3}\cdot900$	0·0756	0·0247	6		$\bar{2}\cdot09$		15·0	0·0184	0·47—0·60
	Group B.—Suspensions of Non-spherical (Rough) Particles											
Dispersed	Emery A	—1·79	$\bar{2}\cdot307$	0·0399	0·0130	11		$\bar{2}\cdot39$	0.72	12·2	0 0248	0·15—0·38
	Emery B	—1·78	$\bar{2}\cdot099$	0·0212	0·00704	5		$\bar{2}\cdot18$	0.72	9·6	0·0152	0·15—0·40
	Emery E	—1·92	$\bar{2}\cdot883$	0·0404	0·00924	11	Particle	$\bar{2}\cdot86$	0.68	64	0·0730	0·25—0·48
Flocculated	Emery A	—1·83	$\bar{2}\cdot160$	0·0423	0·00602	4	shape	$\bar{2}\cdot22$	0.71	12·2	0·0248	0·25—0·40
	Ground glass 1	—1·46	$\bar{2}\cdot468$	0·0433	0·00941	5	not	$\bar{2}\cdot52$	0.82	22·9	0·0497	0·35—0·48
	Ground glass 2	—1·54	$\bar{2}\cdot405$	0·0969	0·0226	6	known	$\bar{2}\cdot48$	0.79	22·0	0·0454	0·32—0·48
	Ground glass 4	—1·66	$\bar{2}\cdot132$	0·0854	0·0293	9		$\bar{2}\cdot10$	0.75	14·2	0·0187	0·32—0·50
	Ground glass 5	—1·57	$\bar{3}\cdot675$	0·0858	0·0334	10		$\bar{3}\cdot68$	0.78	8·9	0·00817	0·30—0·50
	Group C.—Suspensions of Non-rigid Particles											
Dispersed	Red blood cells	—1·00	$\bar{4}\cdot072$	0·0439	0·0284	11	See text	$\bar{4}\cdot121$		5·2	$1\cdot32\times10^{-4}$	0·08—0·42

NOTE 1. The standard error of coefficient is a measure of the variability of the slope. (a) In *Group A* none of the slopes differs significantly (at a probability level of 0.05) from the expected slope. (b) The observed slopes for Emery A dispersed and flocculated do not differ significantly; the pooled slope is 1.80 with standard error 0.0320 on 17 degrees of freedom. (c) The observed slopes for ground glass 1, 2, 4, 5 do not differ significantly. The pooled slope is —1.58 with standard error 0.0406 on 36 degrees of freedom; this gives a value of ψ of 0.79 with a standard error of (approximately) 0.013 also on 36 degrees of freedom.

NOTE 2. The residual standard error is a measure of the scatter about the regression line. To test whether the observed intercept differs significantly from the expected value, it is necessary first to test the linearity of the regression. The only sufficiently extensive data is that for tapioca spheres and in this case there is no significant difference between observed and expected intercepts. If the *assumption* is made that the regression is linear in the other cases also, then there is no significant difference between the observed and expected intercepts, except in the cases of the flocculated glass spheres 1 and 2 for which the reported particle sizes are in any case very uncertain.

NOTE 3. The reported Stokes' diameters are measurements by the Wagner turbidimeter except for (a) glass spheres 1 and 2, for which only air permeability measurements are available and (b) Emery E, which was prepared by sieving for which the Stokes' diameter was computed by using the observed value of the sphericity.

4.2. *Other data on sedimentation of spheres*

Some measurements on suspensions of spheres are reported by Camp[34] and by Lewis, Gilliland and Bauer[35]. Unfortunately, the spheres were of large diameter and the Reynolds numbers $Re_o = \rho d_s U_o/\eta_o$ at infinite dilution exceeds the limiting Reynolds number ($Re_o < 0{\cdot}1$) for Stokes' settling. However in a suspension, the presence of other spheres would tend to damp out inertia terms and equation (13) might be expected to apply at Reynolds numbers exceeding 0·1, at least for the more concentrated suspensions.

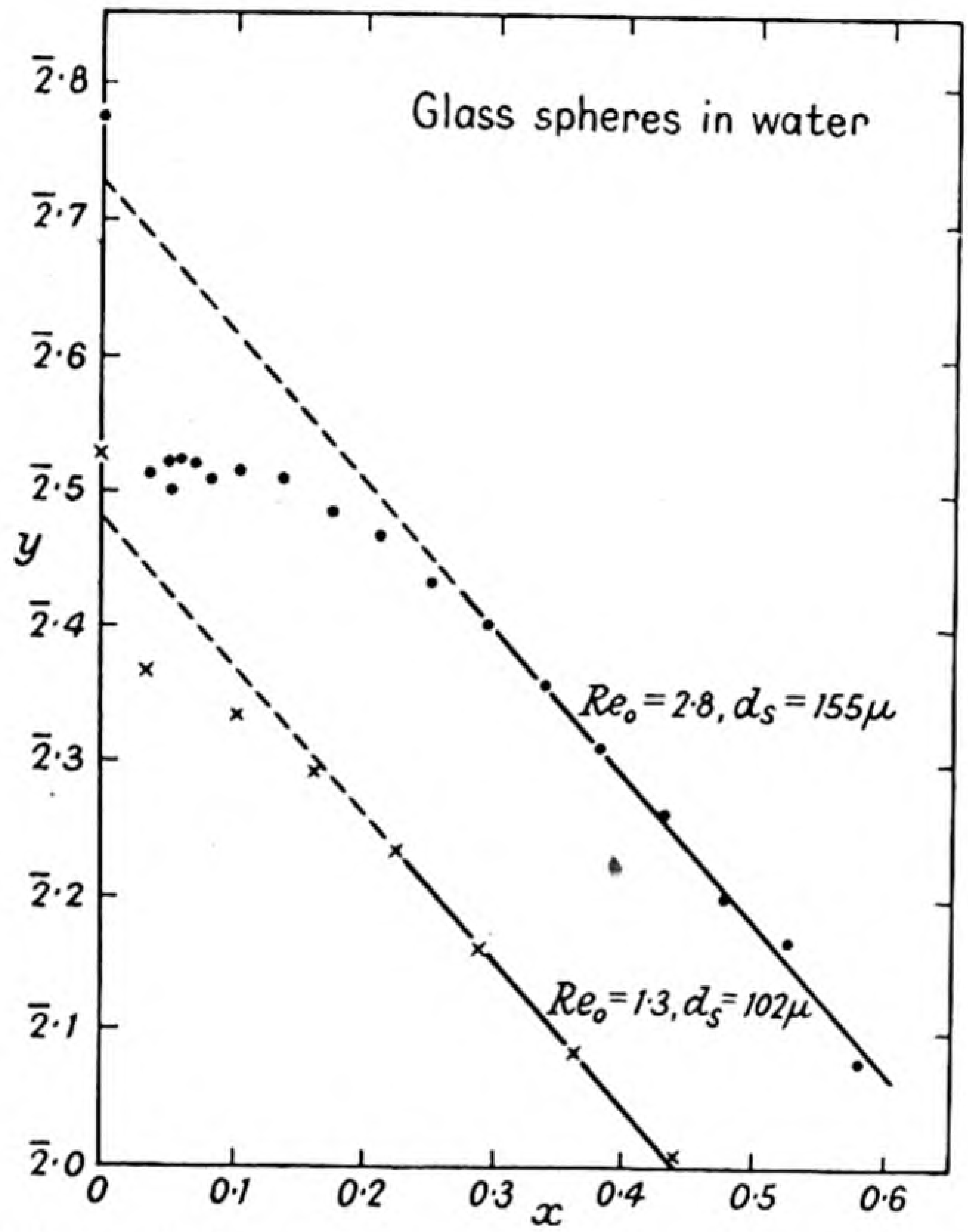

Fig. 4.—Settling of dispersed suspensions of spheres outside Stokes' range.

The data of Lewis, Gilliland and Bauer[35] are therefore plotted in Fig. 4 in the form of equation (18). There is good agreement with theory. The slope corresponds to $k = 2{\cdot}5$ and the free falling speed (corrected[1] for departure from Stokes' law) calculated from the reported average diameter, is comparable with the extrapolated value. The departure from the linear relation occurs at a higher concentration ($c = 0{\cdot}23$) for the larger spheres as would be expected. The measurements of Camp[34] (not given here) for $Re_o = 0{\cdot}6$ agree down to 10% concentration.

4.3. *Non-rigid Particles (red blood cells).*

Some data of Whittington's[22] on the sedimentation of red blood cells dispersed in normal saline with sodium citrate have also been examined (Group C of Table 1 and Fig. 5). There is good agreement with theory over

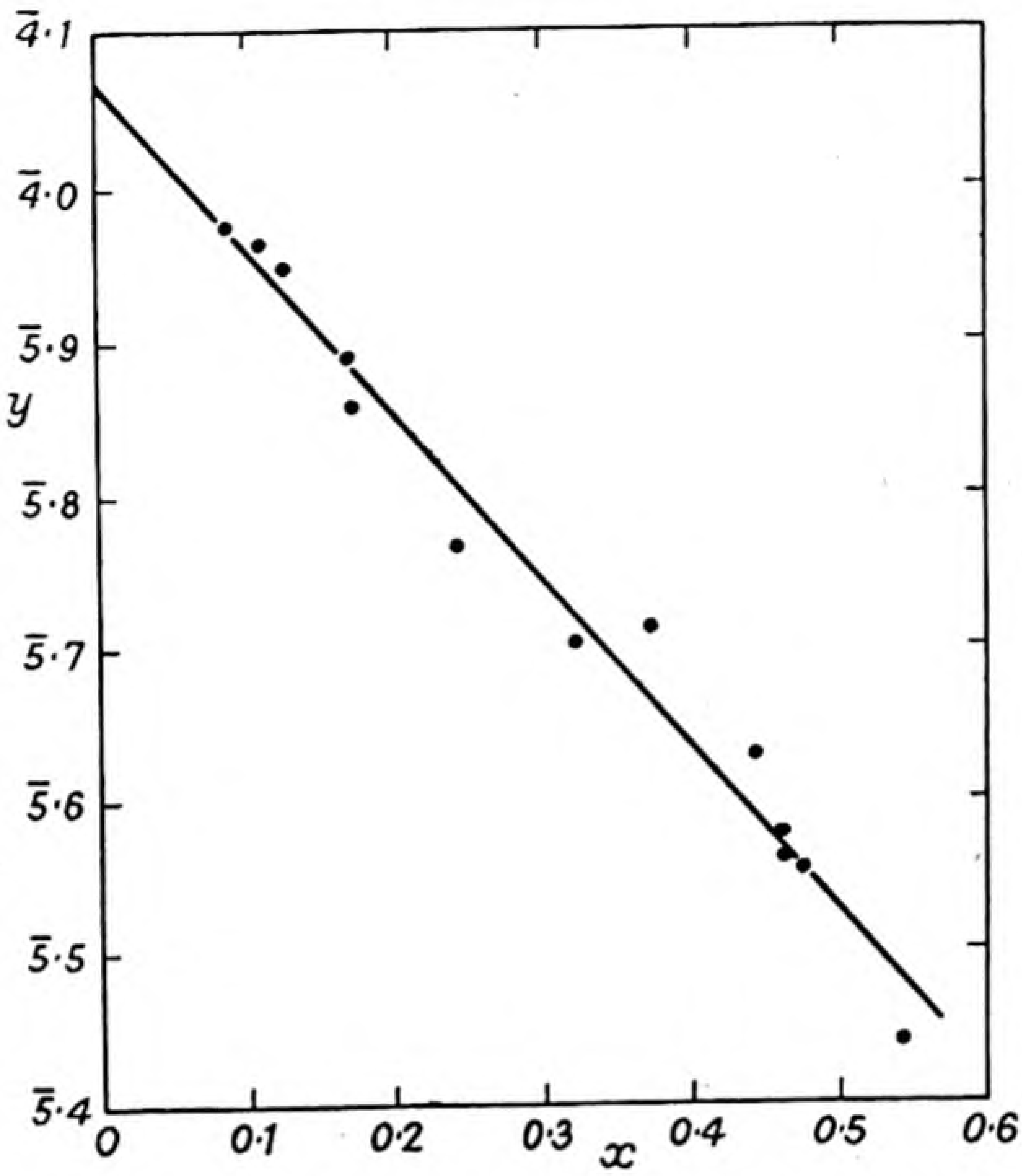

Fig. 5.—Settling of dispersed suspensions of red blood cells.

the whole 8—42% range of concentration. The slope is —1·09 corresponding to $k = 2\cdot5$. Blood cells are smooth bi-concave disks of thickness about 1/6 the diameter and are highly flexible. As discussed in Section 3.2, no substantial increase above $k = 2\cdot5$ would be expected for smooth particles and for these blood cells, it may be supposed that a slight increase due to their shape is balanced by a slight decrease(13) due to their lack of rigidity.

A value for the intercept $a = \log_{10} U_o$ has been obtained by computing the dimensions of the blood cells from the measured volume ($86\mu^3$) and reported axial ratio 6. The average viscous drag is obtained for an oblate ellipsoid in random orientation by use of Davies'(1) computations on the motion of ellipsoids. The free falling speed is then found on equating the viscous drag with the gravitational force computed from the particle volume. The observed constant agrees well with the calculated value.

5. FLOW THROUGH POROUS MEDIA

The flow of fluids through porous media, such as packed beds of equal spheres, is not an essentially different process from the settling of a suspension. The driving force is an externally applied pressure gradient instead of the weight of the particles. The spatial arrangement of the particles is however fixed and it is necessary to include an orientation factor ζ as for flocculated suspensions. It may be shown that equation (13) then becomes :

$$U_c = \frac{d_s^2 \Delta P}{18\eta_o L} \cdot \frac{\zeta(1-c)^2}{c\, exp(kc/1-Qc)} \tag{19}$$

where U_c is the velocity above the bed (volume flow per unit area of empty tube) and $\Delta P/L$ is the mean pressure gradient.

The factor ζ is not independent of concentration. At the higher concentrations the approximation $\zeta = \overline{\sin^2\phi} = 2/3$, is probably adequate. But as the concentration decreases, the fluid streamlines become more and more nearly parallel to the direction of bulk flow and ζ will tend to unity. For most packed systems however, the concentration range is 0·2 to 0·6 and the value $\zeta = 2/3$ should be reasonably satisfactory.

5.1. *Packings of Spheres*

The above expression may be compared with that developed by Kozeny[36] and others,[37, 38] which has been shown to agree reasonably well with experiment:

$$U_c = \frac{\zeta}{2\,k_o}\;\frac{(1-c)^3}{c^2}\;\frac{d^2\Delta P}{18\,\eta_o\,L} \tag{20}$$

This form is due to Sullivan[39]. The factor ζ has the same meaning as above, and k_o/ζ is an empirical constant having a value of about 5 over the concentration range 0·2 to 0·6[37, 40]. The particle size is defined here by the specific surface diameter, d. Equations (19) and (20) are identical if

$$k_o = \frac{(1-c)exp(kc/1-Qc)}{2c}\,.\left(\frac{d}{d_s}\right)^2 \tag{21}$$

Assuming for the moment that we are dealing with a system of spheres so that $d/d_s = 1$, we can compare k_o from equation (21) with the experimental constant $k_o/\zeta = 5$ (Table 2 and Fig. 6). It is seen that over the range 0·2 to 0·6 k_o is substantially constant at about 3. This compares well with experiment if ζ is assumed to be 2/3 over this range. Equation (21) may be regarded as a theoretical estimate of the Kozeny-Sullivan content k_o.

TABLE 2. *Comparisons of theoretical and experimental value of k_o.*

Concentration c	Theoretical values of k_o according to: Emersleben	Brinkman+	Equation (21)	Experimental values of k_o ‡
0.01	*	41	51	
0.02	*	22	26	
0.05	c.10	10.2	10.7	
0.075	7.3	7.3	7.5	
0.1	6.0	5.8	5.9	
0.15	4.45	4.3	4.3	3.3 ± 10% (0.15–0.6)
0.2	3.5	3.6	3.5	
0.3	2.05	2.9	2.9	
0.4	1.25	2.8	2.8	
0.5	Decreases	3.7	3.0	
0.6	rapidly	9.5	3.5	
0.7		negative	4.5	
0.74†		negative	5.1	

* The graphical solution of Emersleben's equation is very uncertain at low concentrations. The theory is not applicable at high concentrations.
\+ Assuming a value of $\zeta = 2/3$.
‡ Experimentally k_o is constant over range 0.15 to 0.6; at lower concentration k_o increases but experimental data is slight.
† Limiting concentration for a packing of equal spheres is 0.7405.

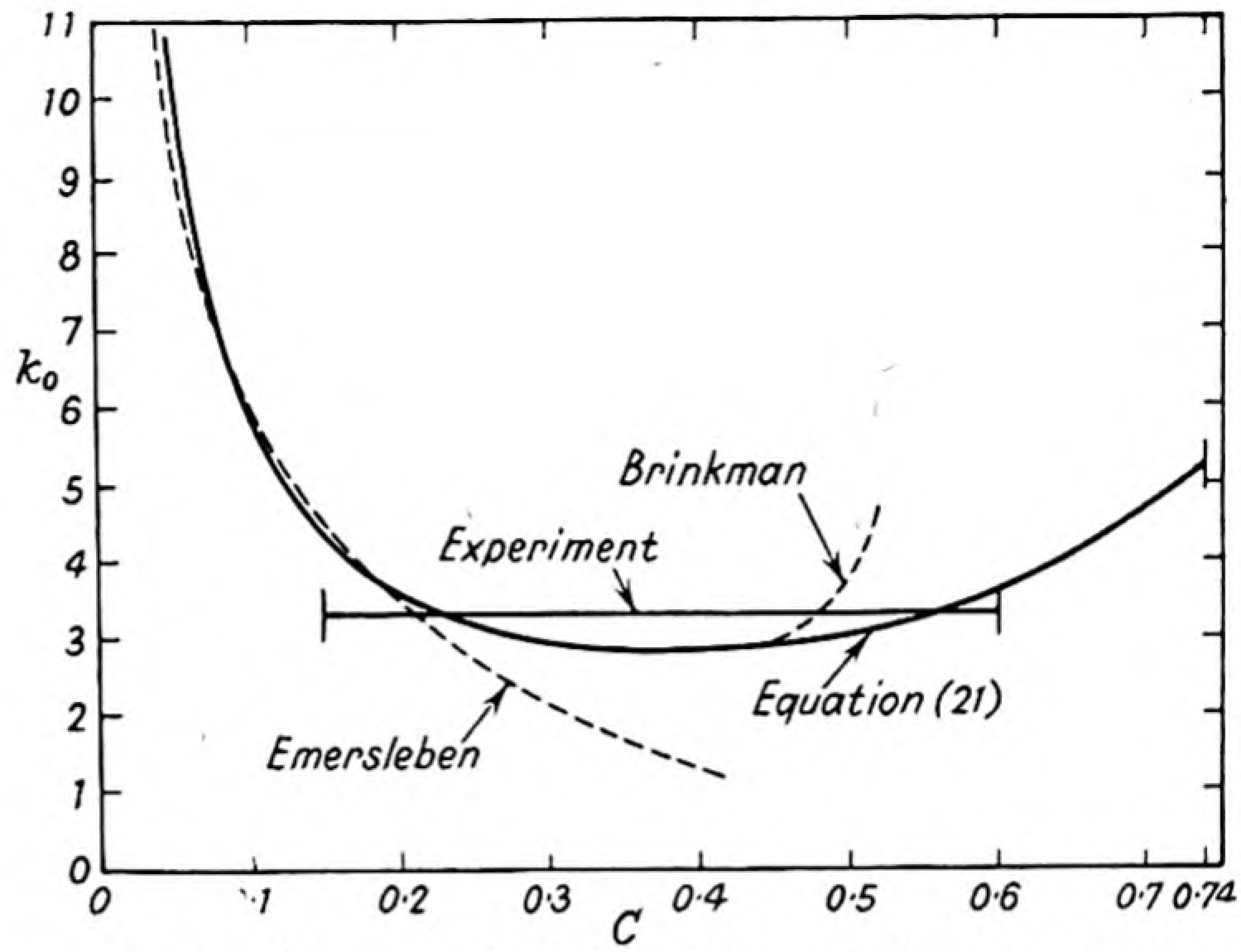

Fig. 6.—Theoretical values for Kozeny-Sullivan constant k_o.

Another theoretical estimate of k_o is provided by Emersleben[41] who solved the case of viscous flow through a packing of cylindrical rods whose axes are parallel to the flow, the rods being uniformly spaced in a square lattice arrangement. This analysis was applied by Sullivan[39] to a packed bed. The values of k_o are given in Table 2 and plotted in Fig. 6. Over the range 0·05 to 0·2 there is very close agreement with k_o from (21). Emersleben's analysis refers only to very porous systems and the divergence above 0·2 is not surprising. Unfortunately, numerical evaluation of Emersleben's theory becomes extremely inaccurate at low concentrations and comparison cannot be made below 0·05. This comparison is independent of the variation of ζ with concentration.

A further comparison can be made. Brinkman[2, 42] and independently Debye and Bueche[43] have examined a model consisting of a sphere separated by an infinitely thin shell from a porous mass. The drag on the sphere is calculated by imagining the fluid to be subject to a uniform damping force of magnitude determined by the permeability of the porous mass; the value obtained is a function of the permeability and reduces to Stokes' solution for infinite permeability. The driving force is given by D'Arcy's law[37, 40] and, on equating with the drag, the permeability is eliminated giving

$$U_c = \frac{1}{4}\left[3 + \frac{4}{c} - 3\sqrt{\frac{8}{c} - 3}\right]\frac{d_s^2}{18}\;\frac{\Delta P}{\eta_0 L}$$

comparison with (20) gives

$$k_o = \frac{2\,\zeta(1-c)^3}{c^2}\cdot\left[3 + \frac{4}{c} - 3\sqrt{\frac{8}{c} - 3}\right] \tag{22}$$

To compare k_o from equation (22) with that from (21) a value of ζ has to be assumed. If $\zeta = 2/3$ is taken, it is seen from Table 2 and Fig. 6 that there is excellent agreement from $c = 0{\cdot}45$ down to 0·075. Brinkman's theory is not applicable at high concentrations (equation (22) becomes

negative for $c > 0{\cdot}67$). Brinkman[44] later modified his model by permitting a variable thickness to the spherical shell to obtain exact agreement with equation (20) but this complication seems unnecessary since the experimental evidence indicates k_o to be only approximately constant.

To sum up: a comparison of equation (19) with the theories of Emersleben and Brinkman shows good agreement over the concentration range 0·075 to 0·6 if the approximation $\zeta = 2/3$ is accepted. At the higher concentrations equation (19) appears to provide a better representation of the experimental results than either Emersleben's theory, which fails above $c = 0{\cdot}2$, or Brinkman's, which departs seriously above $c = 0.45$.

5.2. *Packings of non-spherical particles*

The above comparison refers to systems of spheres. In practice the loosest packed bed of spheres has a concentration not much below $c = 0{\cdot}6$. Experimental results for packed systems at lower concentrations have been obtained by employing non-spherical particles. It was remarked above that k_o/ζ is experimentally constant when the particles size is defined by the specific surface diameter. It is necessary in order to justify the comparison of equation (19) with experiment, to enquire to what extent equation (21) is independent of particle shape. For non-spherical particles k from equation (16) is approximately $\psi^{-3/2}$ times k for spherical particles (providing ψ is not very different from unity), whilst the ratio $(d/ds)^2$ can be shown from (15) to be of the order of $\psi^{3/2}$. Thus k_o should not be much affected by particle shape.

5.3 *Sedimentation and Permeability*

From the discussion above the relative velocity for flow through a packed bed is about 2/3 of that for a dispersed settling suspension down to concentrations of about 10%. At lower concentrations, the difference becomes smaller. An examination of the process of "fluidisation" (suspension of particles by an upward fluid current) shows in fact that there is a re-arrangement of particles from the condition $\zeta = 2/3$ in the packed bed to the condition $\zeta = 1$ in the just fluidised bed. (The process of fluidisation is more complex than that of sedimentation, and the relation (13) cannot be expected to apply except in the very earliest stages of expansion of a fluidised bed). Precise quantitative treatment is limited by the absence of any tests at sufficiently low Reynolds numbers. Care is necessary therefore in applying theories developed for flow through packed beds to the settling of a suspension or to the process of fluidisation.

6. CONCLUSIONS

(i) The discussion of the settling of suspensions in Sections 2 and 3 leads to the following expression for the rate of settling of dispersed suspensions of spheres in which the particles have reached an equilibrium arrangement.

$$U_c = U_o\,(1-c)^2\,exp(-kc/1-Qc) \qquad (13)$$

where U_o is the terminal velocity of a single sphere at infinite dilution, c is the volume concentration and k and Q are numerical constants having the values 5/2 and 39/64 respectively.

For a flocculated suspension at sufficiently high concentration the same relation is applicable if multiplied by the factor $\zeta = 2/3$.

(ii) For non-spherical particles the terminal velocity U_o may be *estimated* (if a direct measurement is not available) from

$$U_o = \psi^{\frac{1}{2}} (\sigma - \rho) g \, \delta^2 / 18 \eta_o \tag{15}$$

where $(\sigma - \rho)$ is the density difference, η_o the viscosity of the pure fluid, g the gravitational constant, δ the volume diameter and ψ the sphericity. Also for non-spherical particles the shape factor k exceeds 5/2, probably only slightly for smooth particles, but appreciably for irregular (rough) particles when its value may be *estimated* from

$$k = 5/2 \psi^{-3/2} \tag{16}$$

With these substitutions the rate of settling of irregular particles can be obtained from the relation derived for spheres.

(iii) For viscous flow through packed beds, the expression :

$$U_c = \frac{d_s^2 \Delta P}{18 \eta_o L} \cdot \frac{\zeta \, (1-c)^2}{c \, exp(kc/1-Qc)} \tag{19}$$

where $\Delta P/L$ is the mean pressure gradient and d_s the Stokes' diameter, is shown to agree numerically with the theories of Emersleben and Brinkman for concentrations above 10% (voidage less than 90%), when the factor ζ has the value 2/3. The expression is also in agreement with the Kozeny semi-empirical relationship, which is known to give a good fit with experimental observations over the concentration range 20—60% (voidage 40—80%).

7. ACKNOWLEDGMENT

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8

Fluid Flow through Beds of Granular Material

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ABSTRACT. The general equation of fluid flow through granular beds is developed by dimensional analysis and the results of the Author's investigations of the relationships between the various dimensionless groups is discussed; comparison being made, where possible, with the results of other workers.

The equation produced has maximum generality and is believed to be of reasonable accuracy while an effort has been made to show the theoretical and practical limitations of the treatment.

Some interesting points upon which data is insufficient or lacking are mentioned and suggestions for future investigation are made.

NOTATION

A	=	surface area of material in bed
A_c	=	surface area of container
A_h	=	area of hole penetrating particle
A_m	=	average cross-sectional area of particle (Volume of particle ÷ length of hole)
D	=	diameter of bed tube
d	=	diameter of spherical particle
d_e	=	equivalent diameter of non-spherical particle
e	=	height of roughness of bed material
f	=	voidage of the bed
g	=	acceleration due to gravity
H	=	head of fluid lost across bed
h	=	depth of bed
K	=	a coefficient arising from non-sphericity of the particles
L_g	=	greatest dimension of a particle
L_l	=	least dimension of a particle
N	=	number of particles in a composite bed
P_0	=	pressure below bed
P_1	=	pressure above bed
P_x	=	pressure corresponding to standard conditions
R	=	gas constant
R_e	=	Reynolds' number $(vd\rho/\mu)$
T	=	absolute temperature of the gas
U	=	a dimensionless parameter defining size distribution
V_p	=	volume of particle
v	=	velocity of flow (=Volume rate of flow ÷ Cross-sectional area of bed tube)
V_x	=	velocity of flow measured at standard conditions
Z	=	dimensionless parameter defining shape of bed material
μ	=	absolute viscosity of fluid
ρ	=	density of fluid
ψ	=	resistance coefficient
ν	=	kinematic viscosity of fluid

ϕ denotes " a function of "

Dimensionless groups are enclosed within round brackets thus (h/d) except when the group is represented by a single symbol such as R_e or ψ.

INTRODUCTION

During the past century a steadily increasing interest has been taken in the laws governing the flow of fluids through beds of granular material, and, as a result of this interest combined with the increased demand for information brought about by advances in technical science, many theoretical and experimental investigations have been undertaken in order to establish the true relationship existing between the different variables. Study of the literature relating to the subject, however, indicates but little agreement between the findings of the different workers and also that the equations obtained are by no means of general application. This situation arises, in part at least, from the fact that, in general, each research has been undertaken with a definite but strictly limited object in view, and usually no effort was made to obtain a general equation applicable to all fluids at any rate of flow through a bed composed of particles of any shape.

The reason for this limitation of the range of conditions investigated is doubtless the large number of variables, arising from hydrodynamic considerations, from the characteristics of the bed material and from the statistical nature of the packing of the bed, which enter the problem. The experimental difficulties are further increased because many of the variables are closely inter-related : for example, particle surface area and the closeness of packing of the bed are both related to the particle shape and so the effect of particle surface area upon the flow resistance cannot be investigated independently of the effects of variation of the closeness of packing of the bed. Attempts to correlate the results of investigations of different workers are further complicated by the fact that different methods of presenting data are adopted ; which often renders the task extremely difficult or impossible.

Because of these difficulties and the general uncertainty of the state of knowledge of the subject the writer has endeavoured to obtain a general equation, by a combination of experiment and deduction from existing aero and hydrodynamic theory and from the experimental results of other workers, and it is upon these investigations that the present paper is largely based.

DERIVATION OF THE FLOW EQUATIONS BY DIMENSIONAL ANALYSIS

Previous experimental investigation by many workers has shown that the head of fluid, H, necessary to maintain a given nominal velocity of flow, v, through a bed of granular material, is dependent in some way upon the density, ρ, and the absolute viscosity, μ, of the fluid, upon the depth of bed h, the diameter d, of the particles of which the bed is composed, the diameter D, of the tube into which the bed is packed, the porosity f of the bed, and the gravitational constant g. It is also reasonable to assume that the resistance to flow, and thus H will depend upon the height e, of the surface roughening of the particles from which the bed is composed. Finally, it is evident that the resistance to flow will depend upon the shape of the particles and upon their size distribution, but by suitable choice of a system of definition, these two variables may be made dimensionless and will be represented by the two letters Z and U respectively.

If it is assumed that the dependent variable H is a power function of the independent variables, then

$$H = \phi\{v^{\alpha},h^{\beta},d^{\gamma},\rho^{\delta},D^{\xi},\mu^{\theta},g^{\tau},e^{\eta},f^{\lambda},Z^{\sigma},U^{\omega}\} \tag{1}$$

where ϕ denotes "a function of". If now the equivalents, in terms of M, L, and T, of the variables on each side of the equation are substituted and the indices equated, in accordance with the principles of dimensional analysis, no difficulty will be experienced in obtaining the equation of flow for an incompressible fluid

$$\left(\frac{H}{d}\right) = \phi\left\{\left(\frac{vd\rho}{\mu}\right)^{\theta},\left(\frac{dg}{v^2}\right)^{\tau},\left(\frac{h}{d}\right)^{\beta},\left(\frac{D}{d}\right)^{\xi},\left(\frac{e}{d}\right)^{\eta},f^{\lambda},Z^{\sigma},U^{\omega}\right\} \tag{2}$$

In order to limit the number of variables for investigation, the group (e/d) may be eliminated by the use of particles having a smooth surface finish ; the dimensionless parameter U, by use of material having but a small size range, and in the early stages of the investigation, the dimensionless parameter, Z, was eliminated by the use of spherical material only.

With these simplifications the equation becomes

$$\left(\frac{H}{d}\right) = \phi\left\{\left(\frac{vd\rho}{\mu}\right)^{\theta},\left(\frac{dg}{v^2}\right)^{\tau},\left(\frac{H}{d}\right)^{\beta},\left(\frac{D}{d}\right)^{\xi},f^{\lambda}\right\}$$

and this may be written in an even more convenient form

$$\left(\frac{H}{d}\right) = \psi,\ \phi_1\left(\frac{h}{d}\right),\ \phi_2\left(\frac{D}{d}\right),\ \phi_3\,(f),\ \phi_4\left(\frac{dg}{v^2}\right) \tag{3}$$

This equation is rigidly true for incompressible fluids and is approximately true for compressible fluids under small differences of pressure.

For the flow of compressible fluids suffering large volume changes it is necessary to include the forces to accelerate the fluid column since the velocity change within the bed, necessary to allow the widely different volumes of gas at the two ends of the bed to escape through the same effective area, is considerable.

It may easily be shown, Rose[1], that for isothermal flow and large differences of pressure equation (2) becomes

$$\left(\frac{P_1{}^2-P_0{}^2}{P_x{}^2}\right) = \left(\frac{v_x{}^2}{gRT}\right)\left\{2\psi,\phi_1\left(\frac{h}{d}\right),\phi_2\left(\frac{D}{d}\right),\phi_3(f),\phi_5\left(\frac{e}{d}\right),\phi_6(z),\phi_7(U)+\log_e\left(\frac{P_1}{P_0}\right)\right\} \tag{4}$$

By almost identical reasoning the equation for the polytropic flow of a gas through a granular bed may be deduced. Since, however, the equation is then not rigidly true, due to variation of the Reynolds number along the bed, and also since the polytropic condition of flow has received practically no experimental investigation, this form of flow equation will receive no consideration.

Reverting to equation (4), this may be simplified by eliminating, as before, $\left(\frac{e}{d}\right)$, U, and Z and by neglecting the logarithmic term.

$$\text{Thus} \left(\frac{P_1^2 - P_0^2}{P_x^2}\right) = 2.\left(\frac{v_x^2}{gRT}\right).\ \psi.\ \phi_2\left(\frac{D}{d}\right),\ \phi_3(f) \tag{5}$$

The validity of the rejection of the logarithmic term must now be considered and for this purpose the value of the two terms within the bracket of equation (4) is calculated, on the assumption of results to be discussed later, namely,

$$\phi_1\left(\frac{h}{d}\right) = \left(\frac{h}{d}\right);\ \phi_2\left(\frac{D}{d}\right) = 1$$

and that $\left(\frac{e}{d}\right)$, Z, and U are eliminated, for various possible bed conditions and the error due to the omission of the logarithmic term computed. These results are given in Table 1

TABLE 1.

Fluid	Particle Dia. Cm.	Velocity Cm/Sec.	$\left(\frac{h}{d}\right)$	f%	$2.\psi.\left(\frac{h}{d}\right), \times \phi_2\left(\frac{D}{d}\right), \phi_3(f)$	$log_e \frac{P_1}{P_0}$	% Error
Air	5·0	50,000	5·0	75	14	6·30	30
,,	,,	500	,,	,,	,,	0·005	0·05
,,	0·05	30,000	50	40	1260	3·7	0·03
,,	,,	0·03	,,	,,	16×10^6	2×10^{-6}	1×10^{-10}
,,	,,	30,000	,,	90	1·3	0·53	30
,,	,,	0·03	,,	,,	16,000	2×10^{-8}	1×10^{-12}

Inspection of this table shows that for the beds of normal dimensions and density of packing the logarithmic term is negligible and that it is mainly with beds of very open packing and high rates of flow that the term is likely to be of importance ; in fact the term is negligible with large pressure differences but important with very small ones which proves to be the opposite to that which would, on first consideration, be expected. Thus, equations (3) and (5) will prove adequate for the investigation of the problem of fluid flow through beds of granular materials. At this point it should perhaps be mentioned that, in the case of fluid flow through a granular bed the use of Dimensional Analysis does not make the equations obtained other than empirical, even though certain writers have implied the contrary.

Strictly, Dimensional Analysis demands Dynamical Similarity which, in turn, is dependent on Geometrical Similarity of the flow systems considered. However, even in the simple case of spheres packed to different voidages the flow passages are not Geometrically Similar so, even in this case, strictly, Dimenional Analysis is not valid and so any equation obtained must be empirical. Obviously, this argument is even more powerful when applied to the comparison of flow systems composed of particles of different shapes. However, even if the results are but empirical the method is an extremely powerful tool for preliminary examination of a very difficult problem.

THE RELATIONSHIP BETWEEN THE RESISTANCE AND FROUDE'S NUMBER

From theoretical consideration it is easily seen that the relationship between the resistance to flow offered by the bed and Froude's Number $\left(\frac{v^2}{dg}\right)$ should be of the form $\left(\frac{H}{d}\right) \propto \left(\frac{v^2}{dg}\right)$.

This has been tested experimentally and the results, shown in Fig. 1, confirm this conclusion. In this diagram, the points indicated by the same symbol on the different curves are calculated from the results of the same experiment as the different lines are in reality portions of a single curve, the

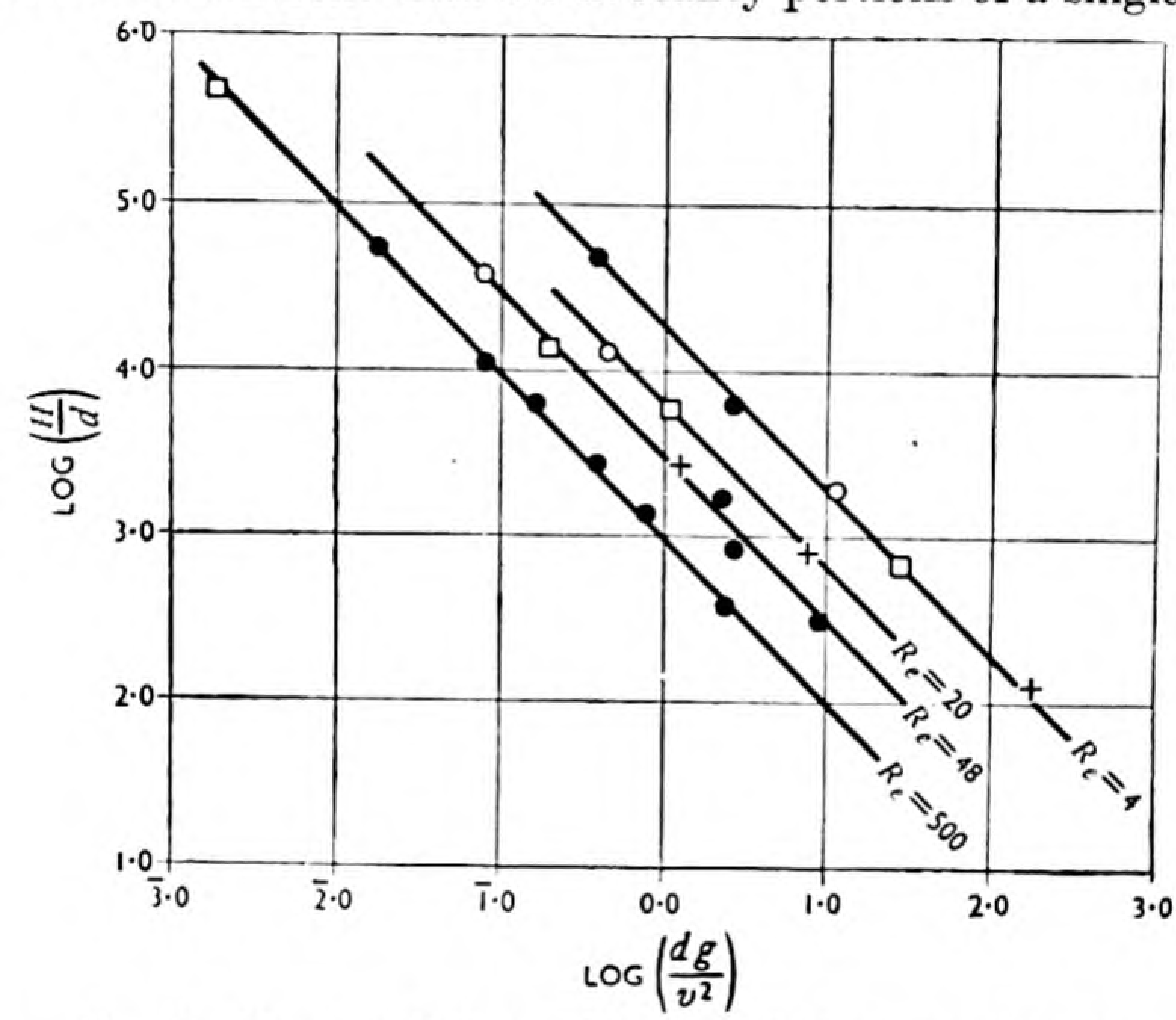

Fig. 1.—Curves relating resistance and the group (dg/v^2).

curve being broken into sections by reason of the different Reynolds number involved. The use of different values of Reynolds number extends the range which would otherwise be limited by experimental difficulties.

THE RELATIONSHIP BETWEEN HYDRAULIC RESISTANCE AND DEPTH OF BED

Examination of the literature shows a fairly general agreement that the resistance to flow is directly proportional to the depth of the bed. Suggestions have been made, however, that certain end-effects exist, these effects arising from energy losses due to sudden contraction of the fluid stream at entrance to, and expansion at exit from the bed.

In view of this uncertainty an investigation has been carried out to determine the relationship between the resistance to flow and the depth of bed, even though it seems highly improbable that bodies so nearly perfectly streamlined as spheres could form an expansion or contraction in the fluid stream which is " sudden " in the hydraulic sense.

Typical results of such test, upon an incompressible fluid, are given in Fig. 2 and it will be seen that the resistance is proportional to the depth of

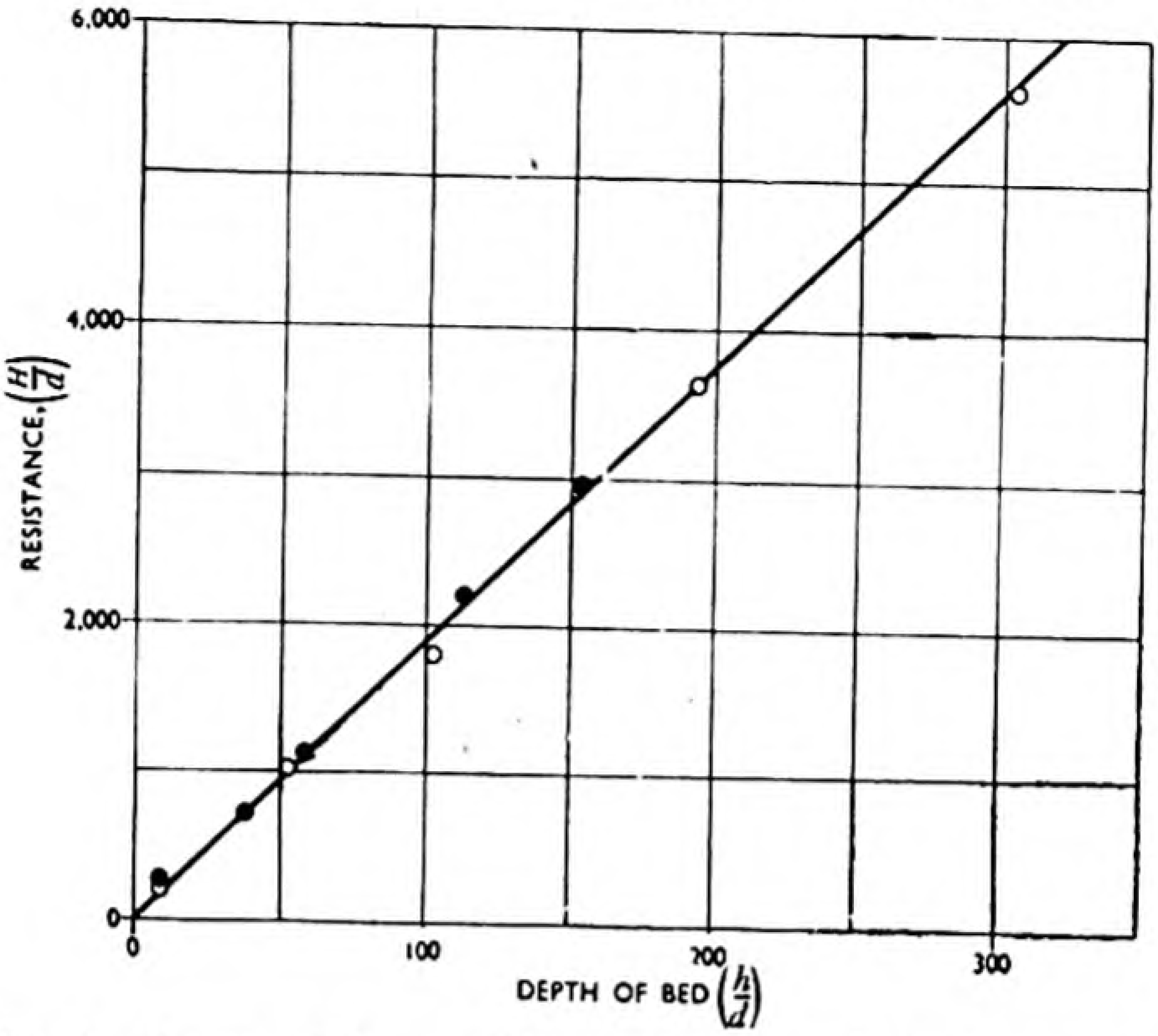

Fig. 2.—Curve relating resistance to flow and depth of bed.
● $(D/d) = 12.2$ O $(D/d) = 21.7$

bed, no " end-effect " being apparent. In this connexion, however, it is necessary to differentiate between the flow of an incompressible and a compressible fluid, for if the conditions are such that the logarithmic term in equation (4) is not negligible, then it is evident that the resistance to flow will not be directly proportional to the depth of bed. It also seems reasonable to expect that the linear relationship will fail to apply when the depth of bed is very small, say $(h/d) = 1$, since the flow will then more nearly approximate to the flow through a plate orifice ; for which a different law is applicable.

Since, however, neither of these conditions are likely to arise in practice, in general it may be concluded that, for an incompressible fluid, the resistance to flow is proportional to the depth of bed and for gases this is true to within the limits of practical measurement.

THE RELATIONSHIP BETWEEN RESISTANCE AND SIZE OF CONTAINER

A study of the work of a number of investigators indicates that the resistance offered to the flow of a fluid by a bed of granular material is not independent of the size of the tube into which the bed is packed, an observation supported by the appearance of the group (D/d) in the general flow equation. There appears to be no agreement, however, as to whether the effect of the walls is to increase or decrease the resistance to flow.

First consider the case of flow at small values of Reynolds' number. The flow will be streamline, and in consequence the energy losses will be due to the viscous forces in the fluid and so will be proportional to the velocity gradient and also to the area of surface upon which the viscous forces act. Thus, it would be expected that the resistance of the bed would be proportional to the sum of the area of the bed material and the area of the container wall.

Carman[2] using a research by Coulson[3] augmented by his own data, shows that a better fit of the experimental data is obtained if but half the area of the container is added to the area of the bed material, and explains this result by the fact that the surface of the particles are presented at an angle to the direction of fluid flow while the container wall is parallel to this direction and so offers relatively less resistance. On this basis Coulson[4] has suggested, for streamline flow, the relation

$$\phi_2\left(\frac{D}{d}\right) = \left(\frac{A + \frac{1}{2}A_c}{A}\right)^2. \tag{6}$$

However, an explanation of this nature will not suffice when flow at high Reynolds' numbers is considered, since the distribution of the particles in the bed, which influences the tortuosity of the bed, will have great effect on the inertia forces, which predominate at high values of Reynolds' number, whereas for streamline flow the particle distribution would appear to be relatively unimportant. Photographs of cross-sections of a bed show that the porosity of a bed is greater in the layers next to the walls of the container than in those nearer the middle of the bed. Furthermore, it is not unreasonable to believe that the passages in the layer of particles in contact with the walls of the vessel are less tortuous than those towards the interior of the bed, because the particles in the layer adjacent to the wall are placed at random in two degrees of freedom only, the wall providing one constraint.

That this annular passage offers relatively low resistance, and so controls the wall-effect, is confirmed by the Pitot-tube survey of the velocity across the exit surface of a bed, made by Saunders and Ford[5], in which it was found that the velocity was approximately constant across a bed except in a ring of about one particle thickness from the wall and in this ring the velocity was about 50% greater than at any other point above the bed.

It has been shown by the writer, Rose[6], that if it is now assumed that the outer layer of particles, to a depth of $d/2$ from the wall, has a voidage of f_1 and an inner core a voidage of f_2 then, on equating the free space in the bed, it is found that

$$f_m = \frac{\left(\frac{D}{d}-1\right)^2 f_2 + \left\{\left(\frac{D}{d}\right)^2 - \left(\frac{D}{d}-1\right)^2\right\} f_1}{\left(\frac{D}{d}\right)^2}$$

where f_m is the mean porosity of the bed, which is also the voidage determined by laboratory methods.

If the flow through the bed is considered to be made up of the flow Q_1 through the outer shell of voidage f_1 and a flow Q_2 through the inner core of voidage f_2 and also Q_m is the corresponding flow through a homogeneous bed of voidage f_m then

$$\frac{\text{Resistance of Composite bed}}{\text{Resistance of mean bed}} = \frac{Q_m}{Q_1 + Q_2}$$

$$= \frac{\left(\frac{D}{d}\right)^2 \bar{f}_m}{\left\{\left(\frac{D}{d}\right)^2 - \left(\frac{D}{d}-1\right)^2\right\}\bar{f}_1 + \left(\frac{D}{d}-1\right)^2 \bar{f}_2} \tag{7}$$

where $\bar{f}_m$, $\bar{f}_1$, and $\bar{f}_2$ are the reciprocals of the resistance—voidage relationship to be given later.

In view of the two probable forms of the wall-effect connection, the relation has been investigated experimentally by Rose and Rizk[7] and the results are given in Fig. 3.*

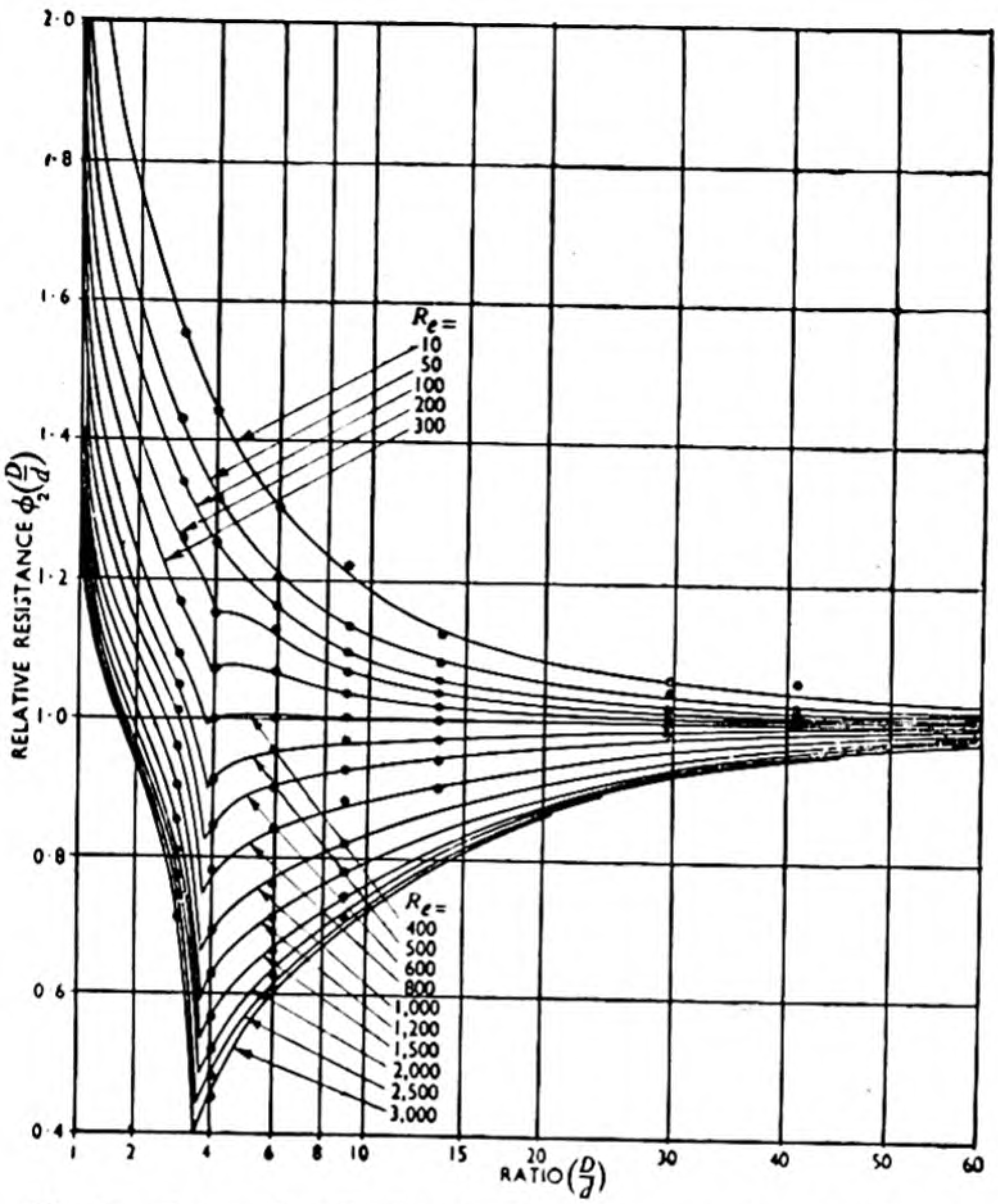

Fig. 3.—Curve of wall-effect for beds of spherical materials.

It will be noticed that for values of Reynolds number below about 400 the curves have the form corresponding to equation (6), as suggested by Coulson, and for Reynolds' Number above that value, of the form corresponding to equation (7), suggested by the present writer. $\{$This can easily be seen if the relative resistance is calculated for various values of (D/d) with $f_1 = 52\frac{1}{2}\%$ and $f_2 = 40\%\}$.

From these curves it is seen that the effect of the walls depends upon the Reynolds number but if (D/d) is greater than about 50 the wall effect is less than the normal uncertainties involved in problems of the type under consideration and so may be neglected.

From these results it follows that the interpretation of experimental data obtained at small values of (D/d) but with wide variations in R_e is subject to considerable uncertainty and it is possible that this is the cause of the diverse findings of various workers.

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*In Fig. 3, abscissae distances are plotted proportional to $\sqrt{\frac{D}{d}}$ in order to enlarge the scale for small values of (D/d).

It may be stated in conclusion that the correction for wall effect depends upon the Reynolds number defining the flow, but for values of (D/d) greater than about 50 the correction may, for most purposes, be neglected; that for Re=400 the wall-effect is practically non-existent and that the minimum of the curves, for $R_e > 400$, occurs at $(D/d) = 3{\cdot}5$ whatever the value of Reynolds number.

THE RELATIONSHIP BETWEEN RESISTANCE AND VOIDAGE OF THE BED

The relation between the resistance to flow and the voidage of the bed is one of the most difficult to investigate since the range of variation of voidage possible with any one material is small and if the range of experiment is increased by the use of materials of different shapes an additional variable, the shape effect, is at once introduced. In addition, the resistance varies as a high power of the voidage so small unavoidable errors in the determination of the voidage of the bed introduce large errors in the calculated value of the resistance of the bed.

Furthermore, it is not unlikely that the effect of voidage depends upon the value of the Reynolds' number defining the flow but the experimental difficulties of investigating this possible relationship are very great so evidence on this point is lacking.

Mathematical analyses carried out by various workers to determine the relation between resistance and voidage of the bed lead to expressions of widely differing form according to the basic assumptions made, while attempts to establish the relationship experimentally have been but little more successful. This is easily seen from Table 2, in which is given some of the many proposed functions.

TABLE 2.

Authority	Proposed Function
Slichter[8]	$f^{-3.3}$
Krüger[9]	$f^{-1.0}$
Zunker[10]	$(1-f)^2/f$
Terzaghi[11]	$[(1-f)^{\frac{1}{3}}/(f-0{\cdot}13)]^2$
Kozeny[12], Blake[13], Carman[14], Fair and Hatch[15]	$(1-f)^2/f^3$
Hulbert and Faben[16]	$(69{\cdot}43-f)$
Hatch[17], Mavis and Wilsey[18]	$f^{-6.0}$
Fehling[19], Kayser[20]	$f^{-4.0}$
Rapier and Duffield[21]	$1{\cdot}115(1-f)\left\{(1-f)^2+0{\cdot}018\right\}/f^{1.5}$

The present writer, Rose[22], has suggested a method, believed to be original, which appears to afford a means of determination of the relationship from the results of a number of simple experiments.

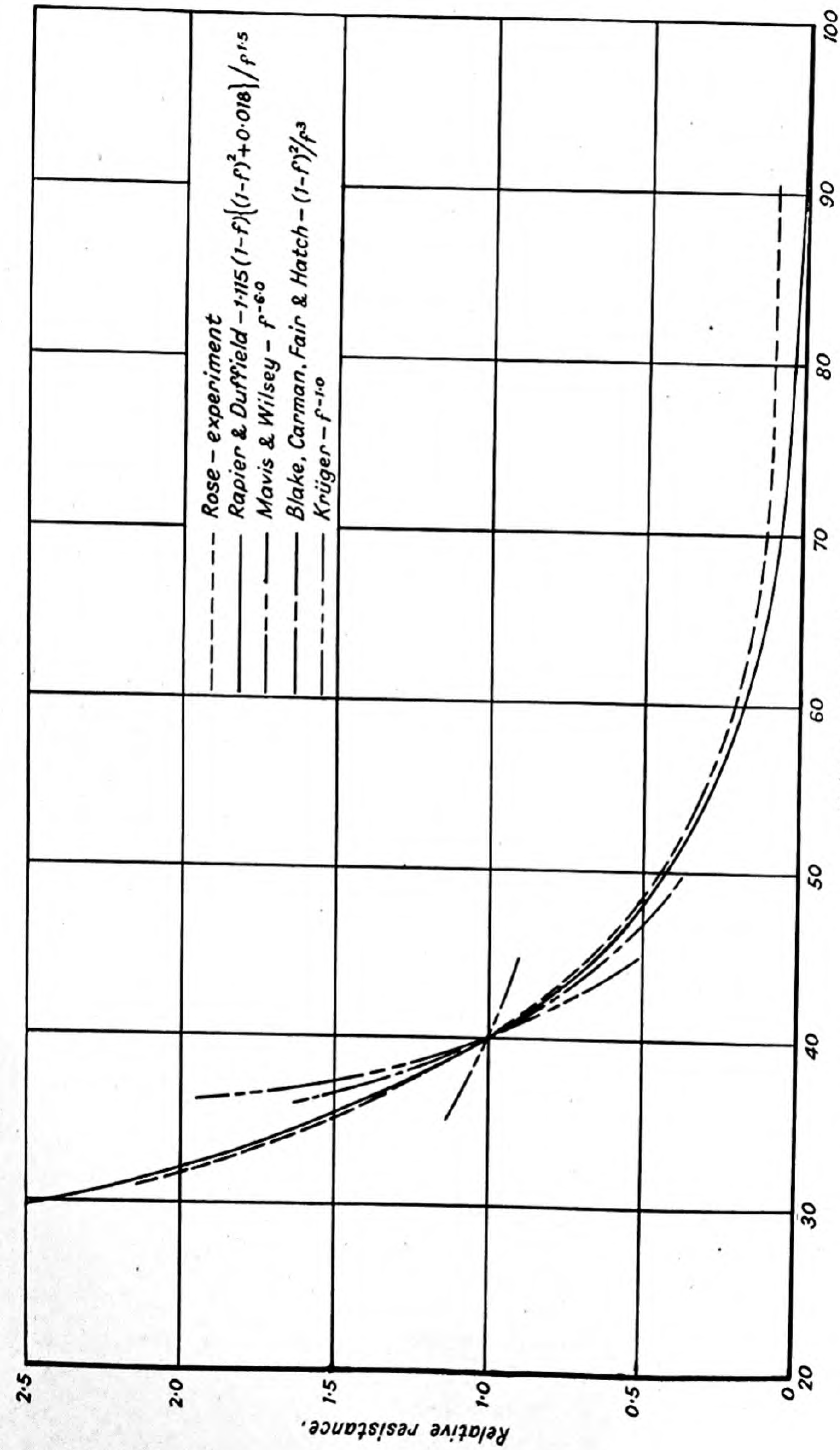

Fig. 4.—Comparison of results relating relative resistance and voidage.

The method is to take a batch of particles, chosen because they are known to pack some desired voidage, say 50%, and to measure the pressure drop across about a dozen beds of these particles ; the beds differing only by the slight differences of voidage obtainable by tapping the bed-tube ; the resulting range of voidage being, say 49% to 51%. The pressure drop, under standard conditions of flow, etc., is plotted against voidage, on logarithmic scales, and so the slope of the " best line " through the points gives this index n in the equation

$$P = Cf^n \tag{8}$$

This procedure is repeated for other materials, to give other values of voidage, and finally a curve of n against f is plotted. A curve derived from this is graphically integrated to give the relation between resistance and voidage required for the solution of practical problems. This method of derivation is mentioned at length since it has the advantage that the parameter C in equation (8), which depends upon particle shape, etc., does not enter the problem.

According to Rapier [23], Rapier and Duffield have algebraically integrated the present writer's results, evaluated the integration constants by modified Carman reasoning and by reference to Stokes' Law and so obtained the equation

$$\phi_3(f) = 1{\cdot}115\,(1-f)\left\{(1-f)^2 + 0{\cdot}018\right\}/f^{1{\cdot}5} \tag{9}$$

This equation is plotted in Fig. 4, together with the writer's graphically derived curve and the curves proposed by Carman and others. Coulson [24], finds, however, that most of the results of his experiments follow the curve of Carman much more closely than that of the present writer but it is also found that one series agrees closely with the curve of the present writer and so these results are inconclusive.

In view of the differences in the results by various workers much more investigation of the effect of this variable is required and in this connexion it is believed that the method originated by the author and extended by Rapier and Duffield should be of value, especially as it should lead to the theoretically correct result of Stoke's Law for a " bed " composed of a single particle. At present, however, equation (9) should be used with caution since the conditions of applicability have yet to be examined and it does not appear to give a reasonable value for voidages approaching 100%.

INVESTIGATION OF THE EFFECT OF PARTICLE SHAPE

The correlation of the results of flow experiments carried out upon beds of spherical and non-spherical material involves the problem of the diameter of the sphere which is hydrodynamically equivalent to the non-spherical particle.

Since, for streamline flow, the energy lost is due to shear forces and so is proportional to the area of the bed material in contact with the fluid and this, in turn, depends upon both the area and the volume of the particle, it seems reasonable to suppose that, at least for streamline flow, the required relation ship would involve both the surface and volume of the sphere and of the particle.

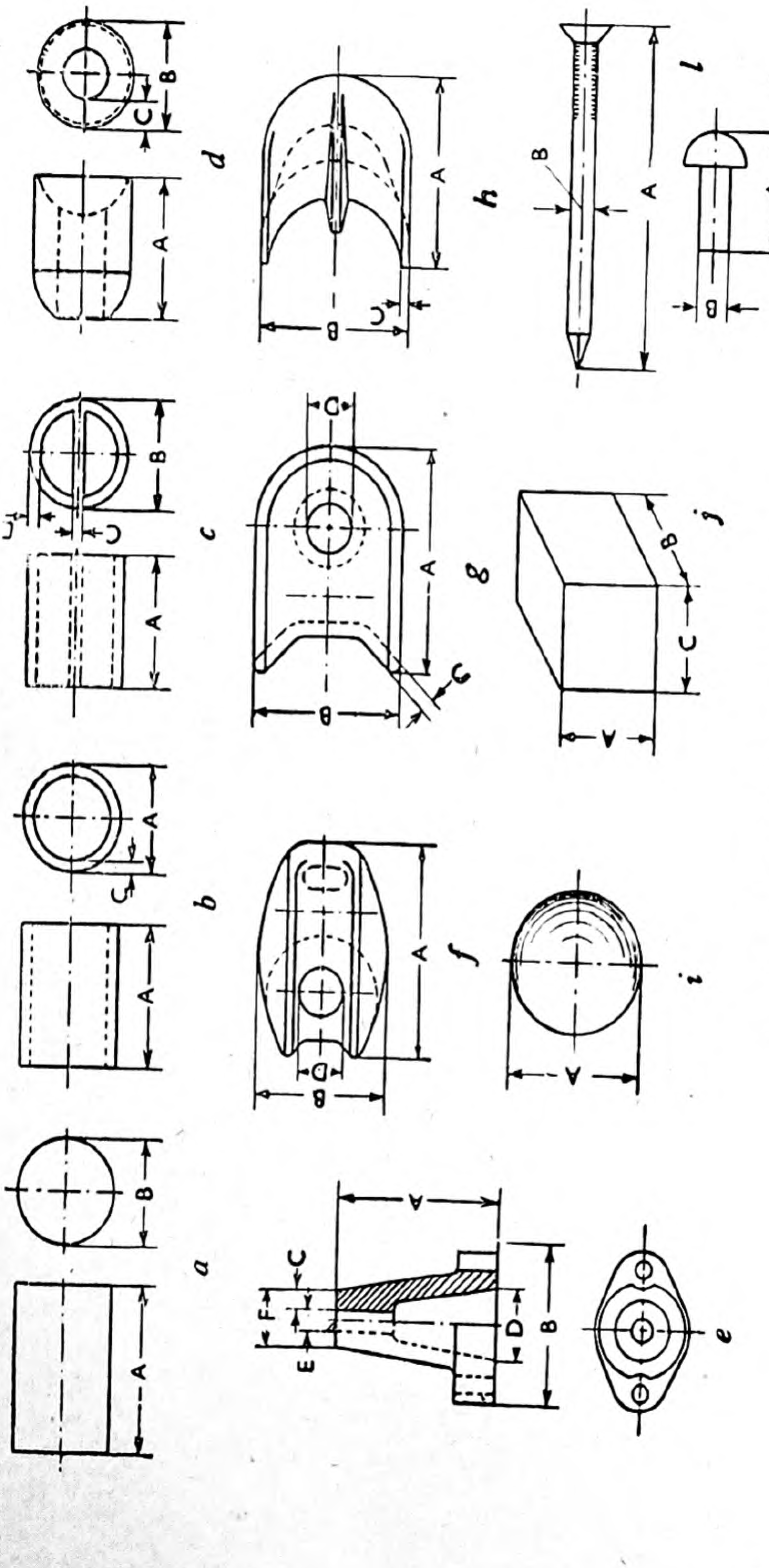

Fig. 5.—The forms of the non-spherical particles.

TABLE 3.

Packing	Material	Code symbol on Fig. 7	Code symbol on Fig. 5	Particle Dimensions A	B	C	D	E	Equivalent Diameter d_e cm	Container Diameter D cm	Ratio $\frac{D}{d}$	Porosity f%	Parameter— $\frac{L_g}{L_l}$	$\frac{A_h}{A_m}$	$\frac{L_g}{L_l}+\frac{A_h}{A_m}$
Spheres	Porcelain	1	i	1.12	1.12	—	—	—	1.12	11.5	10.25	37.3	1.0	0.0	1.0
,,	,,	2	i	2.10	3.1	—	—	—	3.10	11.5	3.7	48.0	1.0	0.0	1.0
Solid Cylinder	,,	3	a	1.42	1.28	—	—	—	1.32	11.5	8.7	38.5	1.11	0.0	1.11
,, ,,	,,	4	a	2.77	2.52	—	—	—	2.70	11.5	4.23	46.0	1.10	0.0	1.1
,, ,,	,,	5	a	,,	,,	—	—	—	,,	,,	4.23	46.5	1.10	0.0	1.1
,, ,,	Steel	6	a	2.00	0.32	—	—	—	0.52	3.8	7.4	50.0	5.3	0.0	5.3
,, ,,	,,	7	a	7.62	1.27	—	—	—	1.76	11.5	6.5	60.4	6.0	0.0	6.0
,, ,,	,,	8	a	8.00	2.54	—	—	—	3.30	11.5	3.46	59.0	3.15	0.0	3.15
Hollow Cylinders	Porcelain	9	b	0.34	0.32	0.09	—	—	0.19	3.8	20.0	52.1	1.05	0.24	1.29
,, ,,	,,	10	b	,,	,,	,,	—	—	,,	11.5	60.0	53.1	1.05	0.24	1.29
,, ,,	,,	11	b	0.66	0.65	0.15	—	—	0.37	3.8	10.4	58.0	1.02	0.40	1.42
,, ,,	,,	12	b	,,	,,	,,	—	—	,,	11.5	31.0	54.0	1.02	0.40	1.42
,, ,,	,,	13	b	0.96	0.96	0.17	—	—	0.44	3.8	8.6	67.5	1.0	0.66	1.66
,, ,,	,,	14	b	,,	,,	,,	—	—	0.44	11.5	25.7	60.0	1.0	0.66	1.66
,, ,,	,,	15	b	2.73	2.66	0.32	—	—	0.87	11.5	13.0	75.4	1.02	1.38	2.4
Thermocouple Beads	Porcelain	16	d	0.7	0.67	0.31	—	—	0.44	3.8	8.8	57.2	1.05	0.21	1.26
Lessing Rings	,,	17	c	2.55	2.52	0.325	0.32	—	0.87	11.5	13.1	66.2	1.01	0.78	1.79
Nails	Steel	18	k	0.89	0.31	—	—	—	0.41	3.8	9.33	48.0	2.87	0.0	2.87
,,	,,	19		,,	,,	—	—	—	0.41	11.5	28.00	47.4	2.87	0.0	2.87
,,	,,	20	l	1.78	0.19	—	—	—	0.27	3.8	14.2	63.2	6.6	0.0	6.6
,,	,,	21	l	2.65	0.203	—	—	—	0.276	3.8	13.9	76.2	9.7	0.0	9.7
,,	,,	22	l	2.67	0.19	—	—	—	0.278	11.5	41.2	71.0	13.8	0.0	13.8
,,	,,	23	l	5.00	0.26	—	—	—	0.40	,,	28.2	75.4	19.0	0.0	19.0
,,	,,	24	l	7.71	0.38	—	—	—	0.57	,,	19.8	79.4	20.3	0.0	20.3
Shell Insulators	Porcelain	25	g	7.0	5.0	0.7	1.0	—	6.14	,,	1.86	81.9	1.4	0.02	1.42
,, ,,	,,	26	g	,,	,,	,,	,,	—	,,	,,	1.86	82.2	1.4	0.02	1.42
Egg Insulators	,,	27	f	3.9	2.9	—	0.8	—	1.33	,,	8.8	63.2	1.35	0.052	1.40
,, ,,	,,	28	f	3.0	2.2	—	0.6	—	1.5	3.8	2.56	65.8	1.36	0.05	1.41
Saddles	,,	29	h	2.95	2.4	0.15	—	—	0.94	11.5	12.0	70.2	1.35	0.0	1.35
Stand-Off Insulators	,,	30	e	4.3	3.8	0.48	1.7	0.5	1.2	,,	9.55	64.7	1.13	0.20	1.33
Cubes	,,	31	j	2.68	2.55	2.5	—	—	2.62	,,	4.56	53.0	1.1	0.0	1.1

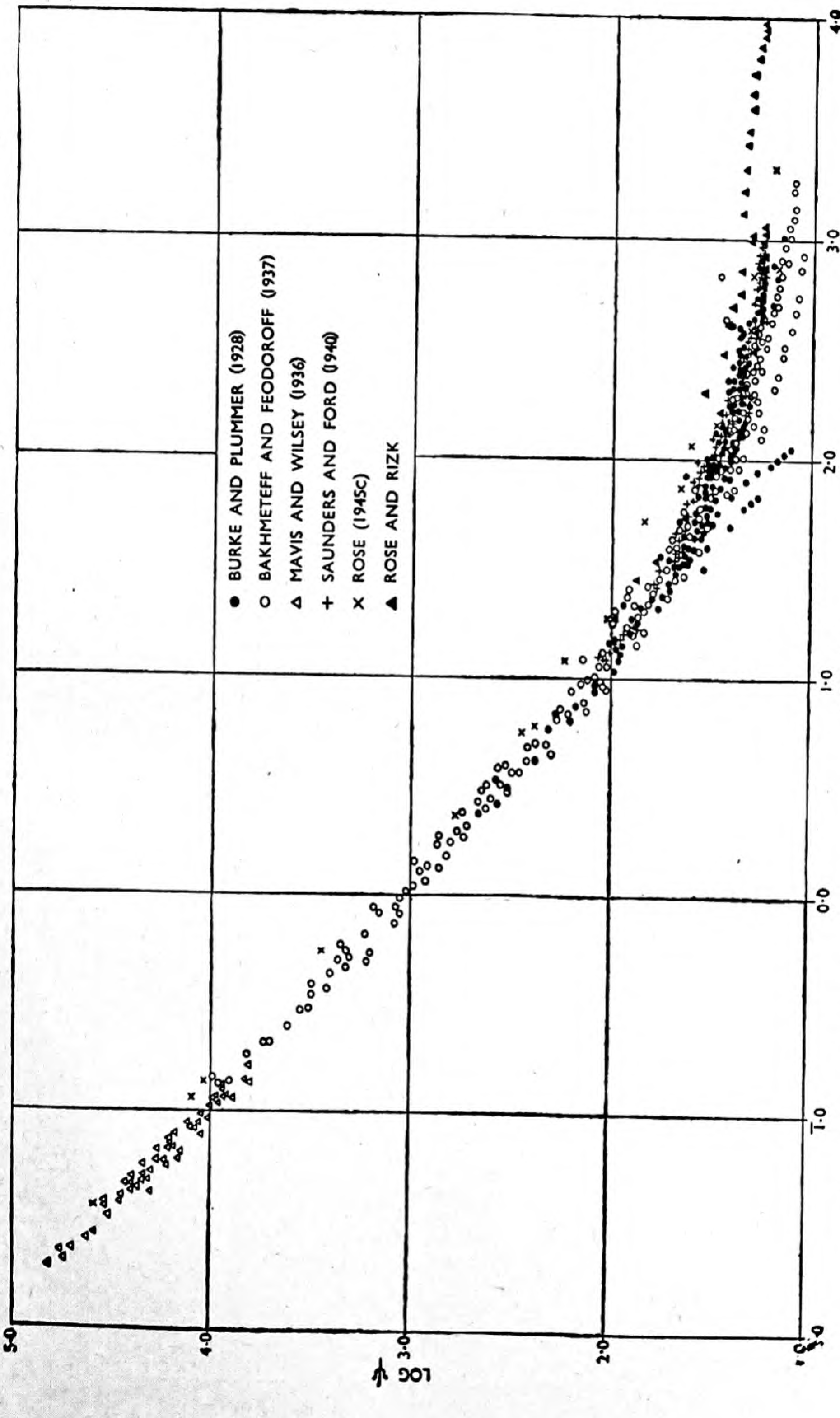

Fig. 6.—Relation between resistance coefficient and Reynolds number for beds of spherical materials.

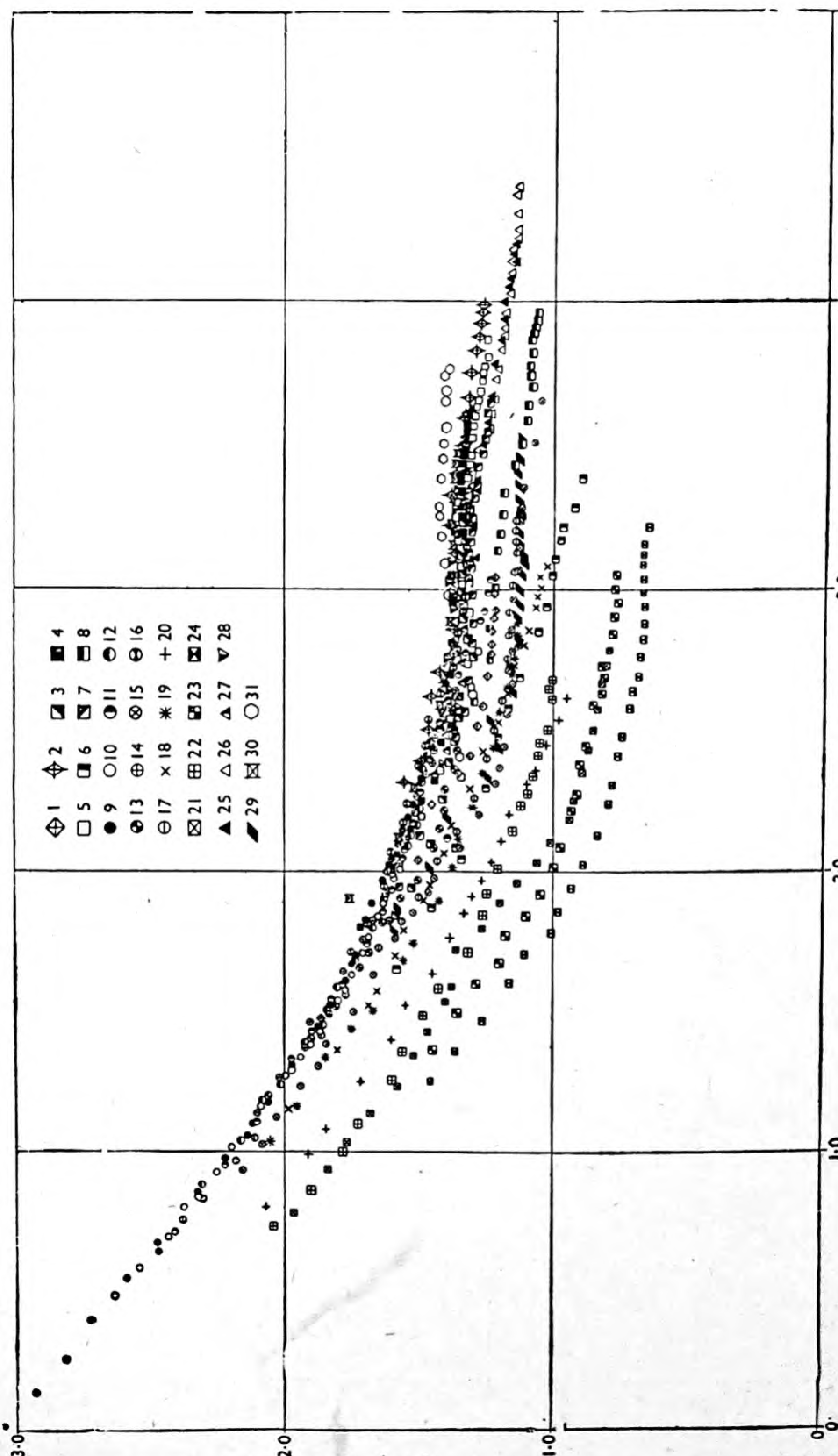

Fig. 7.—Relation between resistance coefficient and Reynolds number for beds of non-spherical materials.

A simple relationship including these variables is

$$\frac{\text{Area of equivalent sphere}}{\text{Volume of equivalent sphere}} = \frac{\text{Area of particle}}{\text{Volume of particle}}$$

or $$\frac{\pi d_e^2}{\frac{\pi}{6} d_e^3} = \frac{6}{d_e} = \frac{A}{V_p}.$$

or $$d_e = \frac{6\, V_p}{A} \tag{10}$$

where V_p and A are the volume and surface area respectively of the particle.

By means of this relationship the results of experiments carried out on beds composed of a wide range of particle shapes and sizes have been correlated with fair success. The forms of these materials are shown in Fig. 5 and their dimensions and the porosities of the beds formed from them are given in Table 3.

The results of experiments by many workers upon beds composed of spherical materials are plotted in Fig. 6 and the results of experiments carried out under the present writer's supervision upon the beds of non-spherical materials, detailed in Table 3, are plotted in Fig. 7 ; the relation given by equation (3) being used for this purpose.

Examination of the curves, Fig. 7, shows that the experimentally determined points fall upon a series of curves having the same general shape as that for spherical material but moved in a direction parallel to the vertical axis. Since these graphs are plotted on logarithmic scales this shift is equivalent to the multiplication of the Resistance Coefficient, ψ, by some factor K, the logarithm of which is denoted by the magnitude of the shift to the scale of the graph. It is possible that an even better fit would be obtained if a horizontal shift were included also, but the assessment of two parameters becomes difficult and, since a vertical shift only brings about fair agreement ; it will be adopted.

It is evident that these "shifts" must arise from physical variables which have not been included in the dimensionless groups of the standard equation of flow equations (3) and (5) and it is also evident that it cannot arise from the groups $\left(\frac{e}{d}\right)$, and U since the materials used are smooth and of uniform size.

Working on this assumption it would be expected that the flow through a bed of cylinders having lengths equal to their diameter would approximate closely to the flow through a bed of spheres while the flow through a bed of cylinders long in proportion to their diameter might deviate widely from it. Thus, the Resistance Coefficient will possibly depend upon the greatest dimension, L_g, and the least dimension, L_l of the particle.

Similarly, when the particle is penetrated by a hole it would be expected that the Resistance Coefficient of the bed will depend upon the ratio A_h/A_m where A_h is the cross-sectioned area of the hole and A_m is the mean projected area of the particle, when projected on to a plane perpendicular to the axis of the hole.

It is then necessary to relate the factor K, to these variables and by a system of trial plotting it is found that a reasonably accurate, while at the same time simple, correlation is obtained when the factor K is plotted against $\left(\frac{A_h}{A_m}+\frac{L_g}{L_l}\right)$. This relation is given in Fig. 8, the points being the

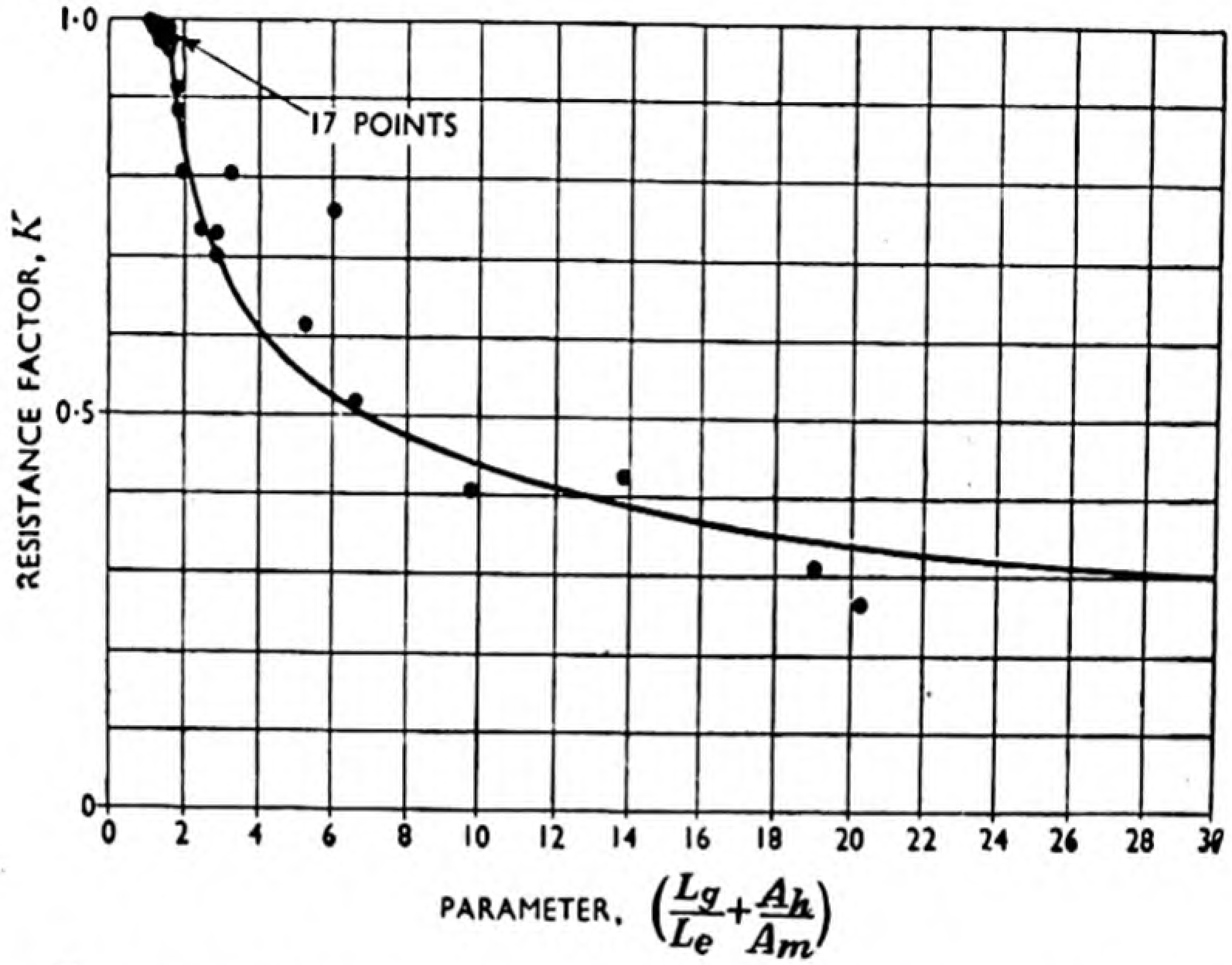

Fig. 8.—Relation between resistance factor K and parameter of particle.

actual values of K as determined by measurement from Fig. 7.

The industrially most important classes of materials fall within the range $1 < \left(\frac{A_h}{A_m}+\frac{L_g}{L_l}\right) < 2$, so, in order to extend the scale within this range Fig. 8 is replotted on semi-log scales to give Fig. 9. It should be

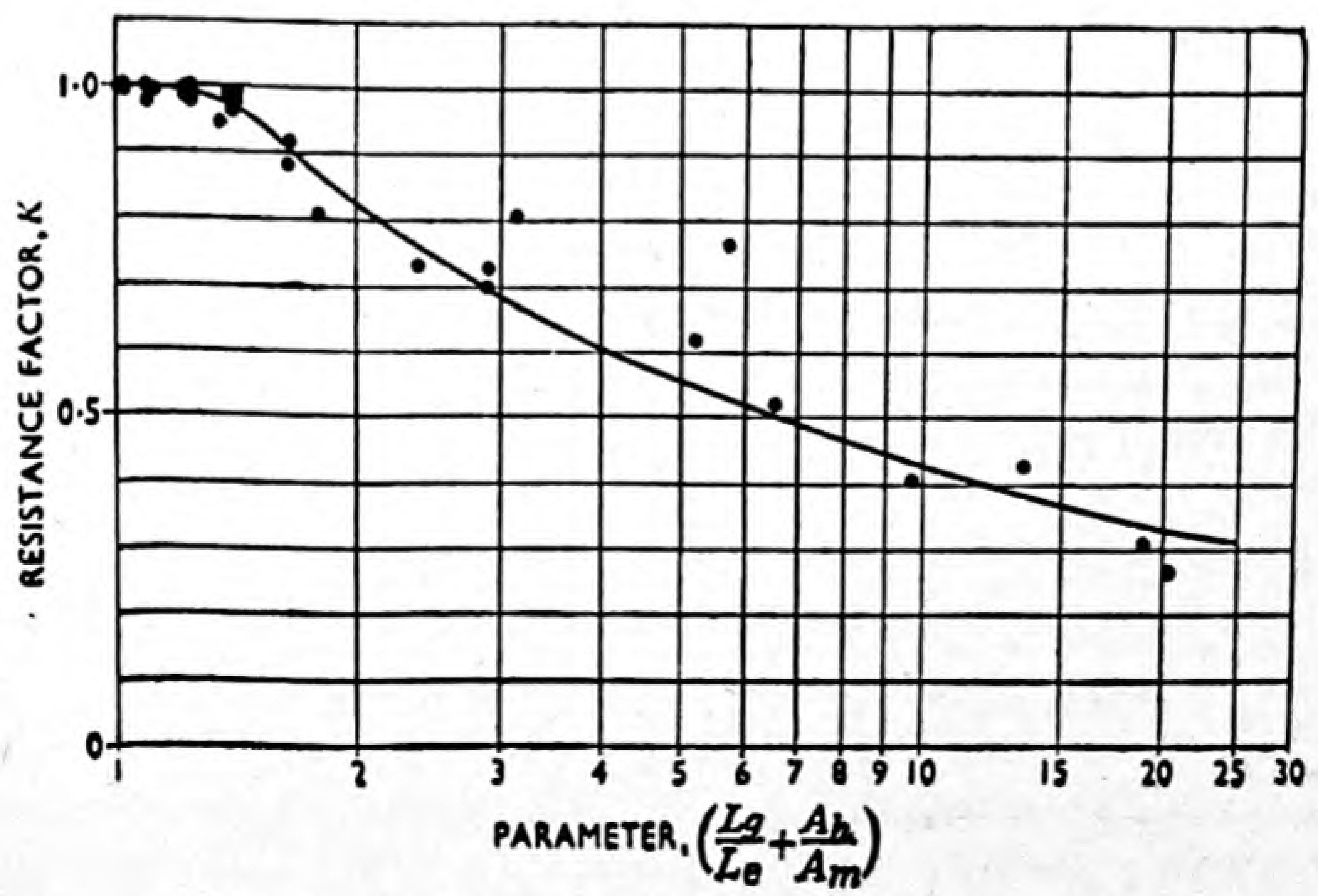

Fig. 9.—Replot on semi-log scale of curve in Fig. 8.

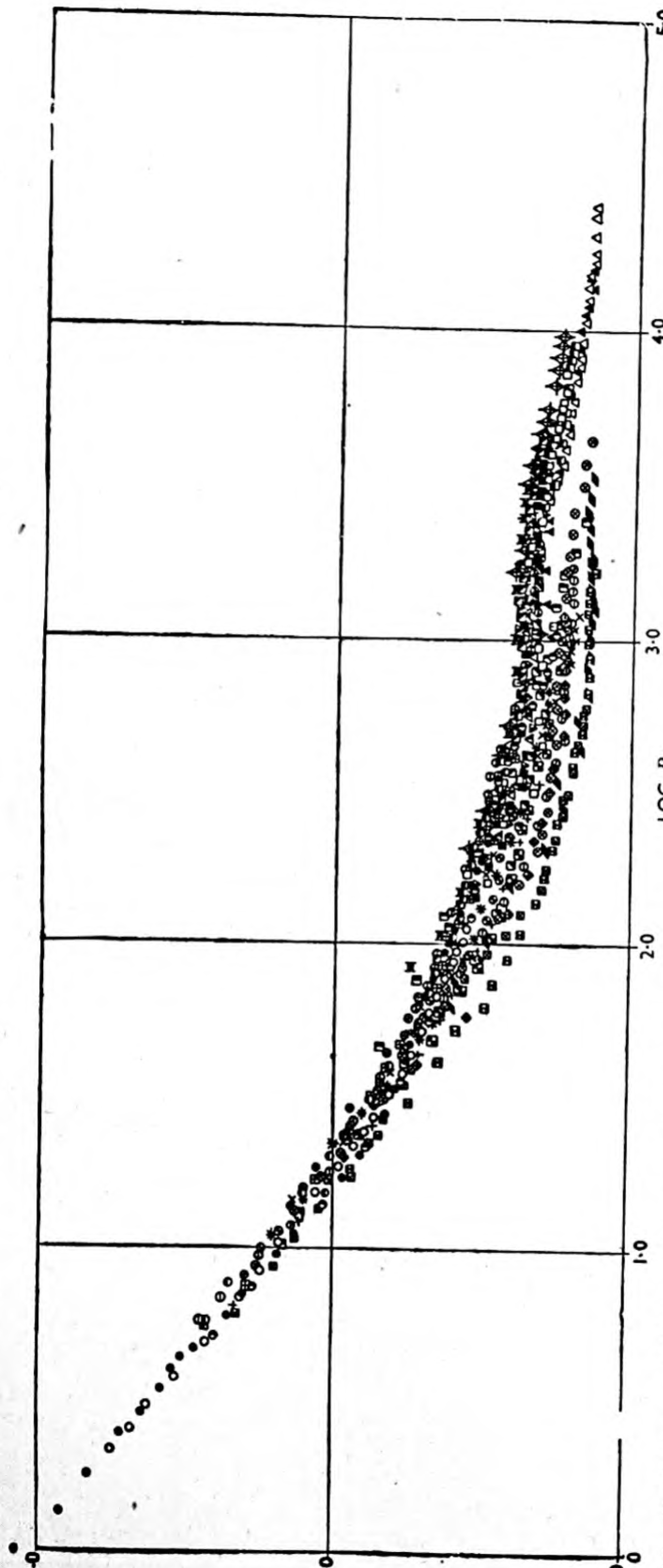

Fig. 10.—Resistance coefficient and Reynolds' number for beds of non-spherical materials after correction. For details of points, see key on Fig. 7 and also Table 3.

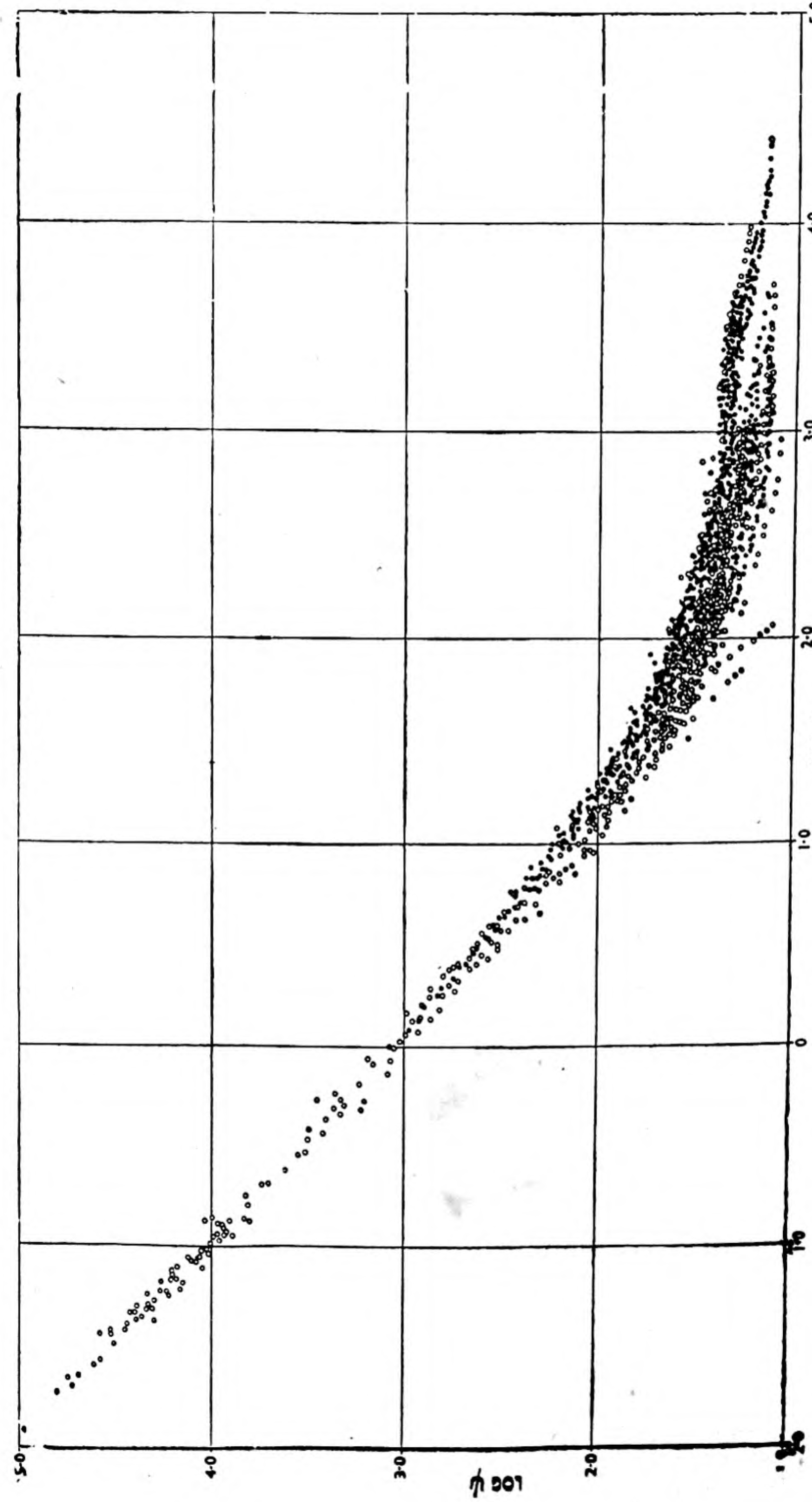

Fig. 11.—Resistance coefficient and Reynolds number for granular beds.

mentioned here that the scatter of the points on these figures possibly arises from the statistical nature of the flow within the transition range, in which range most of the present experiments fall, but this will be discussed later.

The experimental results of Fig. 7 " correlated " by the use of the curve Fig. 9 (the value of the correction being read from the *curve*) are plotted in Fig. 10. It is at once seen that this curve is practically identical in form with that for spherical material, Fig. 6,—there being a tendency to small scatter of the points for high and low values of Reynolds' number and considerable scatter of the points within the transition range, for both curves, and also the shapes of the curves are almost identical.

In Fig. 11 the points of Fig. 6 and Fig. 10 are plotted together, the full circles representing the results of all the tests upon the non-spherical material and the open circles those of all the tests upon the spherical material. It is seen that the correlation is good, having regard for the wide range of shapes of particles used in these tests.

Thus, using these results, it is possible to predict the resistance offered by a bed of material of spherical or non-spherical form with reasonable accuracy, since the functional relationship between the resistance and the various dimensionless groups is also given in the preceding sections.

Various other relationships have been tested tentatively, for example, the equivalent diameter of hollow cylinders has been assessed on the assumption that the whole of the internal surface area is effective in the dissipation of energy, but only half the internal volume promoted flow ; the basic idea being that if a hollow cylinder is situated with its axis perpendicular to the line of flow, then standing eddies would form inside the cylinder, dissipating energy in friction, but the internal voidage would not contribute to mass flow since any flow along the cylinder would be perpendicular to the direction of the main stream. In general, however, these devices offered no significant improvement in the correlation over the simpler method adopted here.

THE RESISTANCE COEFFICIENT—REYNOLDS' NUMBER RELATIONSHIP

The points on the curves of Figs. 6, 7, 10, and 11 have been determined in accordance with the system

$$f = 40\% \quad \phi_3(f) = 1{\cdot}0\,; \left(\frac{D}{d}\right) = \infty \quad \phi_2\left(\frac{D}{d}\right) = 1{\cdot}0\,; \left(\frac{h}{d}\right) = 1{\cdot}0.$$

These values are chosen since they give the functions convenient numerical values for the values of the variables encountered in practical problems. Examination of the curve Fig. 11, in which all the points are plotted on a single sheet, reveals several points of practical and theoretical interest. The first of these is that practically all the plotted points fall within an area bounded by the curves.

Lower Limit $0 < R_e < 100 \qquad \psi = \dfrac{900}{R_e}$

$100 < R_e < \infty \qquad \psi = 12$

and Upper Limit $0 < R_e < \infty \qquad \psi = \dfrac{1100}{R_e} + \dfrac{230}{\sqrt{R_e}} + 16.$

Analysis of the distribution-frequency of the points by a method described elsewhere, Rose[25], shows also that the points fall most thickly along a line

$$\psi = \frac{1000}{R_e} + \frac{125}{\sqrt{R_e}} + 14 \tag{11}$$

except over a range $1000 < R_e < 6000$, within which the most dense distribution of points swings upwards away from the smooth curve given by this equation. A tentative explanation of the scatter within the transition range, based on the analogy between the bed and a nest of tubes, has been advanced by Rose & Rizk[26] but considerably more investigation of this phenomenon is required.

The same writers have also attempted to explain the "hump" in the curve, for $1000 < R_e < 6000$, on the basis of the known characteristics of flow around cylinders but an alternative explanation has been advanced by Tison[27].

In Fig. 10, the points applicable to non-spherical material show a slight tendency to fall above those denoting spherical material. It is felt that this is due to the corners and sharp edges of the non-spherical material tending to promote turbulence. Presumably this effect could be covered by another dimensionless group but it is felt that the correction required is so small as to not justify the additional complication.

The above work establishes that a ψ—R_e relationship of the form of equation (11) fits the experimental data with reasonable accuracy. An additional term would possibly give a better fit over the doubtful range, $1000 < R_e < 6000$ but it appears that such a term would be of quite complicated form and so hardly justifies inclusion.

Forchheimer[28] and others have suggested that the ψ—R_e relationship for fluid flow should have the form $\psi = \dfrac{a}{R_e} + b$, which on combination with the dimensionless group $\left(\dfrac{v^2}{dg}\right)$ gives the form

$$H = a_1 v + b_1 v^2 \tag{12}$$

Some workers have interpreted this equation as meaning that the energy dissipated, at any rate of flow, is the sum of the energy dissipated in viscous friction and in turbulence (apparently implying that viscous and turbulent flow can each exist unaffected by the other in the same flow system).

The present writer feels most strongly that this interpretation is incorrect and, in this connexion it is significant that the data considered here cannot be expressed in the simple two-term form with any reasonable degree of accuracy, an appreciable additional term $125(R_e)^{-\frac{1}{2}}$, being required to give satisfactory agreement in the transition range, so it is felt that this three-term additive form is merely a simple empirical expression which approximates to a more complicated law but does not have any theoretical foundation.

It is believed that the term $1000/R_e$ is an absolute constant for viscous flow through beds of granular material and that the term $\psi = 14$ is an

absolute constant for turbulent flow; the term $125(R_e)^{-\frac{1}{2}}$ then being an empirical correction for the "energies" being not simply additive.

Morcom[29] has found that the expression $\psi = (a/R_e) + b$ relating ψ to Re may be used provided the value of b is varied according to the shape of the material forming the bed. This hardly appears to be in accordance with the general concensus of opinion, however, which seems to favour a definite value for this term independent of particle shape, or with the writer's results which also favour a constant value.

The published curves of Morcom are shown in Fig. 12 (a) and in Fig.

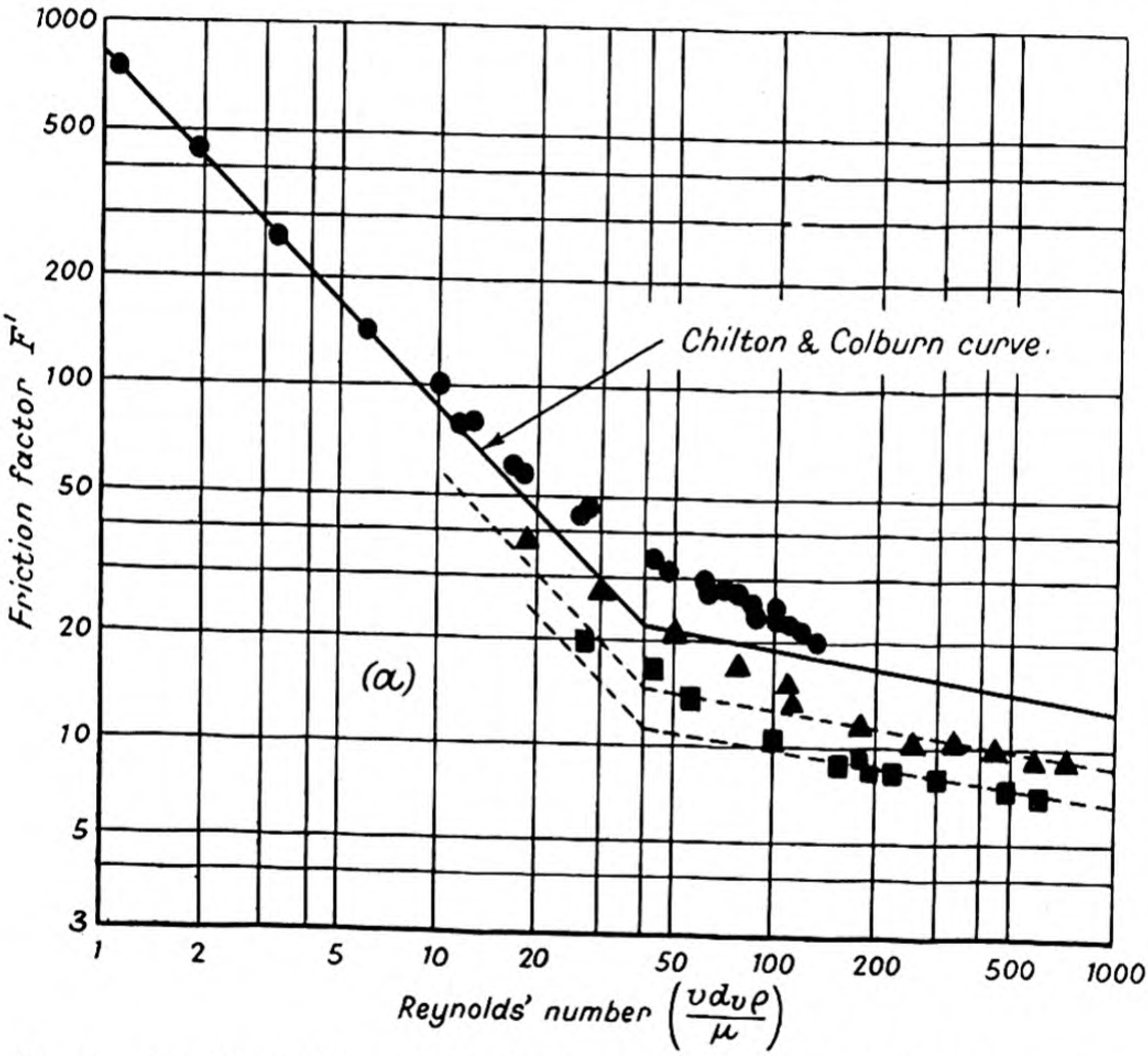

Fig.12A.—Correlation of Morcom's databy the method of present work (Before correlation.)

12 (b), these data, translated to the $\psi - R_e$ form according to the methods of the present paper, are shown. It is at once seen that these points fit the curve represented by equation (11) with considerable accuracy and so the materials of Morcom's tests do not require different values of the coefficient a and b according to the material but fit well into the general equation given in this paper.

It is interesting to note that the term $1000/R_e$ reduces to within 1% of the value 5·0 found for the coefficient in the Kozeny-Carmen equation for the determination of the Specific Surface of a powder.

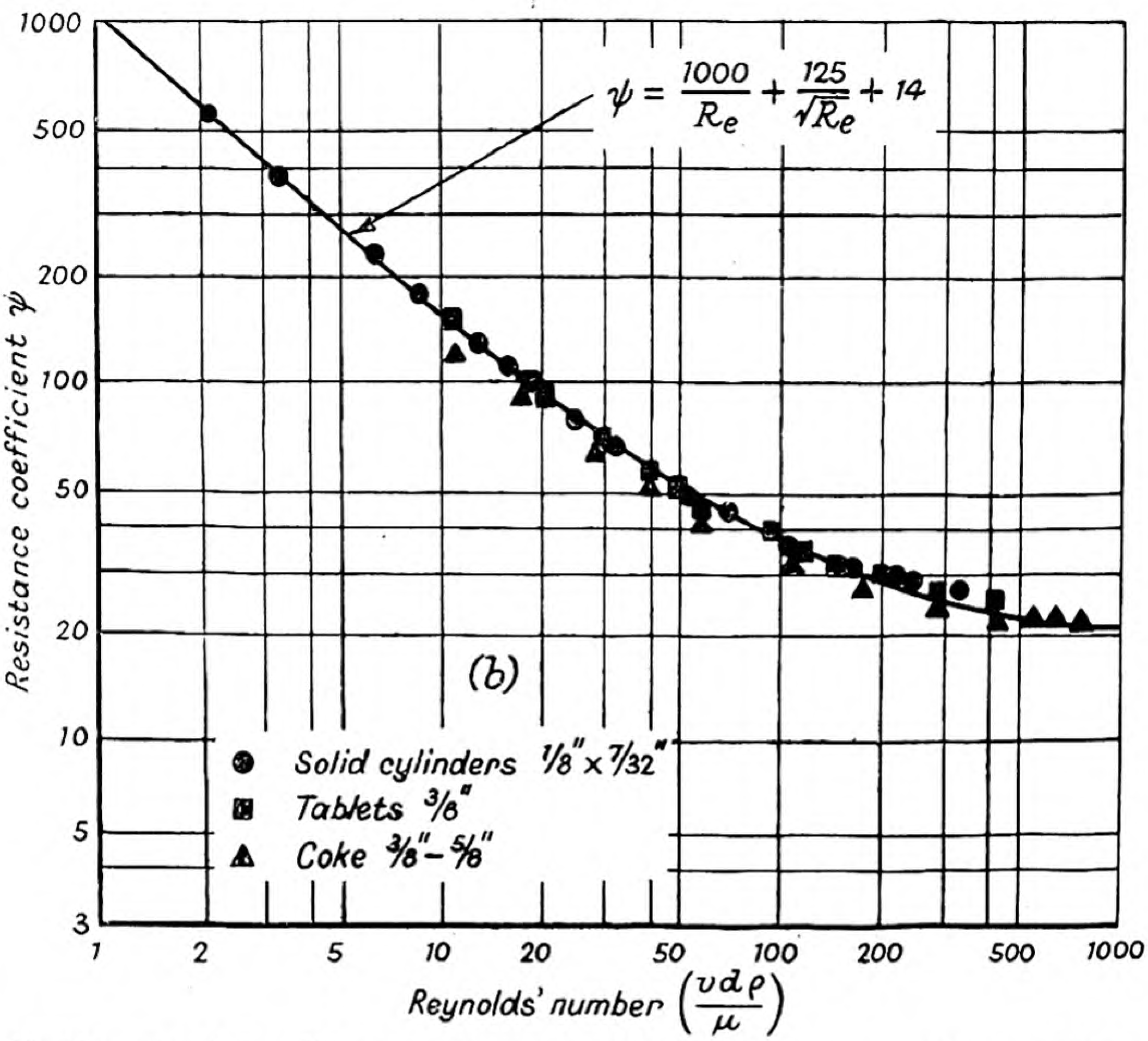

Fig. 12B.—Correlation of Morcom's data by the method of present work. (After correlation.)

Further research is required in connexion with the curve K plotted against $\left(\frac{A_h}{A_m} + \frac{L_g}{L_l}\right)$, for low values of Reynolds' number. The curve given Fig. 8 (or 9), is satisfactory for values of Reynolds' number exceeding about 10, within which range it was determined. For very low values of Reynolds' number, the same curve might be applicable but it seems probable that this correction would not be required, for it would be expected that the shape factor would be absorbed in equation (10) in this case.

ON THE EFFECTS OF SURFACE ROUGHNESS

The effects of this variable have not been investigated experimentally by the present writer. The view has sometimes been expressed that the roughness of the particle is a controlling factor in the case of fluid flow through beds but this opinion does not appear to be generally accepted, and, in fact, hardly appears likely. It seems reasonable to expect that if the irregularities do not break the boundary layer they will have no effect upon the resistance. If they are large, however, they will be included in the assessment of the particle diameter, and so allowed for. Thus it would appear that, in general, surface roughness may be neglected provided the major irregularities are included in the assessment of the particle diameter from equation (10).

THE FLOW THROUGH BEDS OF MATERIALS OF MIXED SHAPES AND SIZES

The tests reported in the foregoing sections were all carried out on beds composed of materials consisting of particles of uniform size and of similar shapes, although the size and shapes varied from bed to bed. Consideration of the theory presented, however, suggests that the equations evolved would be applicable to flow through beds composed of particles of mixed shapes and sizes.

Considering beds composed of a mixture of materials and denoting the components by suffixes 1, 2, 3, etc., and the number of each component in the bed by N_1, N_2, N_3, etc., their equation (10) suggests

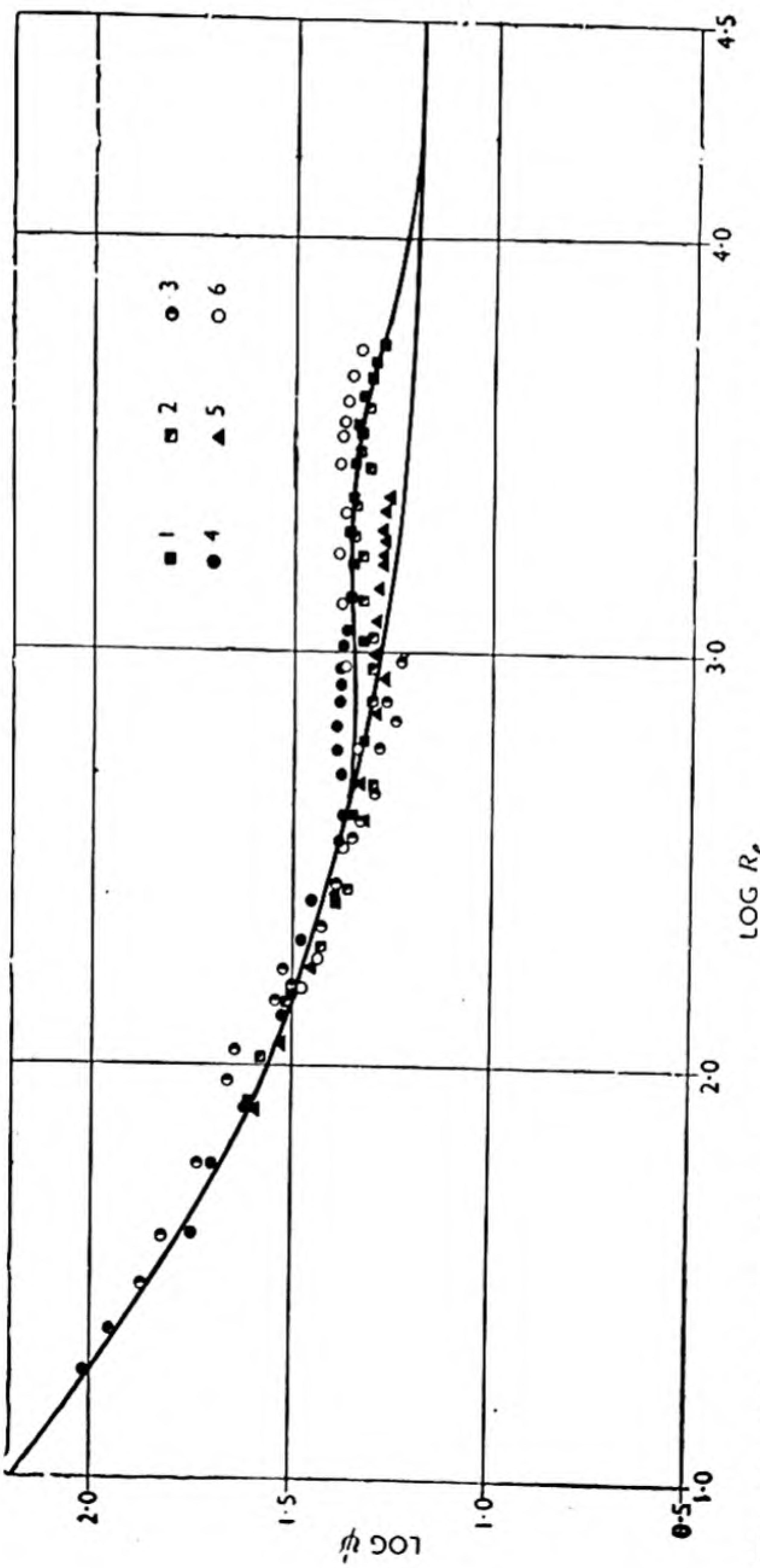

Fig. 13.—Relationship between resistance coefficient and Reynolds' number for beds of mixed particles. For details of points, see Table 4.

$$d_e = \frac{6\{N_1Vp_1 + N_2Vp_2 + N_3Vp_3 + - - -\}}{\{N_1A_1 + N_2A_2 + N_2A_3 + - - -\}} \tag{13}$$

where d_e is the equivalent size of the materials composing the bed ; to be substituted for d throughout the hydrodynamic equations.

Similarly, to estimate the value of the Resistance Factor, K, in terms of the parameter $\left(\frac{L_g}{L_l} + \frac{A_h}{A_m}\right)$ it is assumed that a " weighted mean " is also applicable so

$$\left(\frac{L_g}{L_l} + \frac{A_h}{A_m}\right)_e = \frac{N_1\left(\frac{L_g}{L_l} + \frac{A_h}{A_m}\right)_1 + N_2\left(\frac{L_g}{L_l} + \frac{A_h}{A_m}\right)_2 + - - -}{N_1 + N_2 + N_3 + - - -} \tag{14}$$

where $\left(\frac{L_g}{L_l} + \frac{A_h}{A_s}\right)_e$ is the value corresponding to which the value of K is read from Fig. 9.

In order to investigate the validity of these equations, a limited number of beds composed of two materials have been tested, the data of these tests being given in Table 4 and the results plotted in Fig. 13. It is seen that the points fall as closely along the curves as those applicable to tests upon uniform material, the scatter of which has already been discussed. This is an interesting verification of the equations, for examination of Table 4 shows that without the use of the " weighted mean " correlation would not be obtained.

Thus it may be concluded that the equations are applicable to beds composed of materials of mixed sizes and shapes, although, in view of the limited number of experiments, further investigation is desirable.

ACKNOWLEDGMENT

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TABLE 4.

BED MATERIAL			DATA RELATING TO THE COMPONENT MATERIALS: Material A					Material B					DATA RELATING TO MIXED BED						
Code Letter on Fig. 13	A	B	Code Letter on Fig. 5	Number of Particles	Percentage by Number	$\frac{L_g}{L_l}$	$\frac{A_h}{A_m}$	Code Letter on Fig. 5	Number of Particles	Percentage by Number	$\frac{L_g}{L_l}$	$\frac{A_h}{A_m}$	Equivalent Diameter d_e cms.	Tube Diameter D cms.	Measured Voidage (f) %	$\frac{L_g}{L_l}$	$\frac{A_h}{A_m}$	$\frac{L_g}{L_l}+\frac{A_h}{A_m}$	Resistance Factor K (from Fig. 9)
1	Solid Cyl.	Solid Cyl.	*a*	100	50	1.11	0.0	*a*	100	50	1.10	0.0	2.44	11.5	40.8	1.105	0.0	1.105	1.0
2	" "	" "	*a*	100	25	"	"	*a*	300	75	"	"	2.22	11.5	38.0	1.1	"	1.1	1.0
3	½ in. Wire Nails	Hob Nails	*e*	612	62.3	6.6	"	*k*	370	37.7	2.87	"	0.322	3.8	56	5.2	"	5.2	0.55
4	1 in. Wire Nails	3 in. Wire Nails	*e*	3010	94.83	13.5	"	*e*	164	5.17	20.2	"	0.349	11.5	69.6	13.84	"	13.84	0.38
5	3 in. Wire Nails	Lessing Rings	*e*	160	62.0	20.2	"	*c*	100	28.0	1.01	0.78	0.788	11.5	69.8	13.0	0.218	13.218	0.49
6	Shell Insul.	Raschig Rings (Hollow Cyl.)	*g*	15	15.0	1.40	0.02	*b*	85	85.0	1.02	1.38	1.824	11.5	74.2	1.38	1.54	2.25	0.8

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NOTE ADDED IN PROOF

Comparison of the curves, relating ψ to R_e, given in this paper with those given in the accompanying paper by Mott shows that, owing to the different bases of computation used by the two writers, the values of ψ differ by a factor of the order of 30.

From equation 7 of the paper of Mott it is seen that

$$R_{eM} = \frac{vd\rho}{\mu} \cdot \frac{4}{6(1-\epsilon) + 4d/D}$$

and from equation 2 of this paper that

$$R_{eR} = \frac{vd\rho}{\mu}$$

Thus, if $\epsilon = 0.4$ and $D/d = \infty$ $\qquad R_{eM} = 1.1 R_{eR}$

and if $\epsilon = 0.4$ and $D/d = 10$ $\qquad R_{eM} = R_{eR}$

Similarly, from equation 19 the paper of Mott

$$\psi M = \frac{\Delta P}{\rho L} \cdot 2 \cdot \frac{gd}{v} \frac{\epsilon^3}{6(1-\epsilon) + 4d/D}$$

and by transposing equation 3 of this paper

$$\psi R = \frac{H}{h} \cdot \frac{1}{\phi_2(D/d),\ \phi_3(f)} \cdot \frac{dg}{v^2}$$

Thus, $$\psi R = \frac{6(1-\epsilon)\quad 4d/D}{2.\ \phi_2(D/d),\ \phi_3(f)}\,\psi M$$

since $H = \Delta P/\rho$ and $h = L$

Owing to the different values of $\epsilon(=f)$, D/d and R_e applicable to the original data and to the different bases of correlation adopted by the two writers, it is not possible to evaluate this expression uniquely but by the assumption of values for ϵ, R_e, and D/d and by the use of Fig. 3. of this paper to obtain $\phi_2\ (D/d)$ limits to the value are easily computed.
Thus,

$$\begin{array}{llll} D/d = 10 & \epsilon(=f) = 0.40 & R_e = 1.0 & \psi_R = 26\ \psi_M \\ D/d = 10 & \epsilon(=f) = 0.40 & R_e = 1{,}000 & \psi_R = 36.5\ \psi_M \\ D/d = \infty & \epsilon(=f) = 0.40 & \left.\begin{array}{l} R_e = 1.0 \\ \ 1{,}000 \end{array}\right\} & \psi = 28.2\ \psi_M \end{array}$$

Comparison of the curves of Fig. 5 of the paper of Mott with Fig. 11 of this paper shows that, provided the data of Rose and of Rose and Rizk on Fig. 11 of the paper of Rose is neglected, in order to obtain strictly comparable data, the value of ψ as obtained by measurement from the curves falls well within these limits.

GROUP III

TECHNIQUES

9

Some Aspects of Fluid Flow in Orifices, Nozzles and Venturi tubes

By H. E. Dall.

(George Kent, Luton, Bedfordshire).

ABSTRACT. The aspects of this comprehensive subject on which the author has concentrated are the qualitative physical explanations of the observed characteristic curves of the coefficients of each device, and the advantages and disadvantages attending their use in different flow regions. Particular attention has been paid to the transitional flow regions from viscous to turbulent in which the viscous forces are of appreciable magnitude.

It is stressed that nozzles with compound curves offer no practical advantages over the simple orifice except in some rather special circumstances. Critical flow metering is one of these, and is dealt with briefly. Large Venturi tubes of rectangular cross section are also mentioned, with a field calibration result. Finally, some of the practical difficulties attending the use of differential metering are given.

The most desirable characteristics of any differential device used for fluid measurement are : (1) stability of the discharge coefficient over the widest range of flows or flow regions ; (2) ease of manufacture, reproducibility, and of installation ; (3) maximum recovery of initial pressure. These three headings may be classified as representing physical, mechanical and economic advantages respectively.

Great numbers of differential devices are in use, principally in industrial metering, by no means all of which have been selected after a proper assessment of the overall merits.

COEFFICIENT CONSTANCY

The standard form of the flow formula is based on the Bernouilli proposition of equality of the sum of pressure and kinetic energies in the two flow sections concerned. Changes of pressure energy due to fluid friction are allowed for only in their effect on the discharge coefficient, which is an experimentally determined value. In addition to fluid friction, the discharge coefficient is influenced by, and allows for, other departures from the ideal conditions, e.g., the non-uniformity of velocity distribution in the two sections, contraction of the jet, and the mean curvature of the streamlines at the pressure tapping points. The velocity distribution is determined by initial accelerations and fluid and boundary friction in the approach pipe.

Figs. 1, 2 and 3 show typical coefficient curves for the three principal types of differential device, using in each case the square root of Reynolds' number R as abscissae. This method enables plotting to be continued to zero while giving sufficient range to cover substantially the whole of the transitional region between purely viscous and purely turbulent flows in a compact form.

The validity of the Reynolds criterion (embodying the ratio of momentum and viscous forces) is so well established for pipe line differential devices that the writer is unaware of any careful experimental work which fails

to conform, and in which the discrepancy cannot be explained by geometric dissimilarity, the variation of viscosity with rate of shear or by elasticity or cavitation phenomena.

Referring to the figures it will be observed that the low ratio (i.e., low value of m) thin square edged orifice has the most favourable coefficient constancy of any device in common use, with a maximum departure from the fully turbulent value of 15% down to a Reynolds' number of 20. On the other hand, the high ratio orifice has the least coefficient stability, and its use should be avoided where possible in the region shown in the figure.

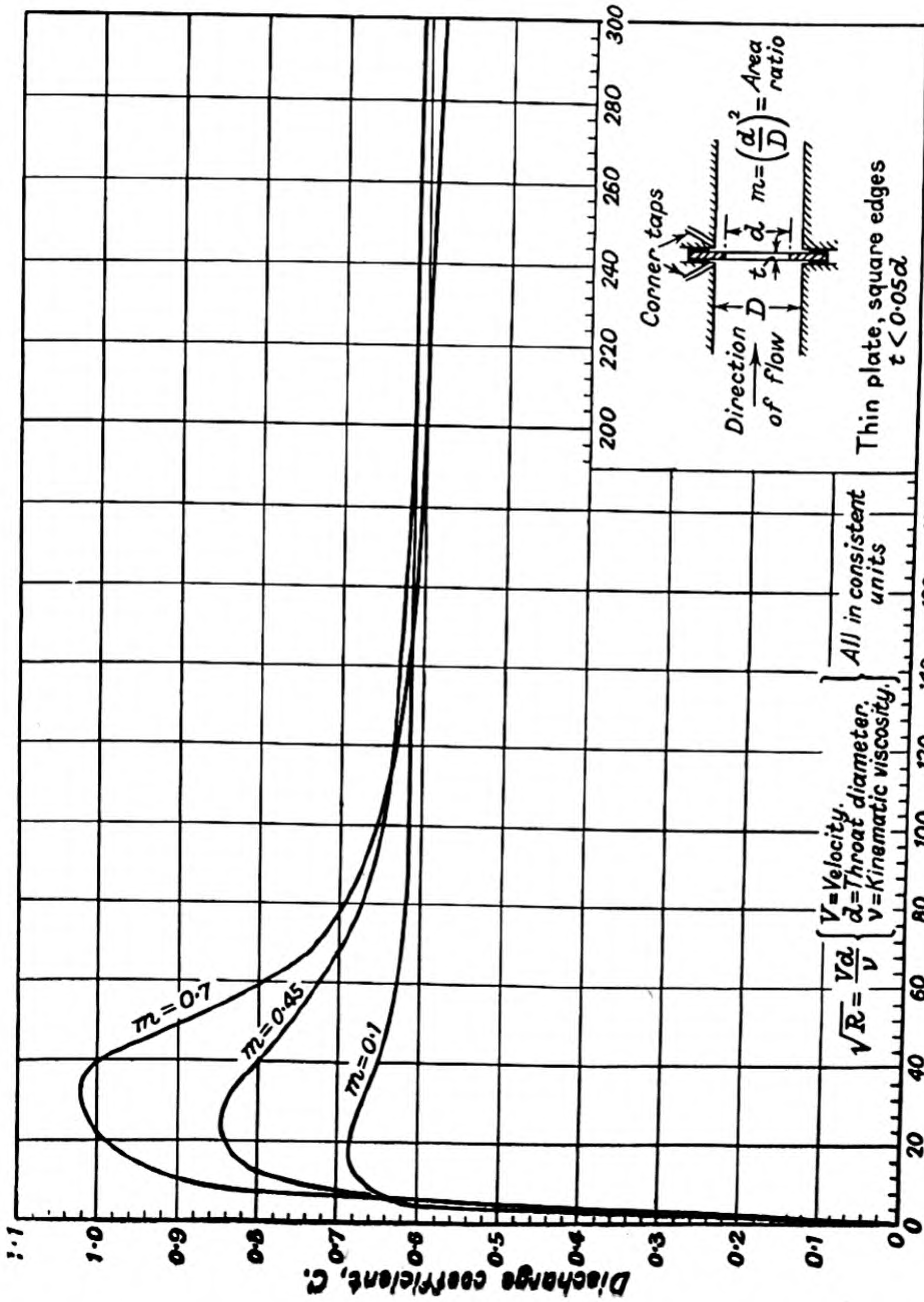


Fig. 1.—Typical coefficient curves for square edged orifices.

It should be pointed out that the major part of industrial metering is concerned with the regions of high Reynolds' numbers, in which coefficients approach within one or two per cent. of the fully turbulent asymtotic value. The minority is nevertheless of considerable importance, and includes much of the research and " pilot plant " regions of fluid flow measurement, for which fortunately greater freedom of choice of the differential device most suited to the duty is usually possible. Industrial metering gives less scope for choice, and the use of high pipe line velocities may at times result in the use of high ratio orifices in regions for which these are least fitted.

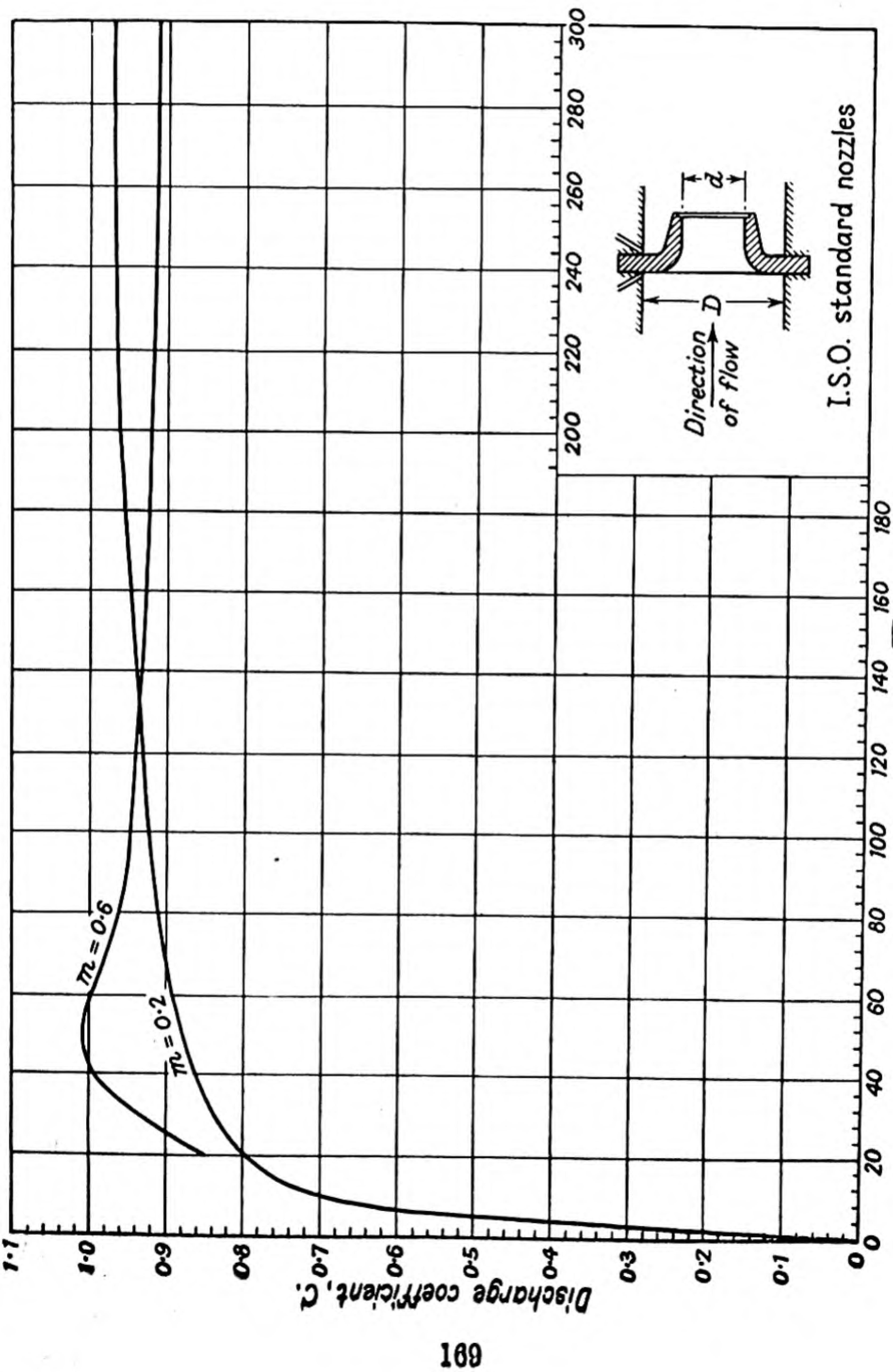


Fig. 2.—Typical coefficient curves for International Standards Organization's nozzles.

Nozzles and nozzle Venturi meters are less suitable than low ratio orifices from the point of view of coefficient constancy at low Reynolds' numbers. This is perhaps not surprising in view of the more intimate jet contact with the boundary walls in the case of the shaped nozzle, hence the greater influence of surface friction. The lower friction of the square edged orifice can be demonstrated by micrometric measurement of the free jet diameter when discharging water into air. The discharge coefficient calculated from this diameter is closer to unity than in the case of a nozzle discharging under corresponding conditions.

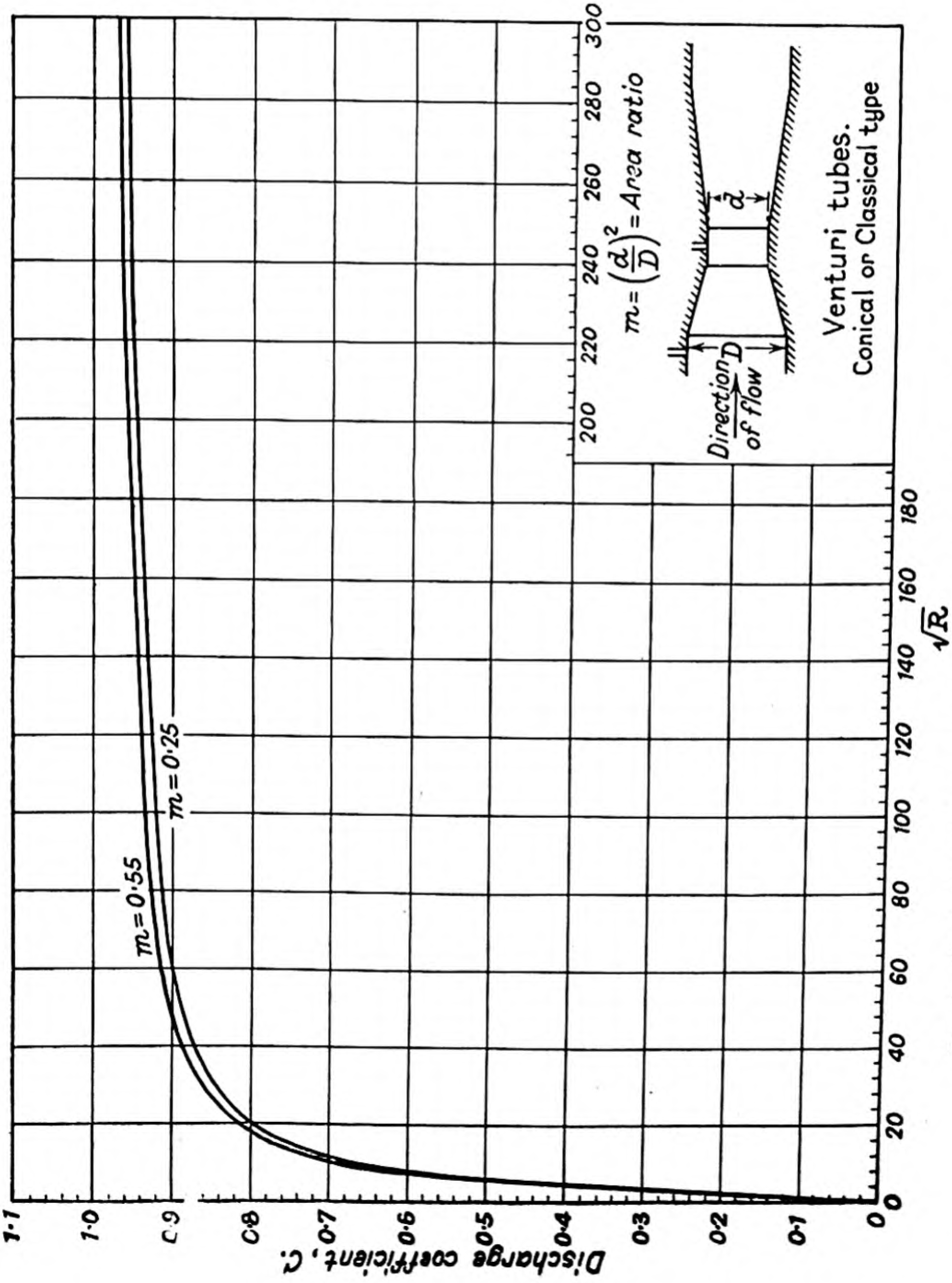


Fig. 3.—Typical coefficient curves for Venturi tubes (conical or classical type).

It is interesting to consider the physical explanation of the humped shape of orifice coefficient curves and the greater height of the hump for those of high ratio. The increasing influence of viscosity shown by lower Reynolds' numbers influences the coefficient in three ways:

(*a*) Internal fluid and boundary friction is increased, leading to greater resistance to shear in the acceleration through the throat, hence a lowering of the discharge coefficient.

(*b*) The velocity distribution profile of the pipe line flow approaching the orifice is changed in a direction which increases the central velocity in relation to the mean. This results in a greater total kinetic energy in the approach flow, and a lower requirement of pressure energy necessary to accelerate through the throat, thus raising the coefficient.

(*c*) Increased wall friction on the upstream face of the orifice, causing retardation of the layers close to the surface which form, by their momentum, the boundary of the *vena contracta*. This results in an enlargement of the latter, hence an increased coefficient.

Predominance of the influences (*b*) and (*c*) over a limited flow phase creates the hump on the coefficient curve. The region extends from that in which the initial effects of viscosity are felt, to the point where viscous forces become of the same order of magnitude as the momentum forces.

In the case of a low ratio orifice with practically zero approach velocity, the hump is due to influence (*c*) alone. The sum of influences (*a*) and (*c*) gives rise to a maximum augmentation of the discharge coefficient of about 15%. It can be reasonably estimated that the (*c*) influence alone is here about 20%, a value which implies a *vena contracta* diameter of 85% of the orifice bore at a Reynolds' number of 200.

At lower Reynolds' numbers, the jet continues to expand and to approach the orifice plate until finally the characteristic symmetrical uncontracted jet of purely viscous flow is given.

Referring again to Fig. 1, the low ratio square edged orifice also shows the least friction loss in the purely viscous region, and it is interesting to note that in the case of a thin plate orifice of ratio m less than 0·05, the viscous resistance is equal to that of a portion of capillary tube of equal bore and of length $0{\cdot}61d$.

For high ratio orifices the pipe line velocity distribution influence (*b*) becomes of primary importance in increasing the coefficient at the hump, and the maximum of the latter occurs at a higher Reynolds' number. Discharge coefficients exceeding unity are given by orifices of area ratio 0·70 upwards; higher, in fact, than for Venturi tubes or for nozzles. It is obvious, however, that the flow region should be avoided for flow metering in spite of an erroneous popular conception that high coefficients imply merit in the device.

For Venturi tubes or nozzles, the absence of a *vena contracta* leaves only (*a*) and (*b*) to influence the coefficient, with (*a*) of a predominant character owing to the much larger surface area in the throat region. As, however, the two influences are of opposite term, it might be expected that (*b*) would

become relatively important at the higher ratios. This is in fact the case, and for m ratios exceeding 0·55 it is found that nozzles and nozzle Venturi meters show a hump on the coefficient curve centred around a Reynolds number of about 2,000. The maximum reached exceeds unity as in the case of orifices for m ratios 0·70 upwards, but these ratios are rarely used even experimentally.

Some irregularities in the curve are sometimes shown in this region, and are probably due to the unstable flow regime in the approach pipe at the transition between turbulent and viscous flows. This transitional instability must also have a disturbing effect on the coefficient of any differential device owing to the large change occurring abruptly in the velocity profile of the approach pipe flow*, but the magnitude of the effect will be appreciable only for m ratios exceeding 0·30.

Various designs of special orifice have been produced in an attempt to deal with relatively viscous flows, by modifying the entrance edge and obtaining coefficients and characteristic curves intermediate between those of the square edged orifice and the nozzle. The desired characteristic is, of course, a constant and stable coefficient down to the lowest possible Reynolds number, an aim which, in view of the complexity of the viscosity effects just described, is impossible to achieve without a considerable variation of entrance shape with aperture ratio.

Low ratios enable the best results to be obtained, giving in the most successful designs coefficient constancy within 1% down to a Reynolds' number of 150; appreciably better results than this cannot be anticipated. Some designs, particularly those with rounded entrances, give coefficient instability (i.e., abrupt changes in value) at moderate Reynolds' numbers, and their use must be confined to a relatively restricted range.

NOZZLES VERSUS ORIFICES

From the earliest experimental work on flow nozzles, it is apparent that the aim of the designers was to obtain a coefficient near unity, and the shape of the curve selected has been based on or influenced by observed jet contours from a freely discharging orifice. This has led to curves of compound radii relatively difficult to manufacture accurately. The same result could have been achieved with a curve of single radius, or even a simple cone. This comment applies particularly to the original German I.G. nozzle (since adopted as a flow measurement standard by the International Standards Organization and by many countries); also to the elliptical nozzle of American origin and adoption.

It can only be concluded that this selection is the result of a misconception because the free jet attains its particular profile only by virtue of its freedom from wall friction. However, once the profile had been decided, a heavy programme of experimental work spread over many years has been necessary to establish the variation of coefficient with Reynolds number, pipe size, throat ratio, pipe irregularities, etc. This undoubtedly gives cogency to its adoption as a standard, in spite of the fact that far simpler devices have greater merits, at least in respect of coefficient constancy.

* STANTON. *Phil. Trans. A.*, **214**. 205 (19).

The square edged orifice has the greatest simplicity and reproducibility, and with experience, has come more general realisation of the fact that a coefficient some 40% below unity does not imply any demerits whatever for flow measurement. In addition, the use of stainless steel has almost eliminated corrosion troubles. Orifices of this material with over twenty years of continuous operation on superheated steam have shown no perceptible erosion of the orifice edge.

Neither has the shaped nozzle any advantages on the score of pressure loss compared with an orifice designed for equal differential. The nozzle has the same susceptibility to errors arising from swirl or other irregularities in the flow. For the measurement of fluid containing suspended solids which are liable to adhere and build-up on the face of the device, the nozzle has some advantage of lower susceptibility to errors providing the bore remains clear. This is not always found to be the case, and in both forms of device some closing up of the bore often occurs from adhering deposits. In this case, the orifice retains the better accuracy owing to the opposite signs of the errors due to face and bore deposits.

The nozzle has definite advantages over the orifice for critical discharge metering of air, gas or vapours, i.e., that form of metering in which a pressure loss of at least half the initial absolute pressure can be accepted. In such cases the flow in the throat attains the acoustic velocity, over the full area in the case of the nozzle but only partially for an orifice. The effect is shown by a coefficient varying with the pressure ratio for an orifice, whereas a constant value is given by the nozzle in the same region.

Critical discharge metering offers the advantage of a simpler technique; a single pressure and temperature reading sufficing to determine the flow. The sensitivity to pulsating flow errors is low, but the large pressure drop necessary restricts its use to limited fields of application.

VENTURI TUBES

Although the Venturi tube was among the earliest differential devices applied to flow metering, it remains substantially in its original " classical " form in many present-day applications. It is equivalent to a nozzle of conical form with coefficient near unity, with the addition of an outlet diffuser cone for reasonably efficient reconversion of kinetic to pressure energy. The pressure recovery diffuser is the primary distinguishing feature of a Venturi tube, and the resulting economy in pressure energy is very well warranted for bulk measurement of pumped or compressed fluids. It is generally a simple problem to decide in any particular case whether the additional cost of the device is justified by the resulting economy in pumping costs.

In the " classical " form, the inlet cone has a total included angle of about 21°, and an outlet diffuser of optimum efficiency, 5—7° total angle. Considerable increase in both angles is now common practice, with resulting reductions in coefficient and recovery fraction.

Departing from the " classical " design, the nozzle Venturi, consisting of the International Standards Organization's nozzle in place of the conical

convergence, gives a further reduction in overall length at the expense of greater manufacturing difficulty and with no other improvement in performance. The discharge coefficient is lower for the nozzle type than for the "classical" type, particularly for high ratio tubes, but this is no disadvantage, and arises from the upstream pressure hole position in the plane of the nozzle face where the streamlines have great curvature.

A 5—7° diffuser cone expanding to zero velocity gives an energy conversion efficiency of about 85% at the higher Reynolds' numbers. This does not, however, represent the whole recovery fraction for tubes of finite ratio, which is augmented by the transfer of momentum from the throat jet, the latter being only partially decelerated in the length of the cone. The augmentation is negligible for low ratio tubes, but brings the recovery fraction up to 93% for tubes of m-ratio 0·50 to 0·55.

Momentum transfer provides the whole of the recovery fraction in the case of orifices or nozzles, and this reaches 66% for the higher ratios in normal use. For all these differential devices, the recovery fraction is a function of Reynolds' number, and drops rapidly at low values, e.g., at $R = 500$, a Venturi tube has no advantage over an orifice in this respect.

RECTANGULAR VENTURI TUBES

Constructional considerations sometimes demand the use of rectangular section Venturi tubes, constricted in one plane only, for dealing with very large flows of water or ventilating air. Information on the appropriate coefficient can be obtained from scale model tests, but extrapolation is usually necessary to the extremely high Reynolds' number of the full scale flows.

An opportunity recently occurred to obtain the coefficient of two 10 ft. × 8 ft. rectangular tubes (throat section 8 ft. × 2 ft.) on site by means of multiple Pitot traverses in the throat section. This section was chosen and provided with suitable insertion points because of the anticipated uniformity of velocity distribution—an anticipation fully realised by the tests, which showed no greater variation than 1½% over the whole section except within 1 in. of the walls. Wall corrections were made by careful exploration of the velocity distribution in this narrow zone, and the overall accuracy of the flow determination assessed as better than 0·5%.

It is of interest to note that the coefficient so obtained (0·978) agreed closely with the value expected for this design of tube.

Rectangular Venturi tubes used for sewage measurement must deal effectively with the hazard of stable air or gas pockets which may form in the roof in or beyond the throat section. Consideration of the pressure gradients here will show how the pockets are formed even if the pressure at roof level greatly exceeds atmospheric. Some fraying and dissipation of the pocket occurs at the downstream end, but building up also takes place from the air and gas carried along in the flow stream, and the condition of equilibrium of a stable air pocket is established. The pocket gives rise to errors of measurement by blockage of the throat area, and thorough venting is necessary to avoid these. A more difficult problem is imposed if the roof pressure at the throat is sub-atmospheric.

Cavitation in high velocity water flows is another hazard more liable to occur with Venturi tubes than with orifices owing to the high differentials usually operative and the high recovery fraction. It is due to the pressure in the throat falling to a value close to the vapour pressure at the local temperature. Air is also released rapidly from solution at these low pressures, and the condition must be avoided in metering practice.

Flow and pressure pulsations constitute one of the major problems in differential flow metering. Pulsations from reciprocating units, etc., give rise to fast errors akin to the analogous r.m.s. errors of electrical practice. No practical means yet exist of eliminating these errors, but they can be reduced to a negligibly low value by damping the flow wave form by means of receiver capacity and throttling.

In general, it can be taken that the errors are negligibly small if the instantaneous maxima of the pulsating differential pressure after damping are not greater than four times the minima. Oscilloscope tests may be necessary to determine this.

10

Techniques for the Study of Fluid Flow

By J. H. CHESTERS, D.Sc.(Tech.), I. M. D. HALLIDAY, B.Sc., A.R.T.C., and R. S. HOWES, B.Met.,

The United Steel Companies Ltd., Sheffield.

ABSTRACT. Aerodynamic similarity of flow in a water model of a gas-fired furnace can be achieved by arranging that the Reynolds numbers at the gas port and air uptakes are equal to those existing in the full size furnace. An approach to similarity for an oil-fired furnace can be obtained by comparing heat input and flame velocity in gas and oil-fired furnaces and making suitable adjustments to the throat size and the rate of flow.

Shallow Perspex trays half-filled with water from one or more jets and fitted with outlet dams at different positions have been used to get information on flow in simple geometrical shapes. Bakelite powder was used as a tracer, photography being by transmitted light. A similar technique enables details of three-dimensional models to be studied. Using a mixture of paraffin wax and lead stearate as a tracer and a light sheet for illumination, studies can be made of the flow in solid geometrical shapes. For example, it is found that if the latter are obtained by revolving a two-dimensional shape the flow pattern is largely predictable.

The technique for scale models of open hearth furnaces (as set out in the *Journal of the Iron & Steel Institute*, August, 1949) is summarized. In this work rounded aluminium particles of air bubbles are used as tracers and a flash discharge tube for photography. A novel feature of the technique is that the direction of flow of particles is indicated, since the track tapers to a point in a direction in which the particle is moving. Work is now in hand on three-dimensional hot models, both of geometrical shapes and of scale furnaces.

Finally it is shown that the flow pattern in actual furnaces determined by the use of wooden tracer blocks or pitch-balls, shows a remarkable similarity to the flow pattern observed in the scale model.

It is concluded that the flow pattern in an open hearth furnace is largely dominated by jet speeds and envelope shape and that buoyancy, combustion and air infiltration are only second order effects.

A detailed account of the work done by The Research and Development Department of The United Steel Companies Ltd. on fluid flow up to the end of 1948 has been given recently. This included a bibliography of some sixty references to previous research on furnace models and allied problems. The contribution to the Conference summarizes the technique used in the above work and adds certain additional information on research carried out during 1949. When the original paper was written no cross-checks between water models and hot furnaces were available, but during the last year several such checks have been made. In particular it has been shown that the flow pattern in a Maerz furnace and in a single uptake furnace are very similar to those previously found in 1/24 scale water models of these furnaces. Furthermore that the flow pattern in a refractory cone fitted with a burner at the apex is very similar to that of the corresponding water model, which in turn shows a pattern deduceable from that of the corresponding triangle.

Fig. 1 shows an imaginary section through an open hearth furnace. The charge is melted on a shallow hearth by means of a flame, the heat from which is conserved by surrounding the hearth by back, front and

end walls, and covering it with a roof supported by girders. The fuel enters through a port at one end and mixes with air introduced at some adjacent point. The waste gases leave by the corresponding ducts at the

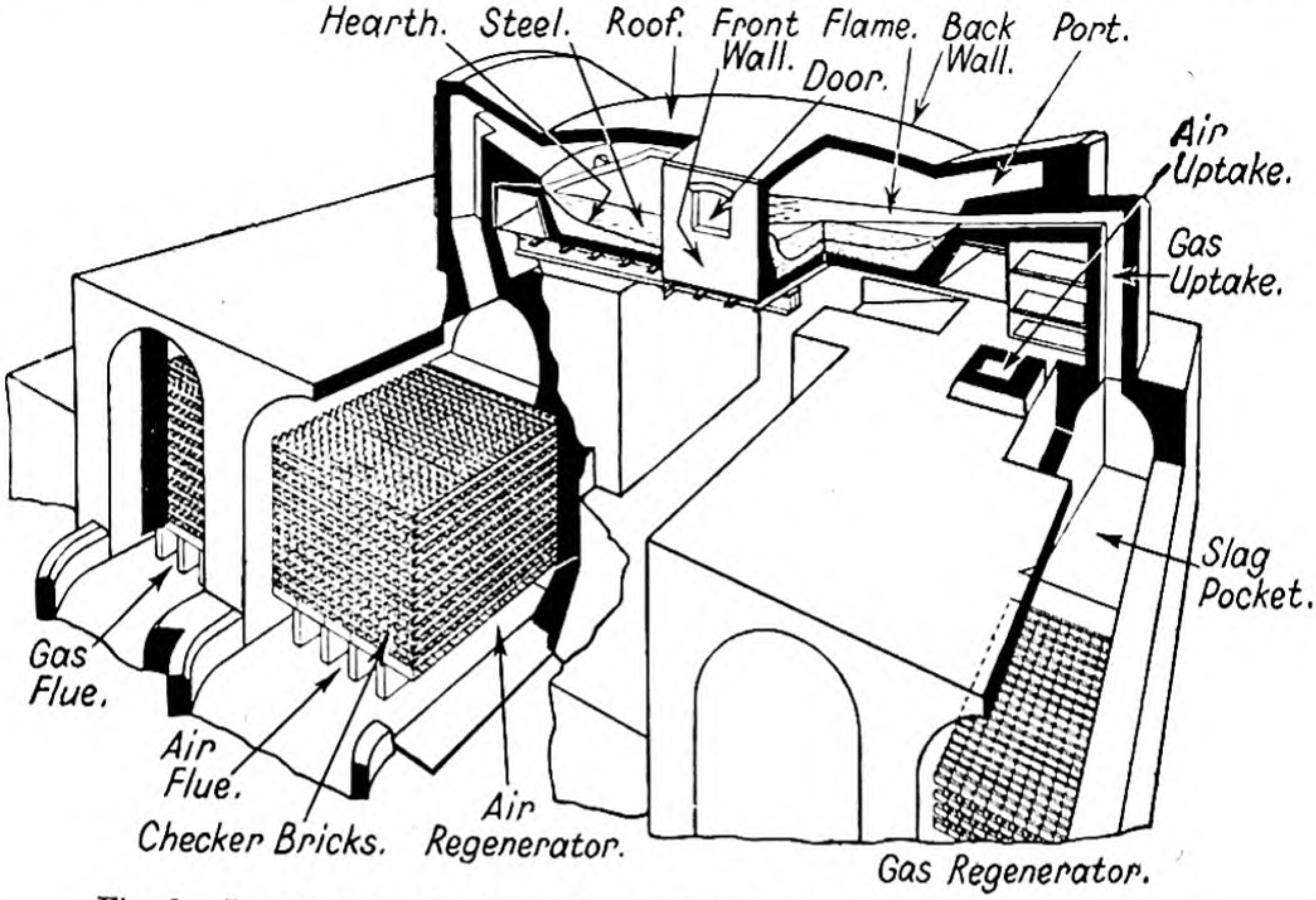


Fig. 1.—Imaginary section through open-hearth furnace. Steelwork removed.
(*Reproduced by permission from "Iron and Steel" by J. H. Chesters, published by Thomas Nelson and Sons, Ltd.*)

opposite end and pass through downtakes and a dust collecting chamber to one or more regenerators. These latter are essentially refractory boxes filled with a checkerwork of bricks. Every 10 to 20 minutes the furnace is reversed, the heat removed from the waste gases by the checkers being returned to the furnace as preheated air. With a gas-fired furnace the gas may be similarly preheated.

It would now appear that the flow pattern in such hot systems is largely dominated by jet speeds and envelope shape and that buoyancy, combustion and air infiltration, although important for other reasons, exert a relatively small effect on flow pattern. These conclusions refer particularly to open hearth furnaces. Where jet speeds are low and buoyancy becomes important the work of Groume-Grjimailo and his colleagues may be more relevant.

SIMILARITY

If it is desired to study the behaviour of a large furnace by means of a model it is necessary to choose operating conditions for the model, that correspond as closely as possible to those existing in full-scale operation. The primary objective was to obtain similarity in 1/24 scale open hearth water models, particularly of the aerodynamic or flow pattern aspects. These latter are mainly dependent on the forces due to momentum and viscosity of the gases flowing in the furnace. Aerodynamic similarity of

flow can be achieved in a water model of a gas fired furnace by arranging that the Reynolds' numbers at the gas port and air uptakes are equal to those existing in the full-scale furnace.

The Reynolds' number R_e represents the ratio of inertia forces to viscous forces in a fluid relative to a characteristic length of the apparatus. It is defined by the equation :—

$$R_e = \frac{\rho V^2}{L} \bigg/ \frac{\eta V}{L^2} = \frac{\rho V L}{\eta}$$

where in consistent units

V, ρ and η are the fluid velocity, density, and absolute viscosity and L is a characteristic length of the apparatus.

Usually the Reynolds' number for the gas flow in a producer gas furnace is of the order of 100,000 at the gas port for normal working flows.

The close checks referred to in the last section of this paper between the results on models and those on producer gas-fired furnaces are attributable to the fact that producer gas and air have similar dynamic properties at the point of entry to the furnace chamber. Discrepancies might, however, have been expected owing to the following factors : (1) buoyancy effects at the incoming end ; (2) physical changes due to combustion ; (3) chemical changes due to combustion, including dissociation ; (4) inleakage of cold air through furnace doors and brickwork.

Buoyancy Effects. The normal levels of preheat of air and producer gas, which are usually about 1350° and 1200°C. respectively, result in the incoming densities being roughly equal. If combustion were instantaneous the expansion of the gases would result in a considerable buoyancy lift, but in fact this is slight since combustion is not complete until the stream of gas is well down the furnace.

At the outgoing end combustion is usually complete and the waste gases fairly homogeneous, so that again buoyancy effects are likely to be small.

Physical and Chemical Changes due to Combustion. In the central portion of the furnace combustion is nearing completion and there is a considerable temperature rise, with expansion of the gases. Whilst the main effect would presumably be forward acceleration of the gases there are likely to be buoyancy effects in this part of the furnace. This is substantiated by the fact that cinematograph pictures taken at the middle door show a somewhat higher level for the flame top than is obtained in the corresponding water model. There may also be slight expansion due to dissociation of water vapour and other gases at these high temperatures, but this is likely to be a very small effect except with particularly high heat release flames, e.g., oil with oxygen for accelerated combustion. These finer points cannot readily be studied in water models, but it may ultimately prove possible to evaluate them by comparing the flow pattern in cold models with that obtained in hot models and in actual furnaces.

An important feature of production furnaces is that the gases leave at a higher temperature than that at which they enter and as a result the

Reynolds' number at exit will differ from that at entry. In view of the largely homogeneous nature of the gases and the observed constancy of flow pattern over a wide range of Reynolds numbers this difference would not seem to have an important effect on flow patterns.

Cold Air Inleakage. This requires special consideration because of the big temperature difference between the inleaking air and the hot furnace gases. On entering the air mixes with the furnace gases and heats up rapidly. Furthermore the flow pattern at entry as observed by watching the smoke from burning sawdust placed on the door sills is very unstable ; the space between the door jambs being filled by a varying mixture of furnace gases and cold air.

It is not possible to represent inleaking air at all precisely in a cold model, since the density of the air changes as it enters and the simulation of its original density would demand the use of a liquid six times as heavy as water.

In general, therefore, flow pattern work refers to fully sealed furnaces. A series of tests have, however, been carried out on a single uptake type furnace model to study the effect of inleaking air. An average temperature of this air, of 400°C, was chosen and a corresponding Reynolds number assumed. If the chosen temperature is too high the air inleakage would correspond to even higher rates of air flow at full scale. The most definite conclusion from this work was that even when the rates of air inleakage are high (comparable in amount to the air fed for combustion), the basic flow pattern in the furnace remains unchanged, apparently because the velocity of the incoming air is low, and, therefore, it has relatively little effect on the momentum balance.

Surface Roughness. No account has been taken in our model work of surface roughness, though complete geometrical similarity would demand a scaled roughness for the interior of the furnace model corresponding to the working face of the brickwalls. Certain workers, e.g., Williamson[2] have suggested that roughness may be of great importance in vortex formation, but our work suggests that under open hearth conditions the precise roughness of the surface is unlikely to have any appreciable effect on the basic flow pattern. It may, however, have a measurable effect on, say, the pressure drop through such restrictions as the incoming gas port barrel.

Similarity Procedure for Oil-fired Furnaces. A first approach to aerodynamic similarity in a water model of an oil-fired furnace can be obtained by comparison with a producer gas-fired furnace. It has been observed for certain of our own furnaces that : (1) the heat release per hour in an oil-fired furnace is roughly equal to that in a producer gas-fired furnace ; (2) the mean surge velocity of an oil flame is approximately two-and-a-half times as great at the first door as that of a producer gas flame. Similarity is based on obtaining the necessary increase in the momentum of entrainment to the fuel jet in the model by an appropriate reduction of fuel port area P and volume of water V supplied to the fuel port. The

necessary equations were deduced by equalising the ratios of the momentum of air entrainment to fuel port Reynolds' numbers, and are as follows :

$$P_o = P_g \times \frac{1}{9}$$

$$V_o = V_g \times \frac{1}{3{\cdot}6}$$

where suffixes o refer to the oil-fired furnace
g refer to the corresponding gas-fired furnace.

It is realised that the above assumption is a very approximate one, but sufficient has been said to show that some approach to similarity can be obtained, particularly for gas fired furnaces and that the results obtained check quite closely with those observed in hot models and full-scale furnaces.

TWO-DIMENSIONAL WATER MODELS

A. Simple Geometrical Shapes. Our work on water models started with the study of scale models of open-hearth furnaces. The interpretation of the complex patterns observed soon forced us to start work on much simpler systems, the preliminary results of which have been described elsewhere[1]. Use was made of shallow Perspex trays half-filled with water from one or more jets and fitted with outlet dams of variable height. Bakelite powder floating on the surface served as a tracer both for visualisation and photography by transmitted light. Fig. 2 shows the sort of pattern obtained with different geometrical shapes and jet locations.

As a result of this work a number of principles were set down that applied to jet systems in containers and which throw considerable light not only on two-dimensional but also on three-dimensional scale models. Above all they demonstrated that recirculation, which at first appeared to be a quite abnormal thing, is in fact to be expected with almost any jet system.

B. Scale Models. Considerable information can be obtained in a very short time regarding the flow patterns in ducts by a study of two-dimensional sections, particularly if the solid duct is a uniform extension of a two-dimensional shape or is a solid of revolution. A good example of the former is given by the downtake system in an open hearth furnace, which consists essentially of a right-angle bend. The pattern obtained (Fig. 3*a*) when water loaded with aluminium powder is used to study the flow round such a bend, shows all the main features of the three-dimensional problem, viz. : the impact of the " waterfall " on the right-hand wall, and the large area of dead space both beneath the top corner and in the area adjacent to the left-hand vertical wall. Not only do such models enable a rough prediction to be made of the " target " area or position of maximum wear, but they afford a ready means of observing the effect on flow pattern of proposed alterations.

The above study for example started because excessive wear had been experienced in the downtakes of an experimental furnace. The first suggestion that this could be remedied by " cutting off the top right-hand corner " was shown to have neglibible effect on the flow pattern since this

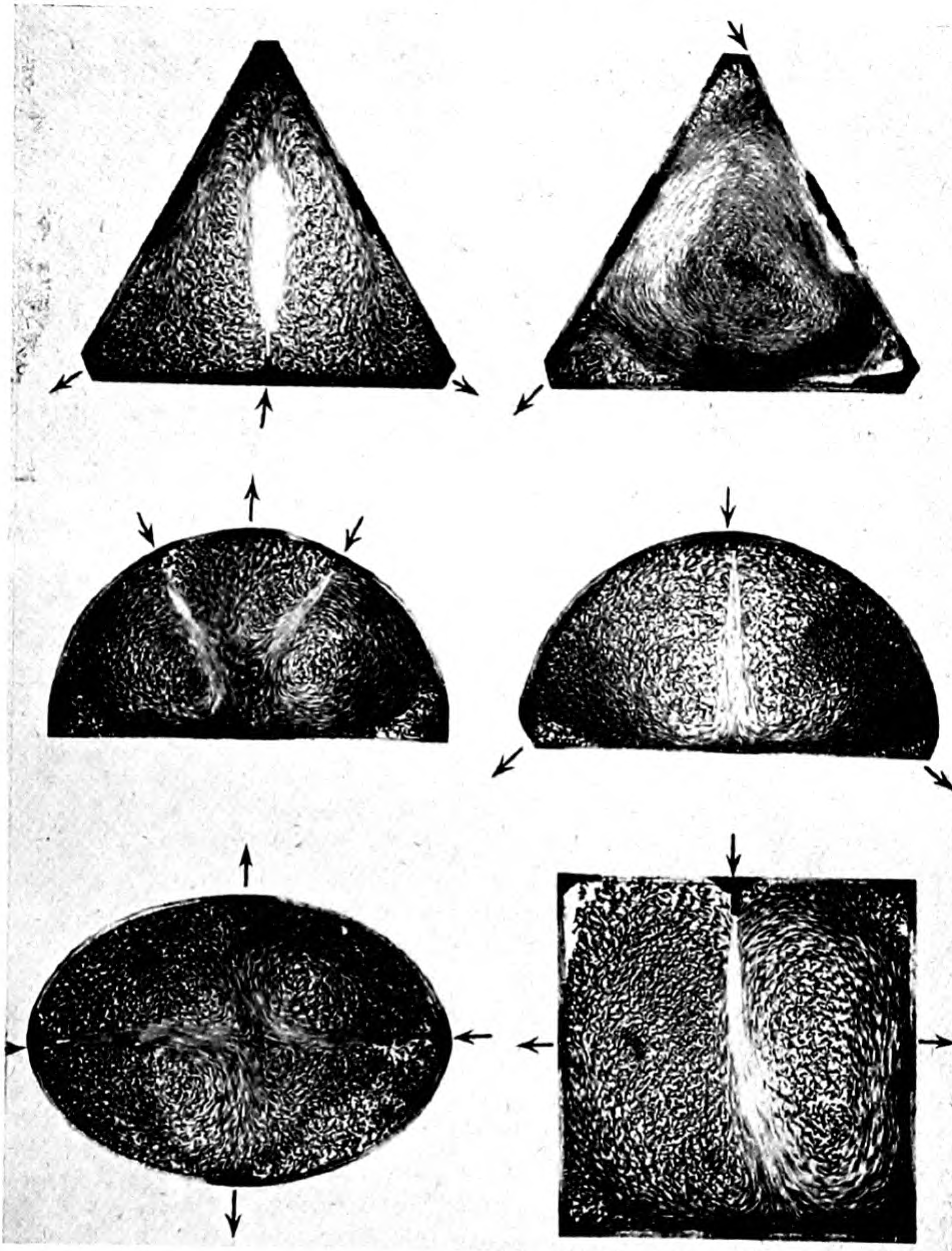
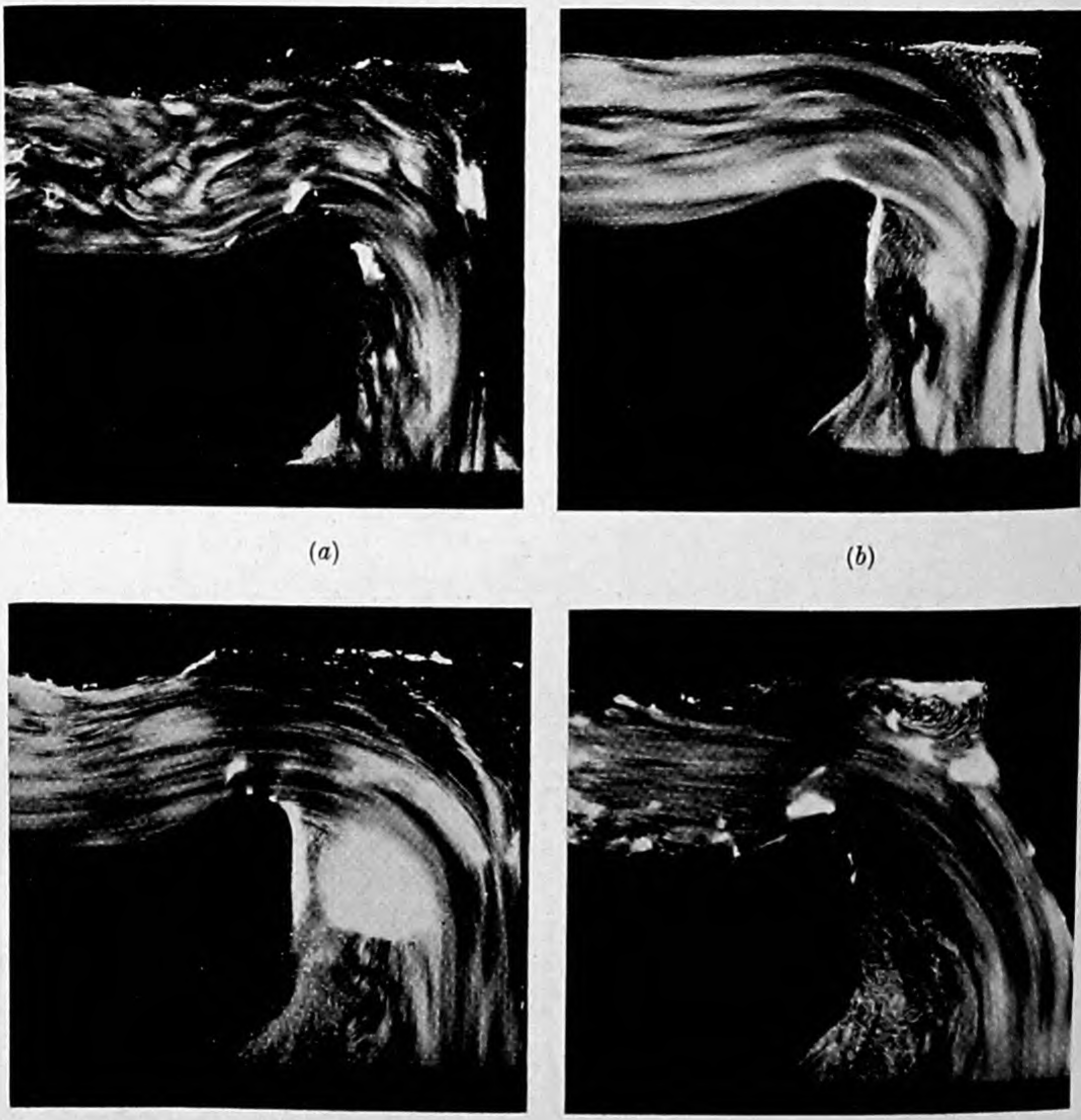


Fig. 2.—Two-dimensional flow patterns observed in simple geometrical shapes.

corner was in any case occupied by a small eddy. Sloping of the end-wall (as in Fig. 3*d*) largely obviated the direct impact and offered promise of far longer furnace life. Other less satisfactory suggestions, such as the broadening of the uptake, are shown in Figs. 3*b* and 3*c*.

The technique has also proved useful in the study of detailed features of actual furnaces, e.g., the effect on the flow pattern of rounding the pillars between the doors. One designer apparently thought this would have a marked effect on brickwork life, and the fact that it did not is in line with



(a) (b) (c) (d)

Fig. 3.—Flow patterns in two-dimensional models of furnace downtakes. Note the varying degree of impact on the end wall.

the observation from a two-dimensional tray, that such rounding does not materially alter the general flow pattern.

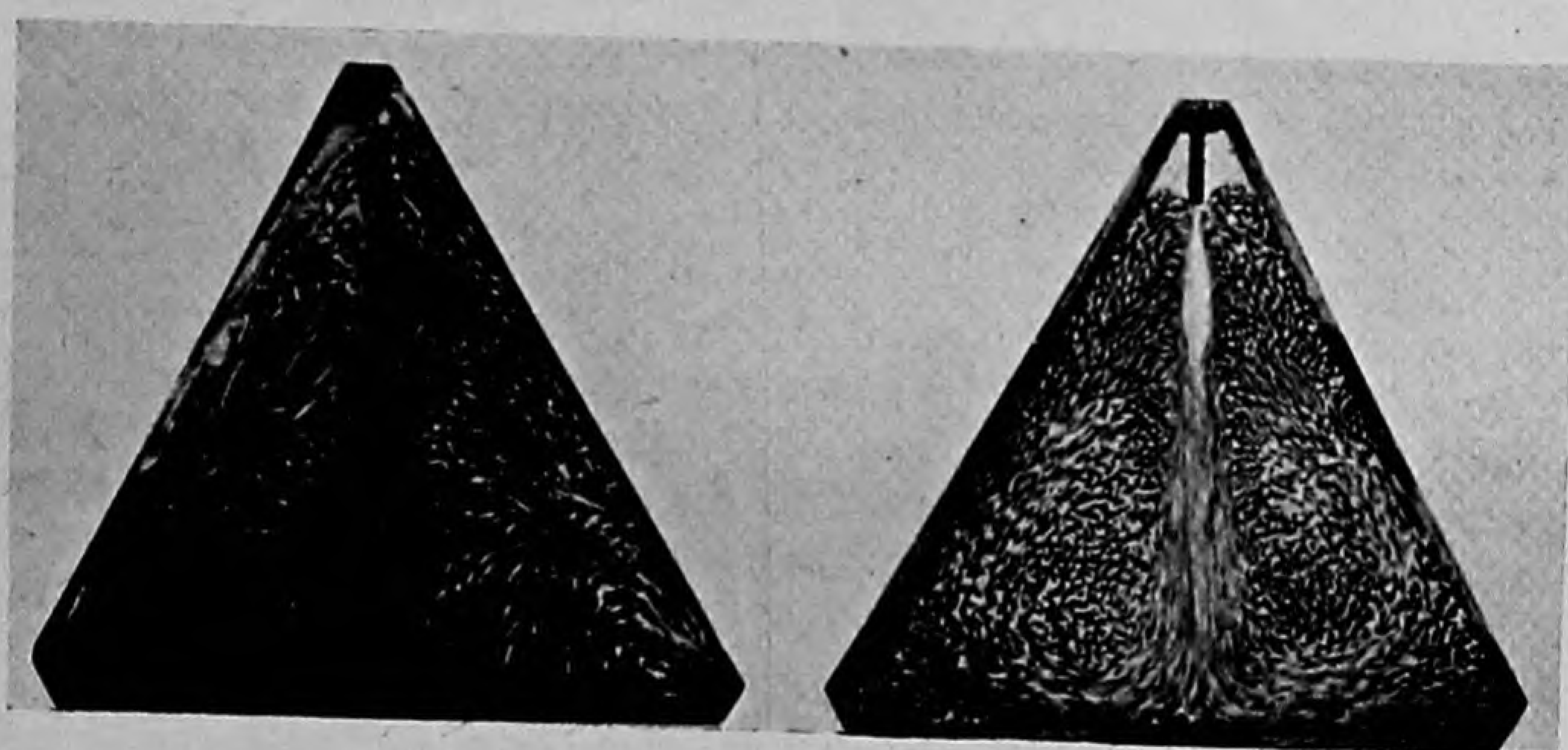
THREE-DIMENSIONAL WATER MODELS

A. Simple Geometrical Shapes. Since the earlier paper[1] was published the flow pattern has been determined in a considerable number of solid geometrical shapes, including a cone, a hemisphere, a sphere, an ellipsoid, a cube and a cylinder. A new technique had to be evolved for this work, since the low velocities occurring in certain portions of the model caused the normally used aluminium particles to sink or air bubbles to rise, thus giving a false picture of the pattern. It was soon evident that the tracer employed should have a density approximately that of water. After studying a number of materials it was found that a mixture of paraffin

wax and lead stearate, prepared by melting these materials together and subsequent grinding and sieving, gave particles of the desired density and size.

Such particles can be readily photographed using the standard slit illumination technique employed in the earlier work. The particles were introduced into the incoming jet from a reservoir of water in a pressurised tank. This method has, however, one serious limitation, viz. : that lead stearate is a poisonous substance, and, therefore, work has started on other tracers.

A most interesting conclusion from this work, as illustrated by Fig. 4, is that if the solid model can be obtained by revolving a two-dimensional shape, then a study of the flow pattern in the latter enables a useful prediction to be made regarding the flow pattern in the three-dimensional



(a) (b)

Fig. 4.—Similarity of flow patterns in section (a) through cone and (b) through triangular dish.

model. Thus the flow pattern in the cone, shown as Fig. 4*a*, is essentially similar to that previously obtained from a triangular tray (Fig. 4*b*). An additional factor is, however, sometimes observed in three-dimensional work, viz. : a rotation of part or all of the flow pattern. This can be seen in the cone by illuminating a section perpendicular to the jet and a few inches away from its origin. All the principles deduced from two-dimensional models, including the instability observed in certain shapes, such as the ellipse and ellipsoid, apply to three-dimensional models. The additional factor of pattern rotation is well illustrated by the cylinder, in which the jet introduced along the main axis tends to wander spirally instead of keeping a centralised position.

B. Scale Models. Full details of the methods used for scale models of open-hearth furnaces have been set down elsewhere[1], but the following summary will illustrate both the problem and the technique used in solving it.

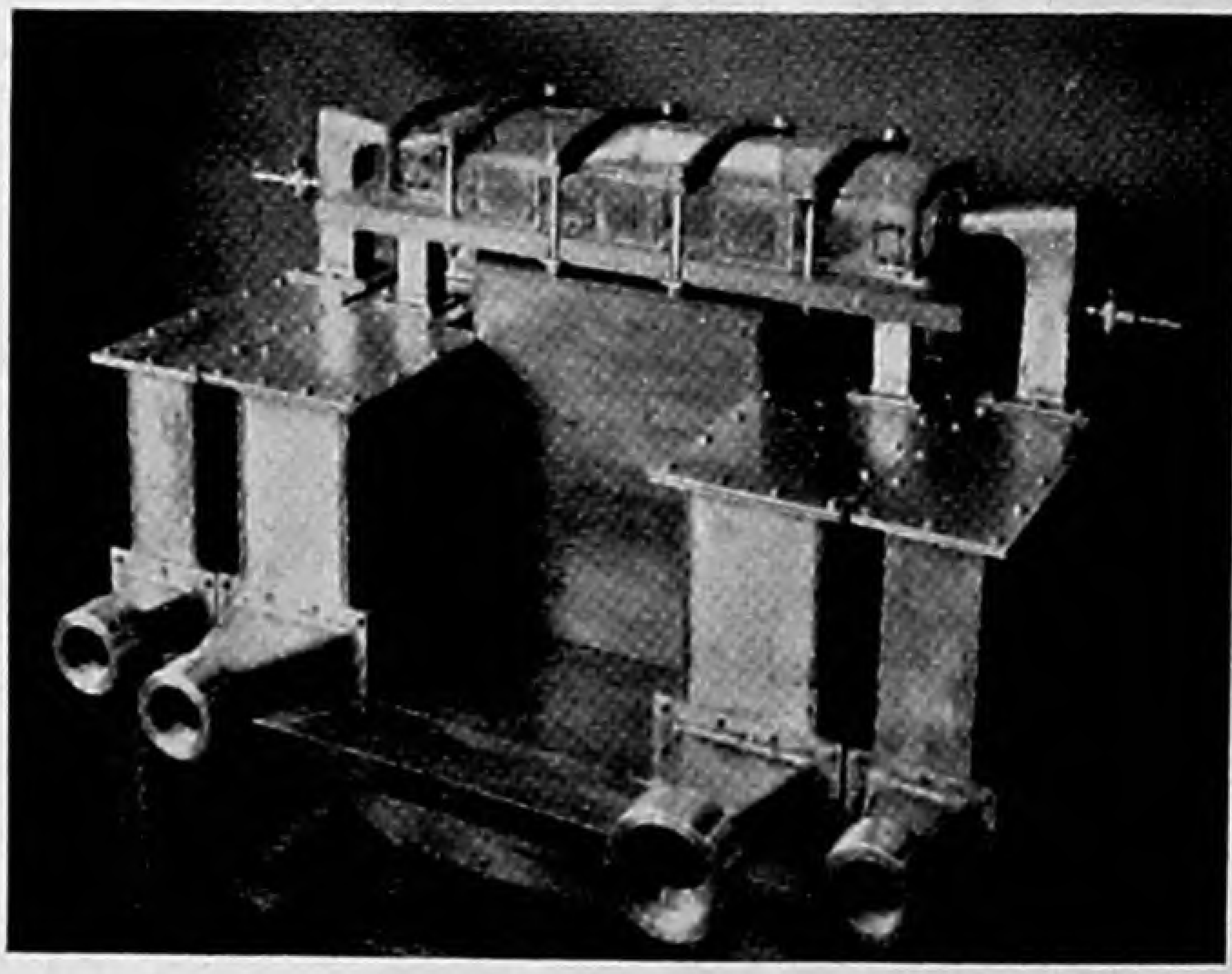


Fig. 5.—Open-hearth furnace model showing furnace chamber with stainless steel downtakes and checker chambers.

Layout. The model layout shown in Fig. 5 represents the checkers, slag pockets, uptakes and furnace chamber, of the " G " Maerz open-hearth furnace at to 1/24 scale. Air and gas flows are represented by water circulated through the system from a 300-gallon tank by means of a 10 h.p. pump, the separate flows to and from the gas and air ports being metered and controlled. The flows employed are calculated to give similarity as far as possible with the corresponding full-scale open-hearth furnace at the incoming or outgoing ports.

Hydraulic System. Water from a supply tank is fed by means of a 10 h.p. pump into a 3-in. pipe. This pipe is fitted with a main control valve, and thereafter divides at a T-piece with double elbows into two 2-in. pipes, which feed the air and gas checkers respectively. These 2-in. pipes each contain an orifice plate and valve for metering and controlling the individual flows. At the outgoing end of the furnace system, separate pipes from the gas and air checkers return the water to the tank, each of these latter pipes being fitted with orifice and valve for metering and control. The supply tank, is fitted with a 2-in. feed and drain, for rapid filling and emptying. This is necessary because of the need for working with very clean water.

Calculation of estimated head loss through the system suggested that, under extreme conditions of operation, the pump should be able to deliver 15,000 gal/hr. against a head of 75/85 ft. Pressures of about 3 atm. exist on the walls of the incoming gas checker and of about 1 atm. on the furnace roof, this latter figure being equivalent to a total upward force on the roof of about 1 ton.

The Optical System. Illumination of the whole model at one time gives a very confused picture since the flow at different levels may have different direction and velocity. An analysis of flow pattern is most conveniently made by building up from the pattern seen at various vertical

or horizontal plane sections, illuminated with "sheets" of light. It was found desirable to employ a light sheet at least 8 in. wide, and to be able to photograph this with exposures as short as 1/2,000 sec.

The optical system consists of an 8 in. double plano-convex condenser, mounted in a light-proof aluminium box, the front of which is fitted with a pair of adjustable shutters to control the thickness of the beam. Behind the condenser in a detachable aluminium lamphouse, is mounted an air-cooled 2 kW lamp with an 8 in. line filament. Normally a slit width of $\frac{1}{4}$ in. is used. To obtain a verticle light sheet, a length of optically flat surface-silvered mirror at 45° is attachable to the condenser box immediately n front of the slit. To avoid distortion due to refraction at oblique surfaces of the model the furnace chamber was surrounded by a rectangular Perspex tank, filled with water to a depth sufficient to cover the entire model.

The use of the 2 kW lamp is quite satisfactory for viewing the flow pattern, but the light output is inadequate for photography with exposure times of 1/100 to 1/200 sec. which give optimum streak length. Special flash discharge tubes with a central portion 8 in. long have been developed for this work, mounted in a housing which is interchangeable with the 2 kW lamphouse.

In operation the condensers discharge about 500 joules of energy in a flash time which may be varied from 1/125 to 1/2,000 sec. Over the shortest exposure time the energy discharge therefore corresponds to an average power output of 1,000 kW. The flash duration is varied by incorporating in series with the tube a variable choke.

Visualisation. In determining flow patterns within the models several techniques have been investigated.

(*i*) *Dyes.* The method first tried was to introduce dyes of different colours into the air and gas streams at the base of the checkers. With such highly turbulent conditions, however, diffusion of the dye in the furnace chamber occurs too rapidly for this to be an effective method. A preliminary attempt has been made to study mixing, using a similar technique, by adding a solution of sodium thiosulphate to the water in the tank and injecting iodine into the gas stream.

(*ii*) *Aluminium.* Aluminium powder, first treated with a wetting agent, is added to the tank and goes into general suspension in the circulating water. With the first type of powder employed, a fairly good visual pattern was obtained, but photographically the method was a failure. The aluminium showed up as extremely short streaks, apparently due to the disc-shaped nature of the particles, which change orientation rapidly as they move downstream.

Eventually a special grade of aluminium powder with approximately spherical particles was adopted, and this gives very satisfactory results (Fig. 6). The concentration normally used is 200 c.c. of powder to 250 gal. of water. The use of aluminium has the advantage that the angle of optimum reflection is about 90° to the incoming light, so that the camera can be placed vertically above the plane on which it is focused, thus avoiding distortion.

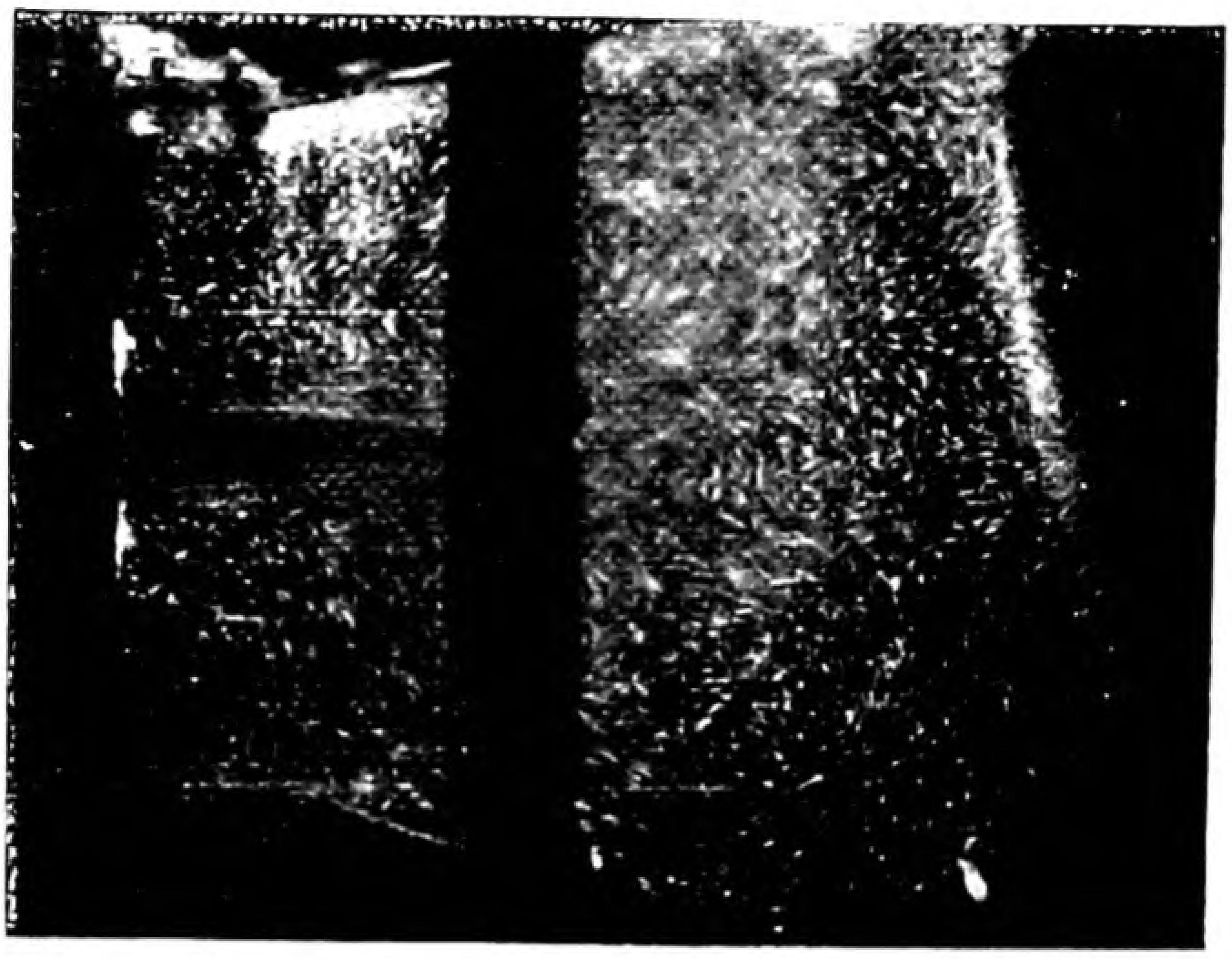


Fig. 6.—Flow pattern obtained on Maerz furnace using rounded aluminium particles.

This method has the disadvantage that air flow cannot be distinguished from gas flow in the furnace chamber. This was overcome by circulating clear water and injecting aluminium into the gas or air stream separately. The aluminium-laden water leaving the furnace was run to waste instead of returning to the tank for recirculation. The resulting photographs are of interest, but the method is not practicable as a routine owing to the excessive water consumption. Incidentally with aluminium powder of less than 120 mesh the tendency for particles to sink in passing through the model is negligible at the water velocities employed.

(*iii*) *Gas Bubbles.* A method has been devised in which tracer bubbles can be introduced into either the gas or the air system separately. Compressed gas, such as air or oxygen from a cylinder, is introduced into the base of the checkers through a fine needle valve. The injected gas appears in the water entering the furnace as a stream of small bubbles which serve as excellent tracers, both for visual observation and photography. The rate of flow of bubbles can be controlled so that the most suitable density of pattern for photography can be selected. With this method it is possible to get some idea of the mixing conditions within the furnace. The rate of buoyancy rise of the bubbles is insufficient to affect materially the observed pattern due to the relatively high water velocities employed. In practice, patterns using lighter-than-water bubbles as tracers show no obvious difference from patterns of the same flow conditions using heavier-than-water aluminium particles.

(*iv*) *Other Techniques.* The possibility of using other means of visualisation has not been overlooked and is referred to in the original paper[1].

Photography. A Leica model IIIA 35 mm. camera and a Stewartry copier are used for the recording. The camera lens is a 5 cm. Elmar of $f3{\cdot}5$ aperture. Focusing is carried out on the ground-glass screen of the copier, using the 2 kW lamp as illumination. Thereafter the flash tube is substituted for the lamp, and the camera for the ground-glass screen,

and an exposure is made by opening the camera shutter in darkness and then discharging the flash tube. Most of the photography has been done on Ilford FP3 film, developed in Ilford ID11 developer at 65°F., and given 50% over-development to increase contrast. Very good results have recently been obtained with Ilford recording film 5G91. This film is much faster than FP3 when used with the flash tube, and it has a higher contrast, but is not of such fine grain. Owing to shrinkage, ordinary bromide paper is unsuitable for producing positives on which streak lengths are to be measured. For this latter purpose Kodak waterproof bromide is used.

Photographs taken with the discharge tube have the advantage over those taken with a continuous light source, that the direction of movement of the particles is shown. The light intensity during a flash rises rapidly to a maximum and then dies away gradually as the charge on the condensers decreases. As a result, the track of an air bubble or aluminium particle appears to taper to a point in the direction in which it is moving. In the study of highly turbulent flows this feature is a great asset.

In the normal procedure, using air bubbles as a tracer, two photographs were taken at each position. The first was a " long " exposure of approximately 1/130 sec., to show direction and velocity of flow by means of streak lengths. The second was a " short " exposure of approximately 1/2,000 sec., in which all motion of the bubbles is " frozen," except near the incoming-gas port. Such a photograph consists of a large number of spots, from the distribution of which a picture of the mixing within the model may be obtained. In this case a tracer is introduced into one system at a time.

With exposures longer than about 1/100 sec., it becomes increasingly difficult to measure streak lengths on a photograph, owing to blurring and overlapping of streaks. This overlapping effect, however, results in more striking patterns in the photographs (Fig. 7) as it accentuates such features

Fig. 7.—Effect of long exposure on Maerz flow pattern photograph.

as vortices. For this purpose exposures of 1/20 to 1/60 sec. are most suitable. Very promising photographs of this type have been produced recently by developing to finality to obtain maximum contrast and threshold speed of the film. These exposures are made on Kodak Panatomic X film, using the 2 kW lamp, not overrun, so that with exposure times, of, say, 1/30 to 1/60 sec., the negatives would be considerably under-exposed, if processed normally. With this technique the resulting negative image shows a high degree of contrast, and excellent prints can be obtained.

Measurement of Films and Representation of Data. The method adopted is to measure streak lengths and directions on the photographs, having regard to certain special considerations. For this purpose it is necessary to know accurately the duration of the discharge-tube flashes for various choke settings.

In measuring streak lengths, certain points must be borne in mind: Firstly, only the component of velocity in the plane of the light sheet can be obtained by this means. To determine the component perpendicular to this plane a second photograph is necessary with the light sheet turned through a right-angle. This procedure has, as far as practicable, been carried out. The velocity component in the plane of light can be further analysed into two parts, e.g., for a plan section of the model, into "downstream velocity" and "transverse velocity." For flow through a simple channel the mean downstream velocity would represent the flow rate, while the transverse velocity would indicate the degree of turbulence. Flow in an open-hearth furnace is complex and of varying direction, so that this simple criterion does not hold. Visual examination of flow through a model furnace will indicate the general direction at any point and the plane in which the principal velocity component at that point will lie. In other words, study of velocity measurements is best carried out with prior knowledge of the general flow pattern as background.

Secondly, the motion of a tracer particle represents, not the motion of an individual molecule of fluid in the model, but the resultant motion of a large number of fluid molecules. A particle thus traces a path intermediate between that of the comparatively random motion of a molecule and that of the overall mean flow, a feature probably much to the advantage of the technique, since both mean flow and visible turbulence can be assessed.

Thirdly, tracer bubbles may not be illuminated for the whole of the duration of the flash. For example a fast moving bubble may be situated above the light sheet at the start of the flash, it may move down and through the light sheet, and continue some distance below before the flash ends.

On consideration it was realised that with a light sheet inclined at a large angle to the general flow direction the errors due to this effect would probably be so serious as to render the method useless. However, with the general flow direction in or near the plane of the light, errors from this cause are sufficiently small to allow comparisons of velocity to be made. For this reason, all velocity measurements have been made on photographs of sections along the length of the furnace. It must be noted that mean

velocity, as measured on the model, tends to be too low, and the error probably increases with increase of velocity.

Fourthly, for an accurate statistical analysis of velocities, a number of photographs would be required of each section studied, because of the limited number of measurable streaks normally found on any one negative. For such an analysis the following points should be considered :

(1) It is not possible to obtain directly equivalent velocity figures for the full-scale, owing to neglect of combustion and buoyancy in the model.

(2) From a few photographs a rough estimate of the general magnitudes of the velocities can be made. As it is most desirable not to strain the interpretation of the model technique by suggesting quantitative results which have in fact no counterpart in the full-scale, it is probably best to use velocity measurements only as indications of the general order of the gas speeds likely to be met in the full-scale furnace.

In addition to photography, it has been found useful to represent the flow pattern in plane sections by means of sketches. This method has the advantage that in a sketch it is possible to simplify the pattern very considerably to show up the essential features. A sketch, therefore, is of considerable assistance for descriptive purposes, especially for the benefit of those who have had no opportunity of viewing the model in operation, and who may have difficulty in appreciating the full significance of a photograph. Incidentally the value of such sketches would be greatly increased if some standard " symbols " were agreed.

Interpretation of results obtained by any of the above methods is complicated by the fact that the flow patterns through the furnace are essentially three-dimensional, whereas only two-dimensional plane sections can be photographed or measured. In practice, a rapid survey of the model with a moving light enables a mental picture of the flow to be built up that is both more dramatic and more representative than that deduced from the study of a series of photographs. To provide a simplified representation of the principal flow features, three-dimensional wire models are constructed using different colours to represent gas, air, partially burnt gases and waste gases.

HOT FURNACES

There would not appear to be any point in studying " two-dimensional " hot models, but three-dimensional hot models have already been shown to be both practicable and useful.

A. Geometrical Models. Little work has yet been done on this subject but a hot cone was recently constructed of approximately twice the size of the water model to see whether the flow pattern was in fact similar. The gas/air flame was introduced at the apex and its path and that of the resulting waste gases traced by means of flame trails from pitch-pellets or pieces of wood introduced at appropriate points. The preliminary answer as shown in Fig. 8 is that the pattern in a hot model is very like that shown by the water model. The flame from pitch placed on the furnace bottom

runs out radially and then turns up the walls in just the way the water does in two and three-dimensional water models. Similarly the flame from a piece of wood inserted through the sloping wall runs upward and parallel to it and is subsequently entrained by the central jet.

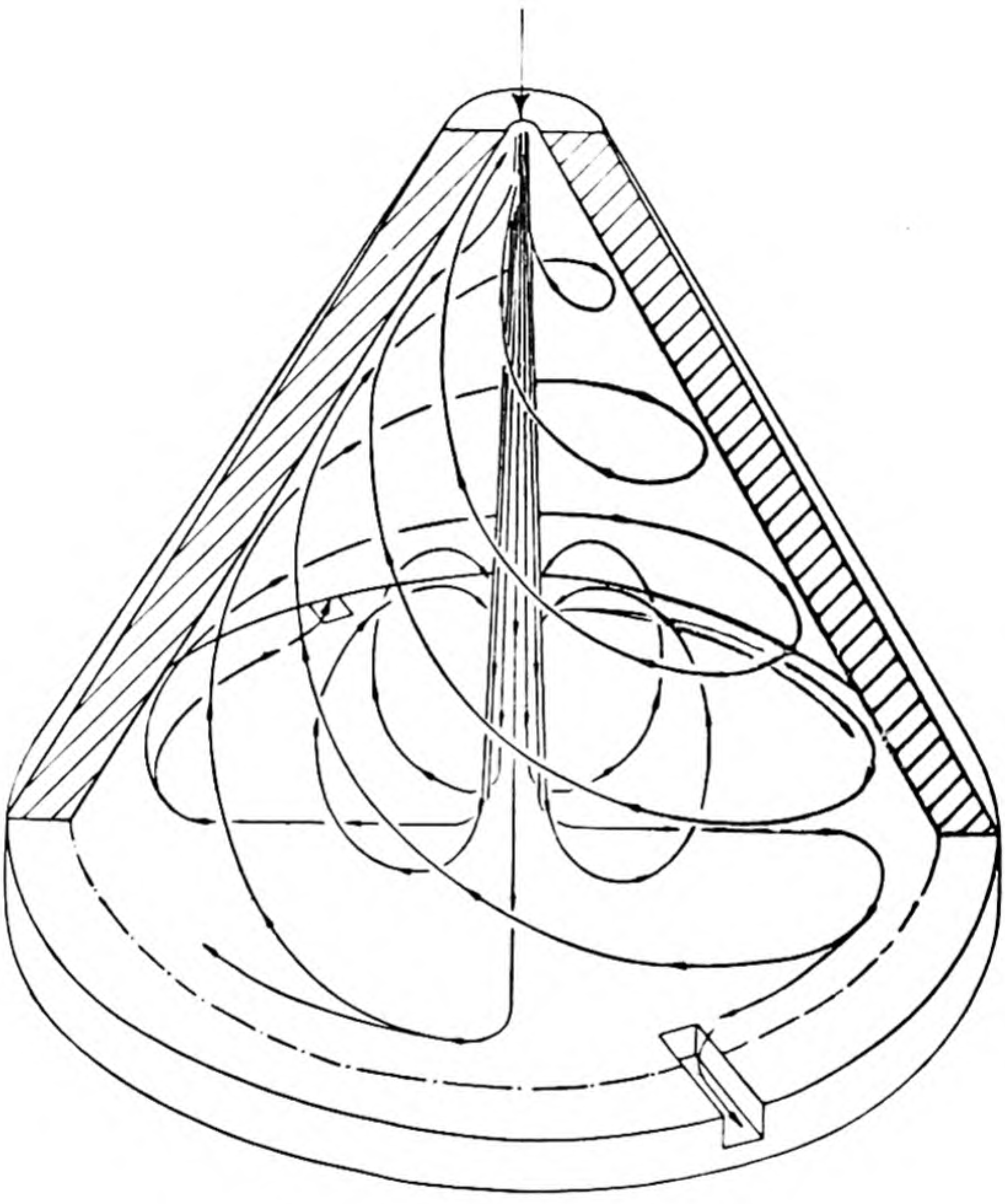


Fig. 8.—Flow pattern in a hot cone.

B. Scale Models. Work has now started on hot scale models, but it is too early to state results. Reference can, however, be made to the extensive work of Leckie[4] and his colleagues, who have made a careful study of the influence of port design and operating conditions on heat transfer in large-scale models. For flow pattern work much smaller models, say 1/12 or 1/24 full size might well give sufficiently reliable results. The problem of similarity is, however, an extremely difficult one and it is unlikely that any one model can adequately represent both the heat transfer and the flow pattern in the production unit. It is likely, however, that envelope shape and the ratio of momenta in the different jets will still be the factors dominating the flow pattern obtained.

C. Full-Scale Furnaces. Since the publication of the water model work by the Iron & Steel Institute a number of checks have been carried out on full-scale furnaces, in particular on the Maerz and single-uptake types. The results on the latter are shown diagrammatically in Fig. 9. The methods

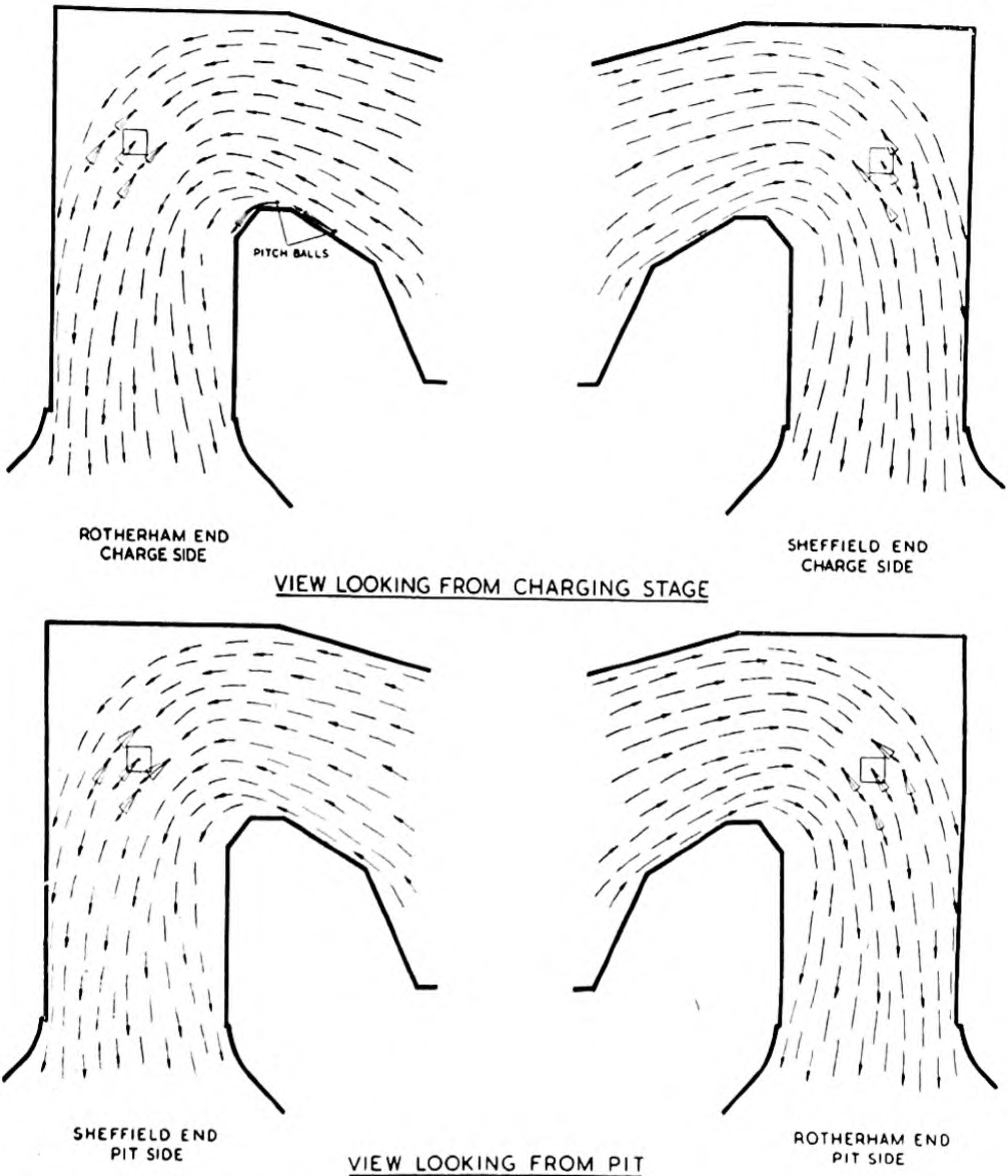


Fig. 9.—Waste gas flow determined on actual furnace by wooden tracer blocks (flared arrows) superimposed on flow pattern determined by water model.

employed in this check are admittedly crude, but the correlation with flow pattern in the model is so close as to demonstrate beyond any doubt the value of model work. Not only was the reverse flow opposite the first door fully confirmed but the converging flow at the exit end and the peculiar effects of air infiltration were as far as could be observed identical with those obtained in the corresponding model. The technique is at present limited to the introduction of tracers such as wooden blocks, either

as received or boiled in tar, pitch-balls and sawdust, though attempts have been made to use a large number of other tracers, e.g., aluminium powder, phosphorus pentoxide and iodine. The results with these special tracers have, however, for the most part been disappointing, though iodine does under certain conditions give a very striking purple trail that may last for several feet before disappearing.

Tracers giving characteristic flame colours, for example the strontium and barium used in the Templeborough trials for measuring gas velocities[3] are not particularly satisfactory, since they only give visualisation if introduced into the flame itself. The explanation of this phenomenon is presumably that the temperatures obtained in the open-hearth furnace although high (1700°C) are insufficient to excite these elements, and that the flame colour is only seen when chemical energy available from combustion causes excitation.

A further correlation between model and full-scale practice is given by a check on the flow pattern in the ends of an experimental furnace. The pattern recorded photographically as Fig. 3*a* was checked on the actual furnace by inserting wooden blocks held in the end of a steel tube through spyholes built on the charge or stage side of each of the four uptakes. Fig. 10

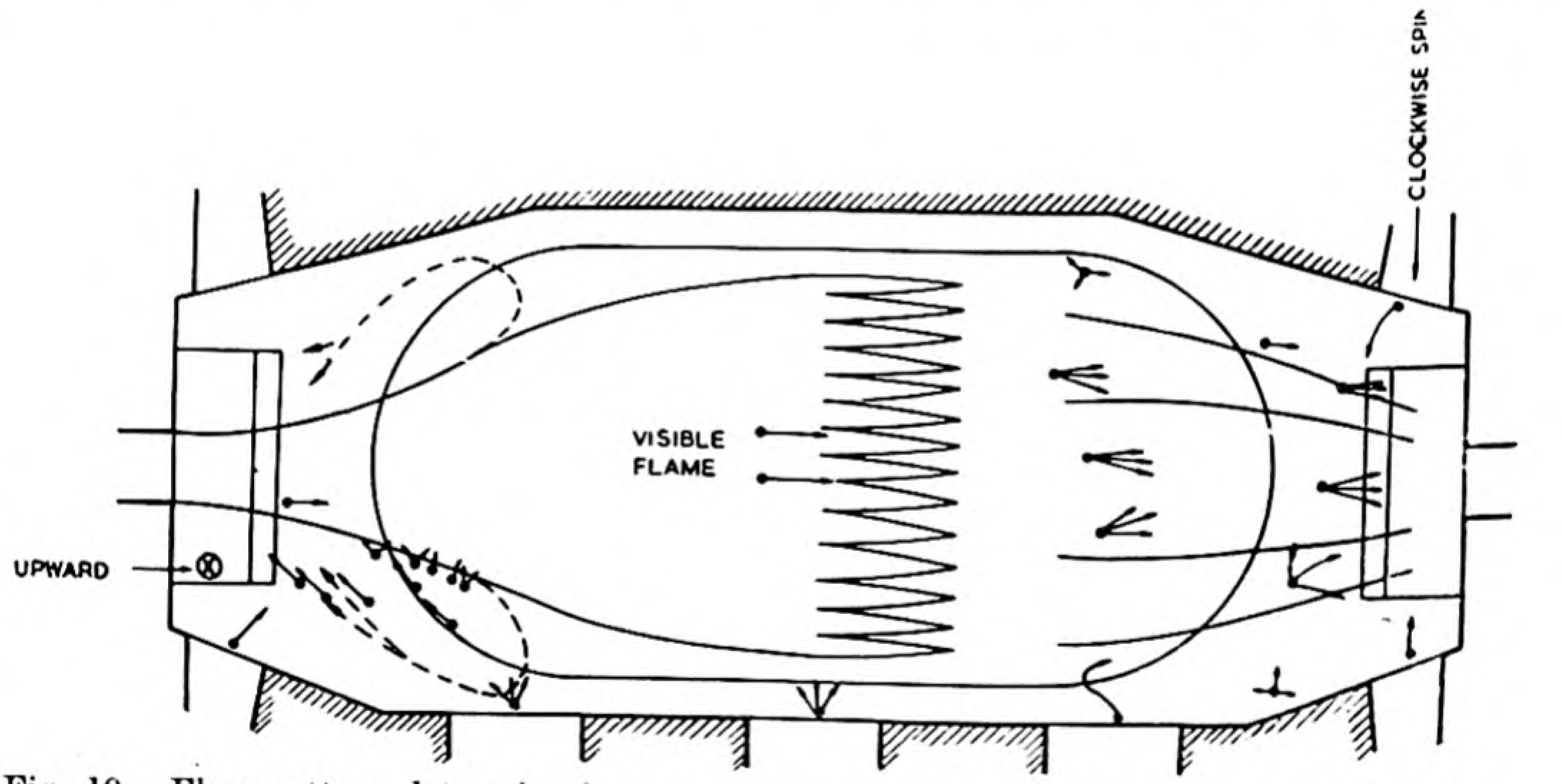


Fig. 10.—Flow pattern determinations on an actual furnace (single uptake) showing close agreement with pattern obtained in water model. Each black dot indicates the position of a tracer block and the arrows the smoke trail direction.

is a composite diagram in which the flame directions obtained by suhc visualisation are compared with the general flow lines as traced from a two-dimensional water model. In practice there was a slight flicker on the flame leaving the tracer block, the extent of which is shown by the fanning-out of the arrow. The check between the two sets of patterns is incredibly close. The pattern shown in Fig. 10 refers to a furnace temperature of 1600°C, but at 1200°C, when the oil, air, and, therefore, waste gas flows, were very much slower, a similar pattern was obtained. This is again in line with model work, where the pattern is not greatly affected by even a large decrease in flow rate.

It will be seen from the above that techniques are now available for the study of flow in two and three-dimensional furnace models, and that the preliminary checks with practice are extraordinarily close. If the flow patterns likely to give optimum results, both as regards combustion and refractory wear, can be specified there is every hope that they can be produced in practice by means of preliminary work on water and hot models of 1/12 or 1/24 scale.

ACKNOWLEDGMENTS

In conclusion we should like to express our thanks to Mr. F. H. Saniter, Director of Research of The United Steel Companies Limited, for permission to publish this work, and to numerous colleagues both in this Department and in our various steelplants for their helpful criticism and guidance.

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11

Fluid Flow in Relation to the Manufacture of Steel

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ABSTRACT. Problems of fluid flow arise in all the main stages in the manufacture of steel; for example in the production of iron in the Blast Furnace, the production of steel in the Open Hearth Furnace and Converter and in the casting of metal.

These processes present a wide field for further research both on the full scale plant and on models which have proved very powerful for investigating flow phenomena.

For investigations on full scale plant new instruments are required and with the development of these to enable accurate comparisons to be made between full scale plant and models the value of work on models will be greatly increased.

1. INTRODUCTION

At almost every stage of the manufacture of steel there are processes which depend upon the flow of fluids either liquid or gaseous at both high and low temperatures. For example liquid steel, water for cooling purposes and combustion gases both before and after preheating.

This paper will consider some of the problems in fluid flow which arise in the various processes of the steelplant and recent work done in connexion with them and especially that carried out by the British Iron and Steel Research Association.

2. IRONMAKING—BLAST FURNACE

In the blast furnace large quantities of air are used to consume coke to form carbon monoxide for reduction of the ore and to provide heat. The main aerodynamic problems arise in connexion with the distribution of the blast within the furnace stack and the cleaning of the gases which leave the furnace (Fig. 1). The furnaces are normally blown with air at pressures up to 15 lbs./in.^{2} and a furnace may take 50,000 ft.^{3}/min. of air. Considerable power is thus used in the blowing of furnaces and the blowing apparatus has to have as high an efficiency as possible and the duct systems conveying the air to the furnace tuyeres have to have as low a resistance as possible. These problems of air supply are, however, not peculiar to the steel industry.

The air is heated to a temperature of the order 400°C.—500°C. before being fed into the furnace and this is accomplished in what are called hot-blast stoves (Fig. 2). The heat is supplied by burning gas from the blast furnace and storing the heat in checkerwork chambers. Later the burning gas is cut off and the blast to the furnace is passed through the stove and heated in its passage through the checkerwork. It is found that if the gas from the furnace, which contains dust having a high percentage of ferrous oxide, is not cleaned, then a considerable erosion of the stove brickwork occurs(1). The expense of repairs is found to be higher than the cost of operating efficient cleaning plant for the gas. When clean gas is used very little deterioration takes place and a stove may be used for a number of years. The effect of ferrous oxide will be dealt with later in considering

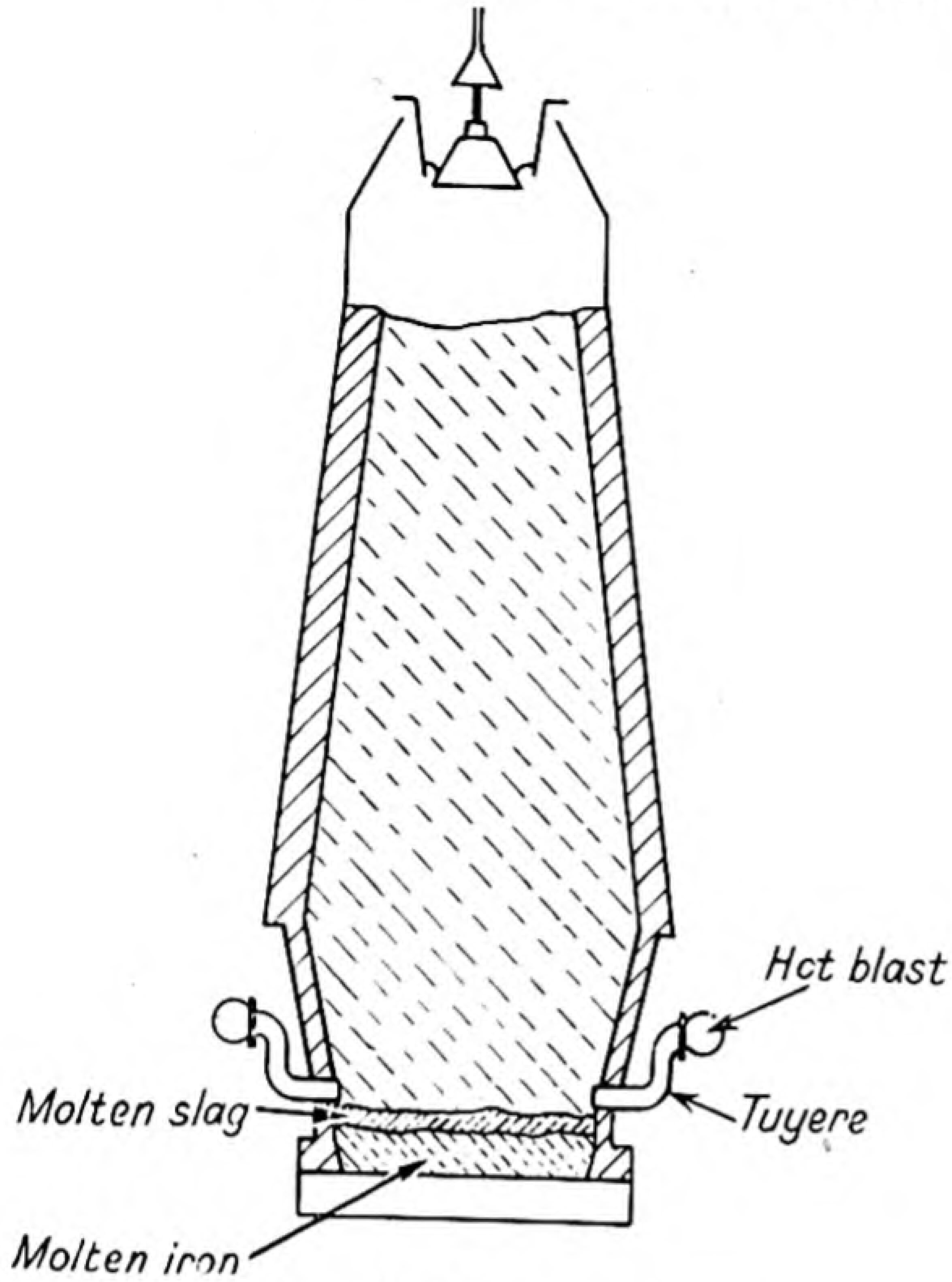


Fig. 1.—Blast furnace.

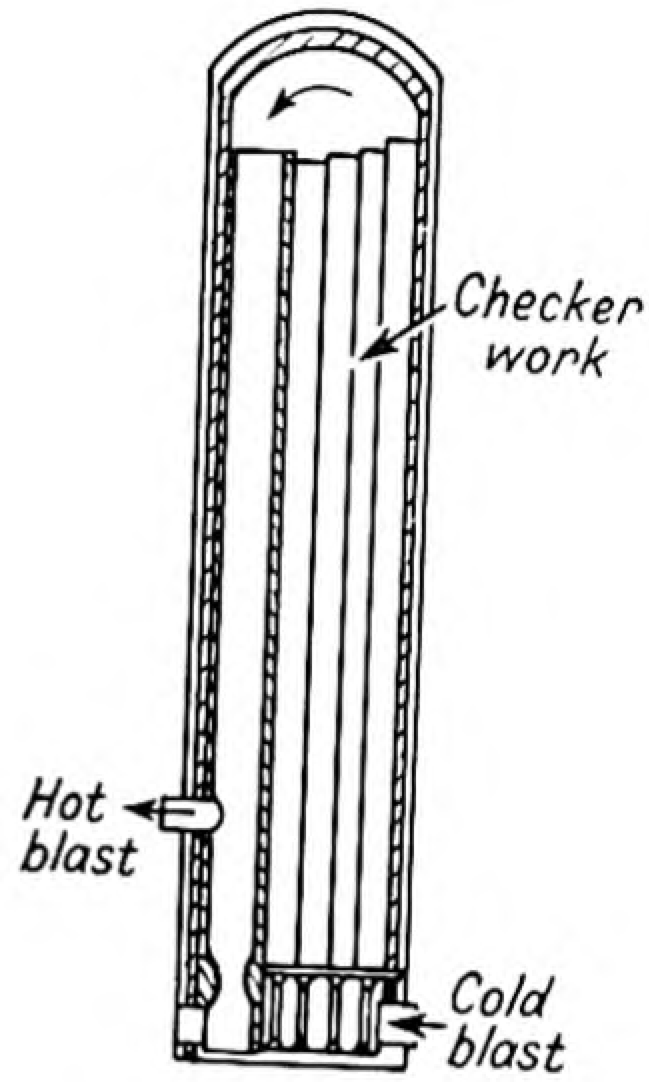


Fig. 2.—Hot blast stove.

the open hearth furnace where similar erosive reactions occur; for the moment we must note the necessity for high gas cleanliness which is needed not only for the gas used in the stoves, but also for that used elsewhere either in the steelworks or outside.

The separation of dust from blast furnace gas usually takes place in several stages.(2) The first stage consists of a large settling chamber in which the coarser particles are removed and this is followed by passage through spray towers followed by electrostatic precipitators or disintegrators. As the volumes of gas which have to be dealt with are so large, as high an efficiency of separation as possible is desirable. The design of cleaning apparatus is limited by the small pressure losses which can be permitted owing to the small positive pressure at which the gases leave the furnace.

The design of more efficient cleaning apparatus is one in which, as mentioned by Mr. R. L. Brown, in the opening session of the Conference, much must still be learned in spite of recent increase in work on this subject. In the stack of the blast furnace itself, a knowledge of the distribution of blast would be of great value in order to understand the chemical and physical processes occurring.

Very little work on this problem had been done until recently; the work by Kinney(3) using pitot tubes driven into the burden is open to grave criticisms both in regard to the calibration of the apparatus and the velocities deduced from the measurements. Recently(4) the use of radon to measure the times of transit of the gases from the tuyere to the top of the stack has shown the complexity of the subject and that considerable variations can

occur according to the condition of the furnace and that in general the gases flow up the walls of the furnace faster than in the centre. The distribution of gases in the stack of the furnace must depend upon the size and shape of the cavities which exist in the burden in front of the tuyeres. Segregation of fine materials may form a barrier, relatively impermeable to the blast, between the walls and centre of the furnace. If the cavity in front of the tuyeres can be made to extend further into the burden it may be possible for the blast to penetrate initially through this barrier and thus give a more even distribution all the way up the stack. Preliminary experiments[5] have shown that in a model employing ammonium carbonate as the burden and using a hot air blast the cavity size can be controlled by the velocity of the blast and further experiments are now under way to determine whether the effect is likely to be of practical value, bearing in mind the limited range of pressures which it is practicable to employ on actual furnaces.

3. STEELMAKING

3.1. *Open Hearth Furnace.*

Much more work has been done to determine the characteristics of the flow processes occurring in the open hearth furnace than in the blast furnace.

Gas from producers, after having been heated in checker chambers below the furnace enters the furnace chamber through a port at the end of the furnace. The gas enters the chamber almost horizontally but with a slight downward component of velocity to produce a long, approximately horizontal flame. The air for the combustion enters the chamber through ports close to the gas port and the flame is produced where the air and gas mix together turbulently. The exact configuration of the ports differs in different furnaces. In many furnaces the gas port projects into the furnace chamber and the air for combustion rises vertically through two ports in

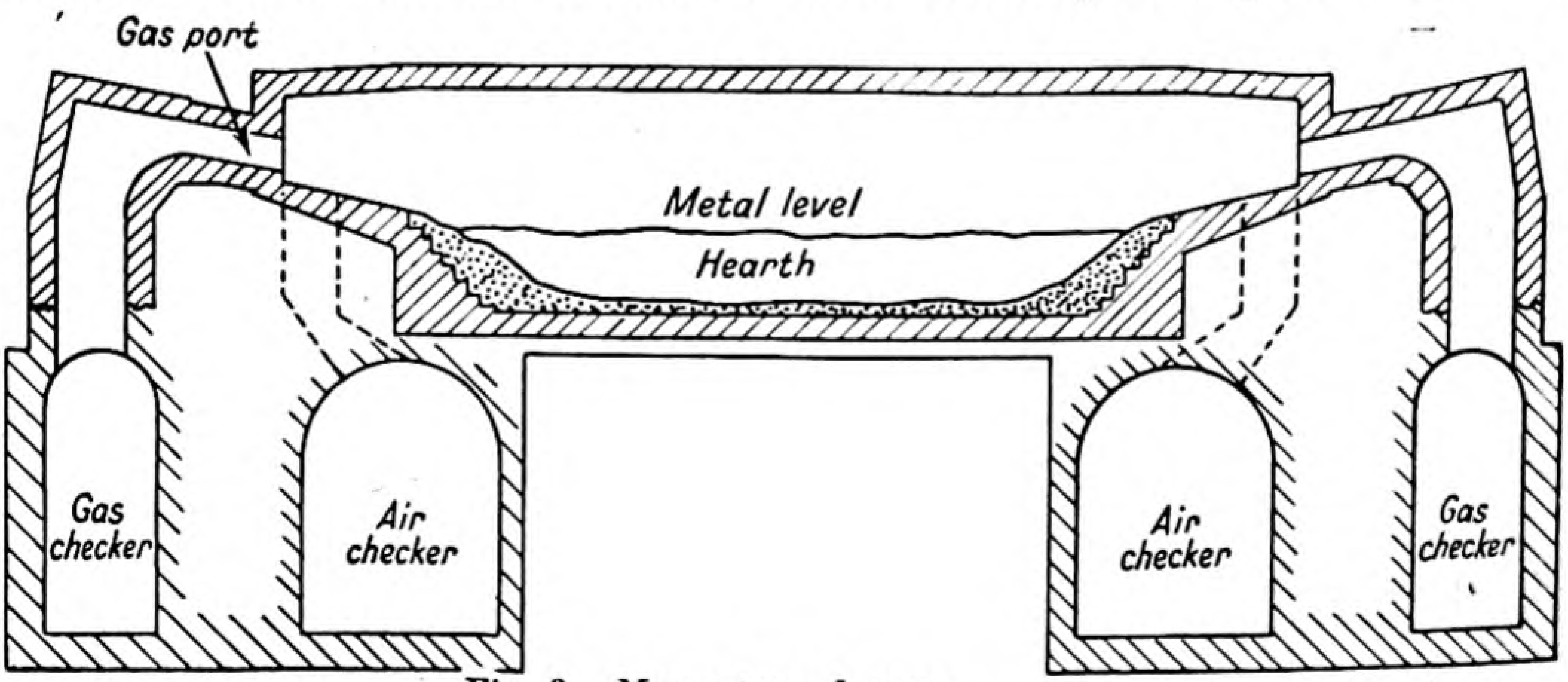


Fig. 3.—Maerz type furnace.

the floor and on either side of the gas port. In the Maerz design (Fig. 3) the gas port is shortened and takes the form of an aperture in the end wall of the furnace. A modern variant of this has a single air port below the gas port.

Another design of furnace is the so-called semi-Venturi furnace in which the gas port is surrounded by a converging chamber or Venturi throat in which the air converges towards the gas from both sides and from above (Fig. 4). Various modifications of these characteristic designs also exist.

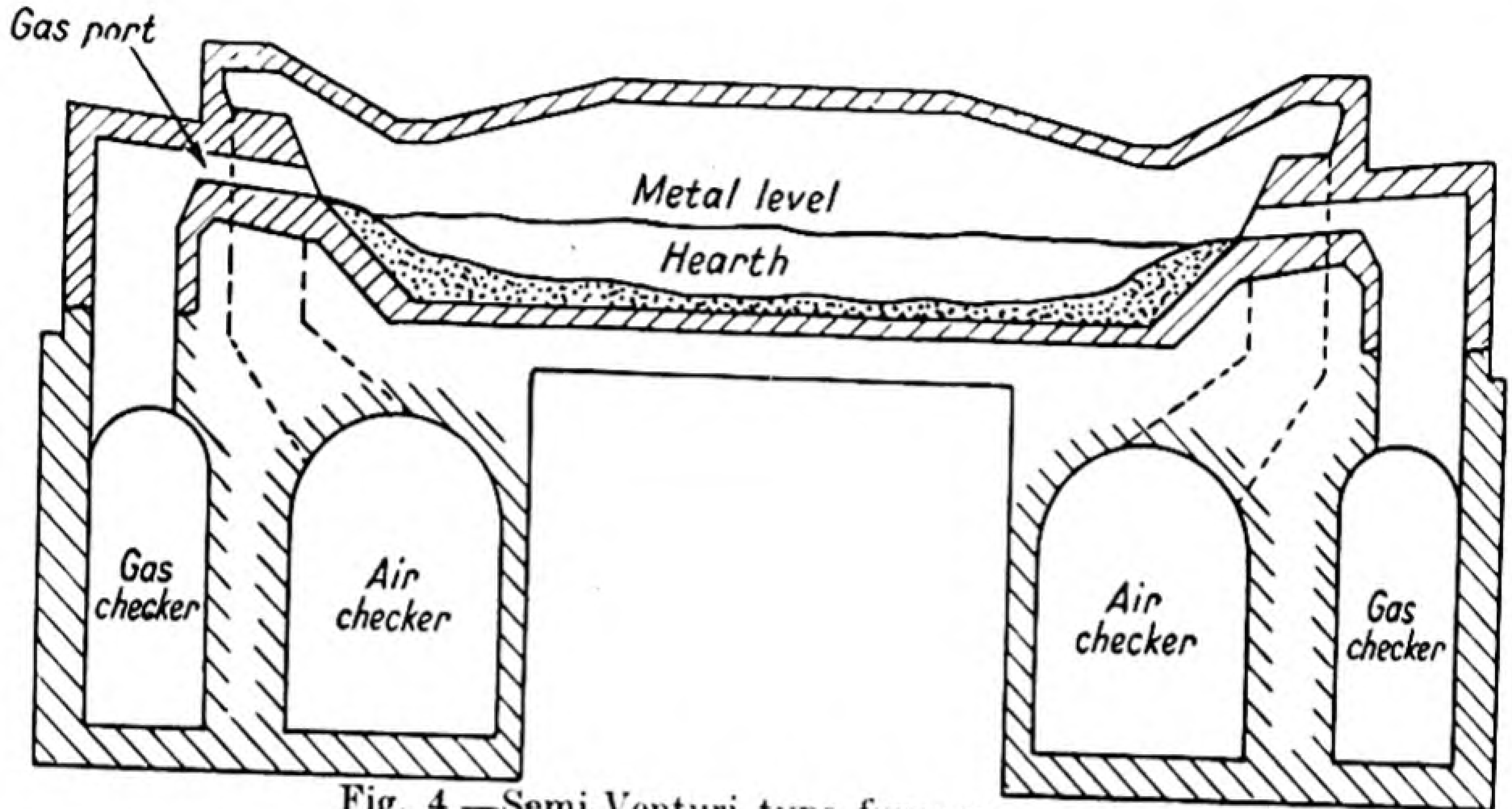


Fig. 4.—Semi-Venturi type furnace.

The essential purpose of these furnaces is to melt steel scrap, pig iron and other materials charged into the furnace and to keep them at a sufficiently high temperature for the refining processes to proceed during which the impurities, especially excess carbon are removed. The furnaces are symmetrical, the outgoing gases being used to heat the checkers. Periodically the direction of gas and air flow is reversed and the heat stored in the checker chambers is used for preheating the air and fuel gas. From the point of view of fluid flow and the interrelation of thermal and aerodynamic processes, these furnaces present a number of interesting problems.

In the first place, very little is known of the flow patterns of the gases in the furnace, or of the effect which the type of flow has on the combustion of the fuel and consequently upon the transfer of heat to the hearth of the furnace. The development of the flame depends a great deal on the mixing of the air and fuel gases within the furnace chamber. This rate of mixing will be determined by the design of the chamber as a whole, and also by the positions of the ports in relation to each other. In particular, the positions of ports and their sizes will greatly influence the rate of mixing of the air and gas within the furnace chamber. From the point of view of heat transfer to the bath, it is generally desirable for the mixing to take place as soon as possible after the gases have entered the furnace chamber. On the other hand, if the mixing is too rapid, and the combustion takes place too quickly, too much heat may be liberated at the ingoing end of the furnace and cause overheating of the furnace structure and of the roof in particular. Great care has to be taken that the roof temperature does not exceed the melting temperature of the refractory materials. On the other hand, the temperature of the roof must be as high as possible in order to transfer as much heat as possible to the bath and as the temperature at which the roof

melts is only a little above the temperature of the steel in the bath, the range of temperatures which can be permitted on the roof is very small. Therefore, for efficient operation, the distribution of roof temperature must be very uniform. This distribution of roof temperature will depend considerably upon the type of flow, which in turn will depend upon the rate of mixing of the air and gas in the furnace chamber.

A very wide field of research still remains to be covered to determine the thermal properties of the flame in terms of the aerodynamic conditions under which the air and gas enter the furnace chamber. So far, two lines of attack have been followed in the open hearth furnace investigations.

One of these has limited itself to determining the flow patterns of the gases in the chamber and the other to determining the rates of mixing between fuel and air. So far, most of these investigations have had to be carried out by the use of models, owing in the first place to the difficulty of carrying out measurements on the full scale furnace when it is in production and secondly, to the fact that no adequate instruments have yet been constructed to measure gas velocity and direction in a furnace at operating temperatures.

3.2. *Mixing Experiments.*

Some experiments on mixing at high temperatures have been carried out mainly by Kofler and Schefels(6) in Germany on the open hearth furnace and more fundamentally by Rummel(7). Much more extensive and accurate work needs to be done in both of these directions. As a result, models have been constructed in the B.I.S.R.A. Aerodynamics Laboratory, to determine mixing(8). The technique which has been applied is to use an infra-red gas analyser to determine the concentration of carbon dioxide in samples of the gas drawn from the model. It operates on air at room temperature without combustion, has carbon dioxide introduced into air passing through the gas port and air is passed through the air ports.(9)

Samples of the air inside the chamber are withdrawn continuously through a moveable probe and passed through one of the tubes of an infra-red gas analyser, and air from the air port is passed through the other tube. Full scale deflexion of the scale on the analyser may be obtained with a concentration of carbon dioxide of only 0·03%. This corresponds in the model which was 1/12 scale to a flow of 2 l. of carbon dioxide per minute when run under conditions of Reynolds similarity. This quantity of carbon dioxide may be safely run to waste in the laboratory provided it is normally ventilated.

Precautions have to be taken to ensure that the gases are aspirating through the two tubes at the same rate and that the transit times of the gases from the apparatus to each of the absorption tubes in the analyser are the same. This precludes unbalance of the system due to variations of carbon dioxide in the supply of atmospheric air. All the air for the three ports is drawn from a single source. The sensitivity of the instrument changes if the absolute concentration of carbon dioxide in the room increases and a check, followed if necessary by adjustment, is made between readings. The model gives an accurate picture of the mixing which occurs on the full

scale furnace in so far as this is governed by dynamic processes. On the other hand, thermal processes will not be taken into account, as the model has to be operated at one temperature which, for convenience, is taken as normal atmospheric temperature. These thermal processes include buoyancy due to local variations of temperature and to changes of gas volume due to combustion.

The results of these experiments have shown that in a Maerz furnace as normally constructed, the mixing between the air and gas is relatively slow, owing principally to the fact that the air stream and gas stream do not impinge directly upon each other. Fig. 5 shows a series of contour patterns of the mean concentration of tracer gas across three successive sections of a furnace the third section being half-way between the two ends of the furnace. The region of active burning on the full scale furnace will occur where the mean concentrations of gas are between 40% and 50%. Outside this belt there will be an excess of air and inside an excess of gas. The mixing rate can be greatly accelerated by the modified design in which the air port is brought directly below the gas port so that the two streams impinge as soon as gases enter the furnace.

Using designs of this latter type, it is possible to obtain almost perfect mixing of the two streams by the time they reach the middle door of the furnace. Fig. 6 shows contours at the same series of planes when a flat gas port is used together with an air port directly below it. Care has to be taken that the vertical velocity of the air is not too great, otherwise the flame will be directed towards the roof of the chamber and the heat transfer to the bath will consequently be considerably reduced. There is also great danger of exceeding the permissible roof temperature with consequent damage to the roof.

Summarising the results of the mixing experiments on the Maerz type furnace, we may say that the rate of mixing between the air and gas can be increased in three ways:

(1) By increasing the perimeter of the gas stream where it is exposed to the surrounding air, so that a larger area is presented through which the air can diffuse. This is obtained in the flat port just mentioned.

(2) By increasing the rate of turbulent diffusion in the gas stream. This occurs when the air port is placed directly below the gas port.

(3) By excluding recirculation of waste gases into the first part of the flame. This is much more difficult to bring about as it depends upon the general shape of the furnace chamber and also on the directions of all the streams entering the furnace. Control of this recirculating stream is brought about to some extent in the semi-Venturi type of furnace where the narrow throat prevents the recirculated gases from penetrating to the beginning of the flame and confines it to the regions where the combustion is already well advanced.

Experiments on models of furnaces of the semi-Venturi type have shown that the general principles for efficient mixing given above for the Maerz furnace are applicable here also.[8] Fig. 7 shows the mixing contours

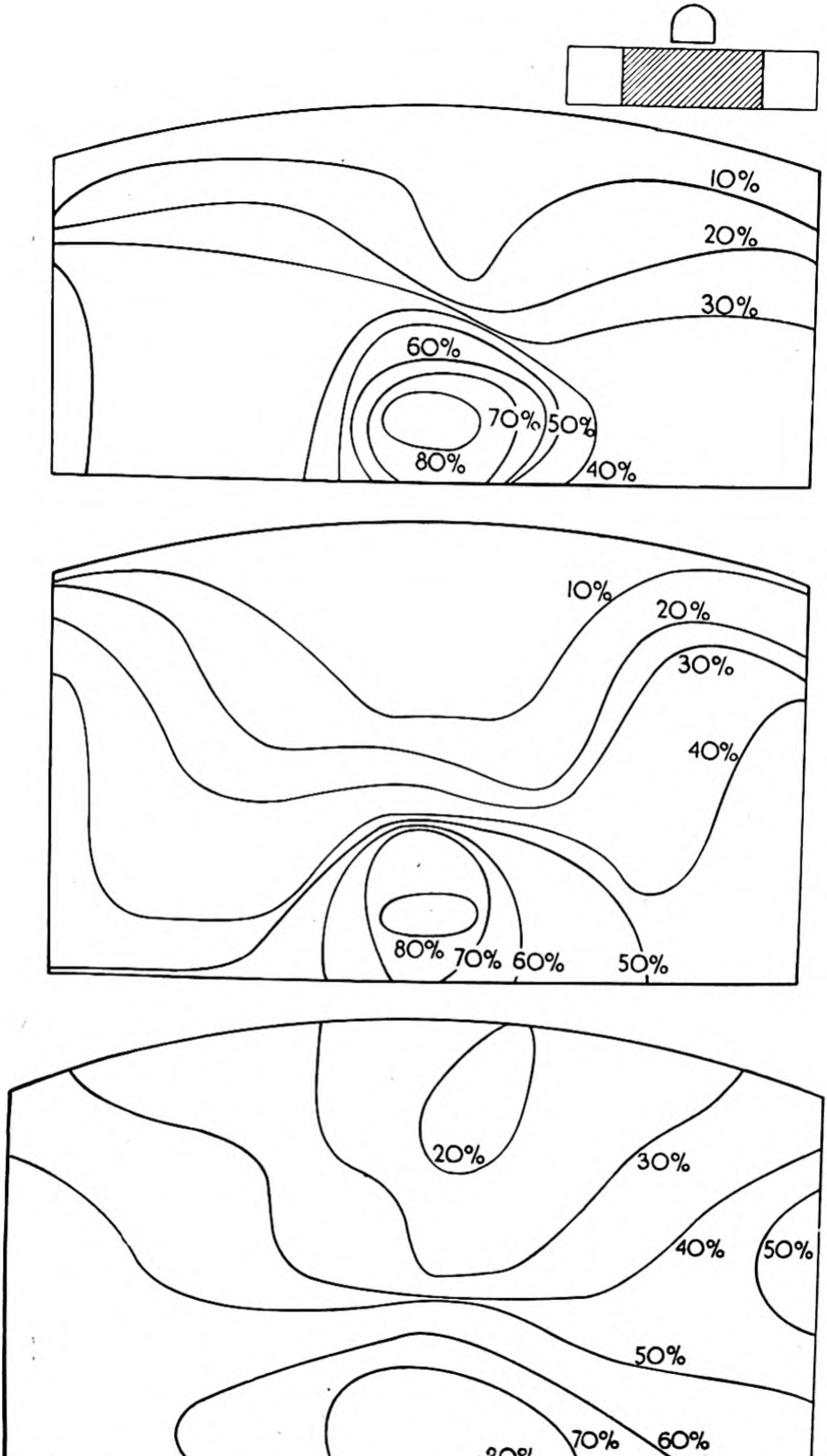


Fig. 5.—Mixing contours in a Maerz furnace model.

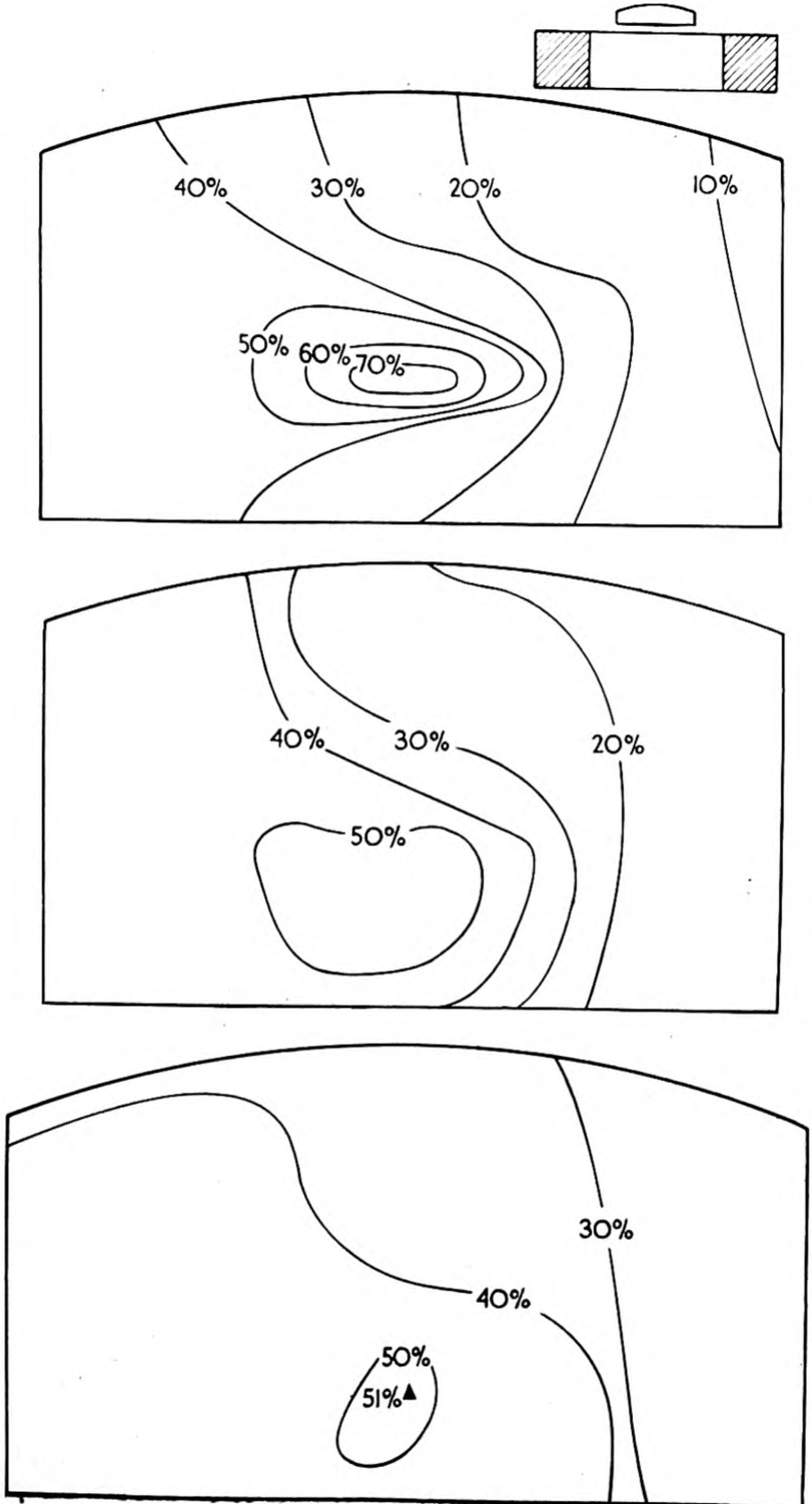


Fig. 6.—Mixing contours in a modified Maerz furnace model.

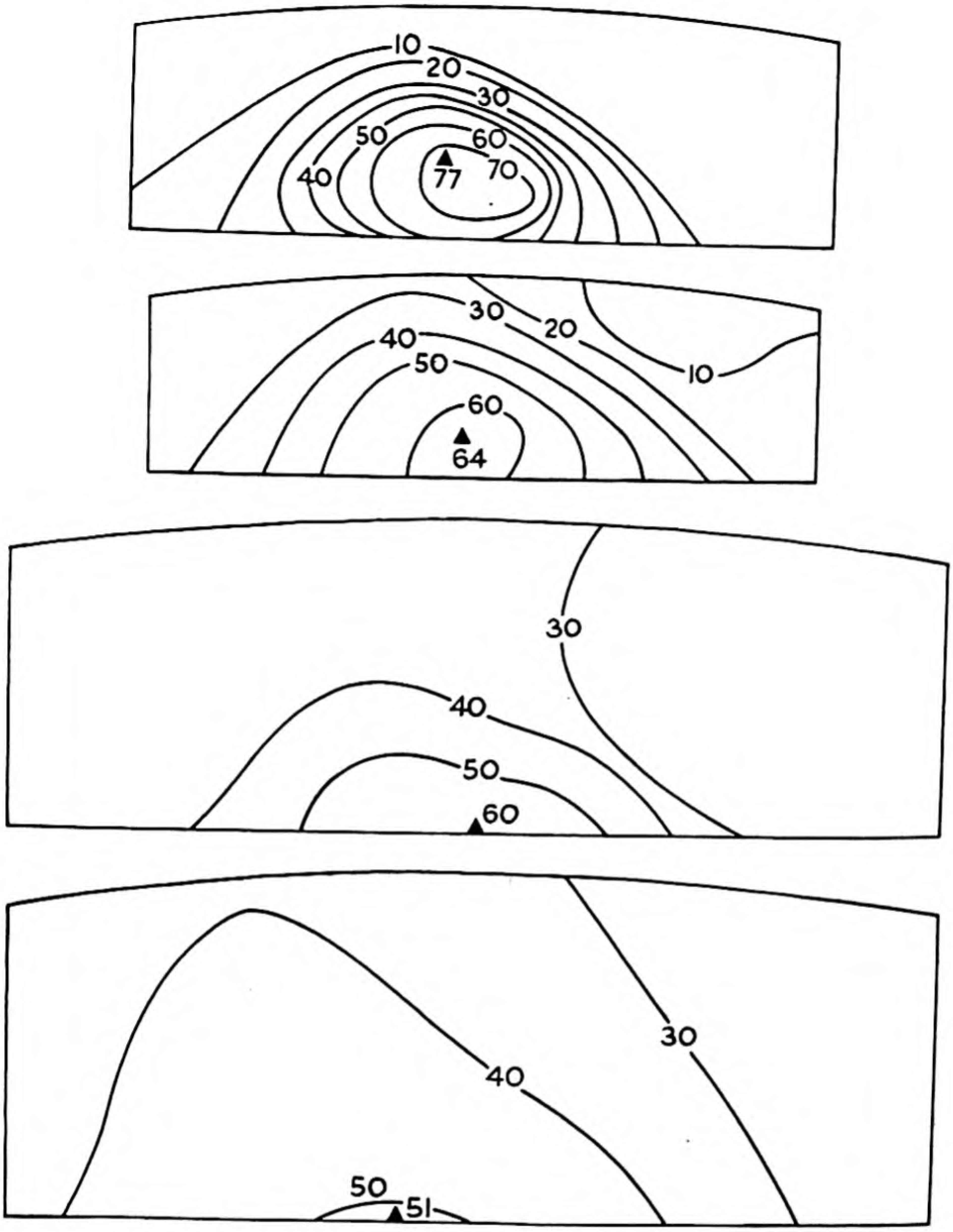


Fig. 7.—Mixing contours in a semi-Venturi furnace model.

obtained in a model of a semi-Venturi furnace. If several gas ports are used, the rate of mixing is very much increased, an effect which is analogous to that obtained with the flat port in the Maerz design. Fig. 8 shows the very great increase in the rate of mixing which is obtained when three equal circular gas ports, evenly spaced in the end wall, are used in place of the normal single port. Their combined area equals that of the original port and the distance between their centres is twice the diameter of each port. Again, if the air is introduced immediately below the gas, a very great increase of mixing rate again results. Considerable increase of mixing rate is also found if the slope of the gas port is reduced. This is because air can penetrate to some extent underneath the gas and diffusion of air

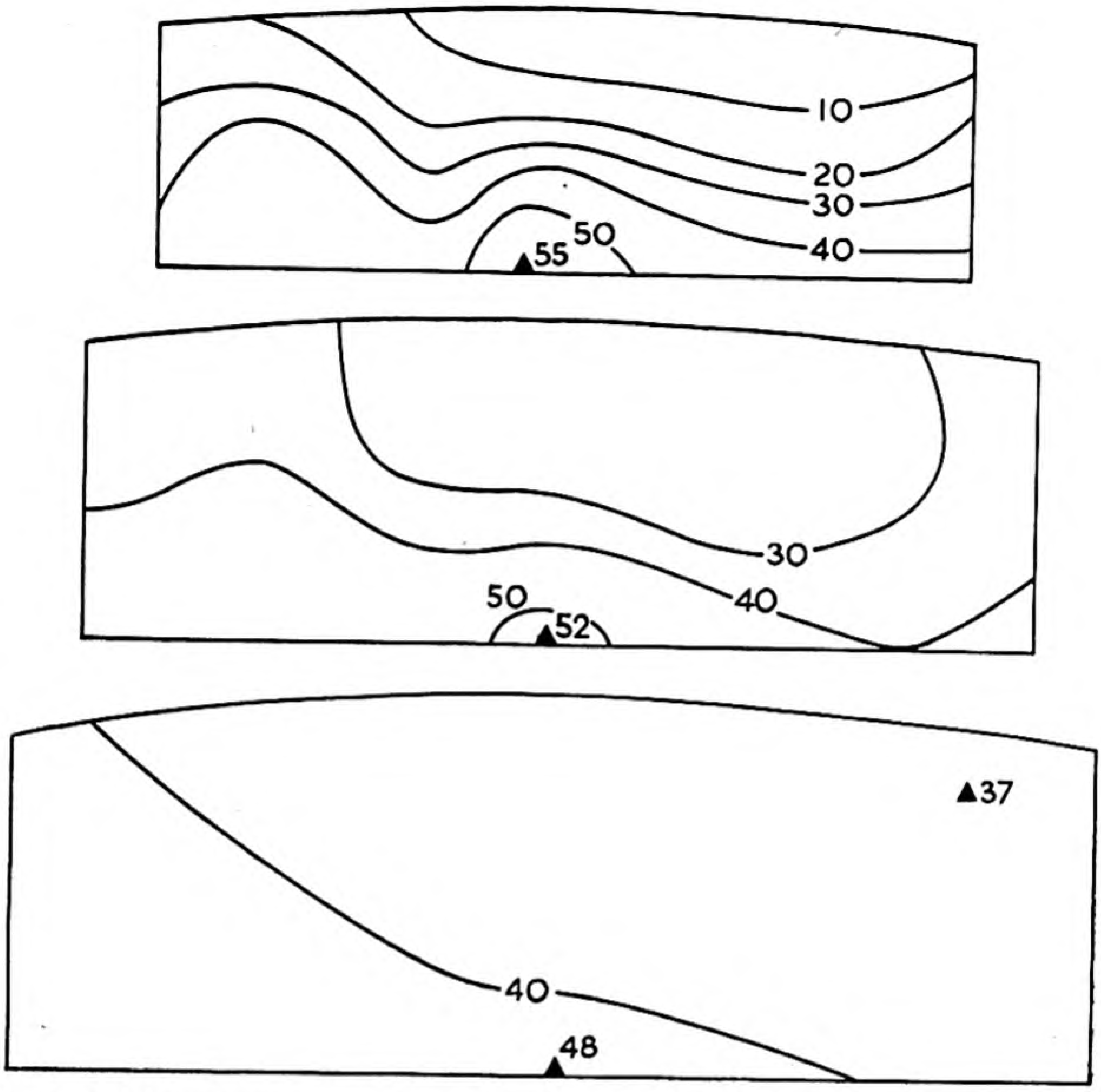


Fig. 8.—Mixing contours in a semi-Venturi furnace model with multiple gas ports.

into the gas can then take place from all sides. Moderate increases in the rate of mixing can also be obtained in the following ways :

(1) By shortening the gas port, which allows the mixing to begin sooner than for a port of normal length and thus increases the degree of mixing at all subsequent points in the furnace chamber.

(2) By lengthening the throat and bringing it nearer to the hearth which improves the general mixing by excluding recirculated gases for a greater distance along the first part of the flame.

(3) By contracting the throat more sharply, which throws air round the gas jet and thus prevents recirculated gases from reaching the gas for some distance downstream of the throat, the effect is thus rather similar to that of No. 2.

Increasing the ratio of air to gas will increase the proportion of air in the recirculated gases and also reduces the amount which is entrained and hence bring about more rapid combustion.

In all the experiments on mixing the conclusion was reached that provided no solid objects projected into the volume which the jet normally occupied no significant change appeared in the entrainment. This is in agreement with the experiments of Rummel[7] on mixing. This result

is important since the flow outside the jet must obviously be considerably altered if solid objects are introduced into it or if the walls of the containing chamber are modified.

As air velocities outside the jet are small compared with those inside we expect to find that the rate at which fluid is entrained at each point and the rate at which the fluid in the jet mixes to attain an even concentration will be dependent only on two factors : the dimensions of the port from which it emerges and the position and shape of any surface against which it strikes.

The throat of a semi-Venturi furnace will not by this theory affect the amount of fluid entrained by the jet or the way in which this entrained fluid mixes with that already in the jet but may have a profound influence on the flow outside it. And if this in turn results in a change of concentration in the region immediately surrounding the jet from which the entrained fluid is drawn then the downstream concentrations in the jet will also be altered, for although the same quantity of fluid is drawn in, its composition is different.

The results obtained are in agreement with this theory. It is found that subdivision of the jet produces an extremely powerful effect of the same order as is found in subdividing a free jet. Next to this the greatest change is produced by altering the angle at which the jet strikes the bath. Increasing the angle makes the jet spread over the floor and excludes air from reaching the underside. On the other hand the deliberate provision of air to this surface counteracted the effect and greatly improved mixing.

Limited improvements could be obtained by changes to the walls and roof of the furnace and these were always accompanied by a change in the concentrations around the jet to account for them. Furthermore, the effect is downstream from the point of modification. The reason why the improvement to be expected by altering the throat dimensions is limited, is that even a wide throat is effective in almost completely reducing back-flow past it while a sharp contraction will only project air around the jet for a short distance. By using multiple jets or throwing the air at the gas jet (which increases the rate of turbulent diffusion as well as excluding back flow) the jet can be made to entrain enough air for combustion before the throat is reached.

Changes to structure which do not affect the concentrations round the jet such as air port area or the outer brickwork of the gas port showed no effect on mixing though they no doubt alter the flow pattern locally and may be of importance in other respects, e.g., erosion, waste gas partition, etc.

At a later date it is intended that a series of experiments shall be undertaken on the rate at which fluid is entrained at each point of a jet which strikes a plane surface, and on its subsequent distribution. With the help of such data it is hoped that a theory of mixing in jets in enclosed chambers can be developed.

The semi-Venturi furnace is a popular design and the reason for this may perhaps be found in the large recirculating stream of gases which occurs below the roof. This may ensure an even roof temperature and also act as a blanket between the roof of the furnace and the flame and enable a larger amount of fuel to be burnt in the furnace chamber.

3.3. *Flow Visualisation.*

In the other line of attack, that is in the determination of the flow patterns in the furnace chamber, a considerable amount of work has also been done(10) and this is being discussed by Dr. J. H. Chesters in this Conference and has also been discussed by him and others elsewhere. These have shown a considerable complexity of flow patterns inside the working chamber of the furnace and the general existence of recirculating eddies of gas from the outgoing end of the furnace towards the ingoing end. The direct consequences of this are not yet understood, but thermodynamically, the recirculation probably increases the efficiency of the furnace considerably by helping to bring about an even distribution of temperature. The relation between the flow patterns and the heat transfer is at present being investigated, but it is not yet possible to give definite results. The importance of recirculation in controlling mixing concentrations by altering the composition of entrained gases has been referred to above.

3.4. *Erosion.*

Investigations of great practical importance concern the relation of the pattern of the flowing gases with the erosion of the refractory bricks from which the furnace is constructed. Sometimes very considerable local erosion occurs in furnaces and this generally seems to be connected with the type of gas flow occurring in the neighbourhood of the eroded area(11). The erosion appears to be largely brought about by the chemical reaction of the refractory materials with ferrous oxide. This material is presumably picked up from the bath of the furnace in the form of fairly small particles and may be deposited on the walls of the furnace particularly in regions where the gas flow changes direction sharply. For example, it is often found in the open hearth furnace that heavy erosion occurs on the roof of the furnace close to where it joins the back wall. This is almost certainly connected with a fast moving stream of gas rising up the back wall and striking the roof. The rising stream of gas having previously flowed across the bath of the furnace will be carrying a certain amount of ferrous material and this, if thrown on to the roof, will be responsible for the erosion. This effect is particularly pronounced in oil fired furnaces where the jet velocities are much higher than in gas fired furnaces and are thus capable of picking up solid matter from the bath more easily.

Considerable research is proceeding at the present time to discover shapes of furnace chamber and positioning of ports which will enable root erosion to be greatly reduced. One method of attack consists in trying of obtain a flame which flows down the centre line of the furnace and which will have a recirculating eddy of similar intensity on each side. All furnaces which have so far been investigated show that the flow is very far from being symmetrical, the greater part of the forward flow occurring towards

the back of the furnace, and the greater part of the returning flow at the front and in the centre. This asymmetry may be due to lack of symmetry in the furnace chamber ; to the fact that the flame is not directed initially in the right direction ; or to the fact that in furnaces which have two air ports, different amounts of air may flow through them. Fig. 9 shows the

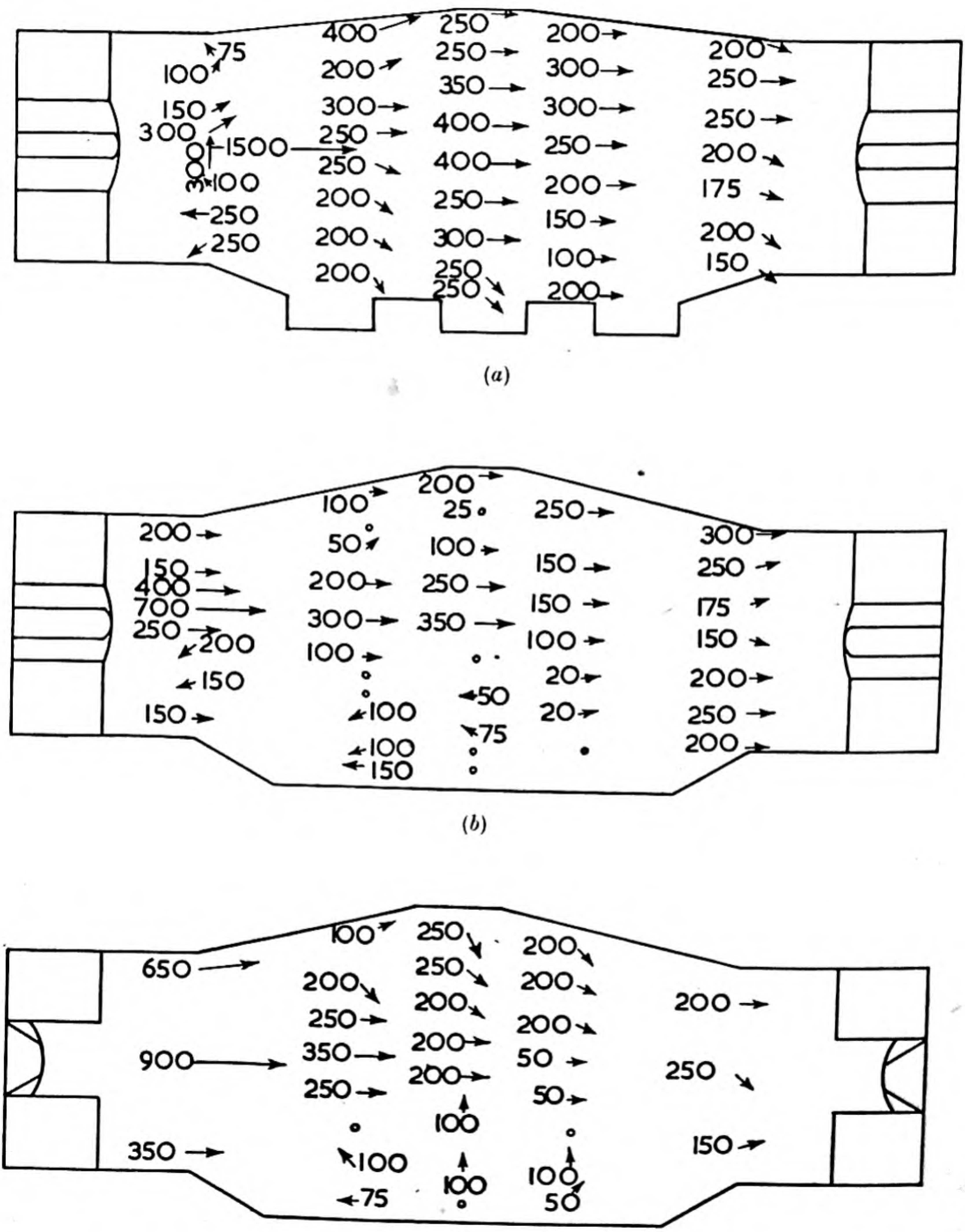


Fig. 9.—Horizontal components of gas velocities (ft./min.), in a cold open hearth furnace. (*a*) 1 ft. above bath ; (*b*) 4 ft. above bath ; (*c*) 6 in. below roof.

circulation of air inside a full scale oil fired furnace without combustion taking place but when compressed air was passed through the oil atomizer nozzle to give momentum similarity. The vectors show the horizontal components of velocity at three different levels in the furnace chamber. The asymmetry of the flow is at once apparent. Laboratory experiments on a model have determined the degree to which the oil atomiser must be pointed off the axis of the furnace in order to induce the large eddy to flow in the opposite direction. The range of angles over which the flow is almost symmetrical is very small and is probably not obtained in practice. It is hoped to carry out work on a full scale furnace with carefully controlled burner direction. The limited checks which it has been possible to carry out on full scale furnaces under operating conditions show that the conditions agree very closely with those found in the laboratory, in spite of the lack of thermal similarity in the models.[10]

This is a fact of very great value, as it means that new designs can be tested out both rapidly and cheaply in the laboratory before the actual furnaces are constructed, and it is to be hoped that there will be a great extension of work by means of models in the next few years to enable the designers to produce more satisfactory results.

In addition to the work carried out on cold models in the laboratory, a considerable amount of work has also been done by Dr. A. H. Leckie and his co-workers on a small furnace operating at lower temperatures than the full scale steelmaking furnaces.[12] These experiments have been mainly directed to elucidating heat flow characteristics but results obtained have been in general support of those obtained on the smaller laboratory models devoted to determining mixing and flow patterns.

3.5. *Oil Firing.*

The oil firing of open hearth furnaces raises problems in the relation of aerodynamics to combustion. The supply of combustion air to the flame can only take place by means of turbulent diffusion and the combustion processes are determined partly by the rate of this diffusion and partly by the rates of evaporation and burning of the oil droplets. Recent papers by Collins[13] and by Cude and Hall[14] have shown that in the steelmaking furnace the flame lengths are such as to indicate that the flames are largely determined by the turbulent flow of air into the steam jet containing atomised oil particles.

Further research is necessary to determine the characteristics of steam atomised flames more accurately and just how important the degree of atomisation is and to what extent the combustion is affected by the temperature and composition of the gases which are entrained.

3.6. *Air Infiltration.*

In the open hearth furnace the gases pass through passages constructed of brick. These permit a certain amount of air or gas to infiltrate between and through the bricks. Work is at present in progress to determine whether this reaches a magnitude which is likely to affect the operation of the furnace appreciably. In particular the infiltration of air into the air

checker has attracted attention. Preliminary experiments at one works showed that the infiltration is only a few per cent. when the air flow is a maximum but rises so that when the normal air supply is cut off the flow of air into the furnace is still about half of that obtained with the air valve fully open.(15)

Further work, however, is necessary in order to check the calibration of the Pitot tubes used in the measurements of the hot air velocities. Experiments to determine the flow of air through the checker walls have shown that the amount does not appear to vary sufficiently to account for the variation of infiltration with the variation of the air flow, and further, that coating the checkers with material sprayed on reduces the amount of air passing through the walls without greatly affecting the total infiltration. The source of the main infiltrated air was thus not established.

4. OIL METERING

Now that many open hearth furnaces are oil fired, problems arise in connexion with the metering of the oil. Normally, this is done by the use of an orifice meter sometimes of standard design and sometimes of special design to give a constant discharge coefficient down to the low Reynolds' numbers associated with high oil viscosity and low feed rates.(16)

Owing to the fact that the viscosity of the oil varies with each consignment and that the temperature may also fluctuate, the Reynolds number in the orifice meter will vary and it is necessary to have a constant discharge coefficient, otherwise the relation between flow and pressure differential will vary in an unknown manner. Certain orifice profiles of the semi-circular type have been shown to have coefficients constant to within 1% down to Reynolds' numbers in the orifice of about 340.(17) This represents a flow of about 1/10 of the maximum flow rate which is sufficient for practical purposes. Fig. 10 shows the discharge coefficient of the best orifice

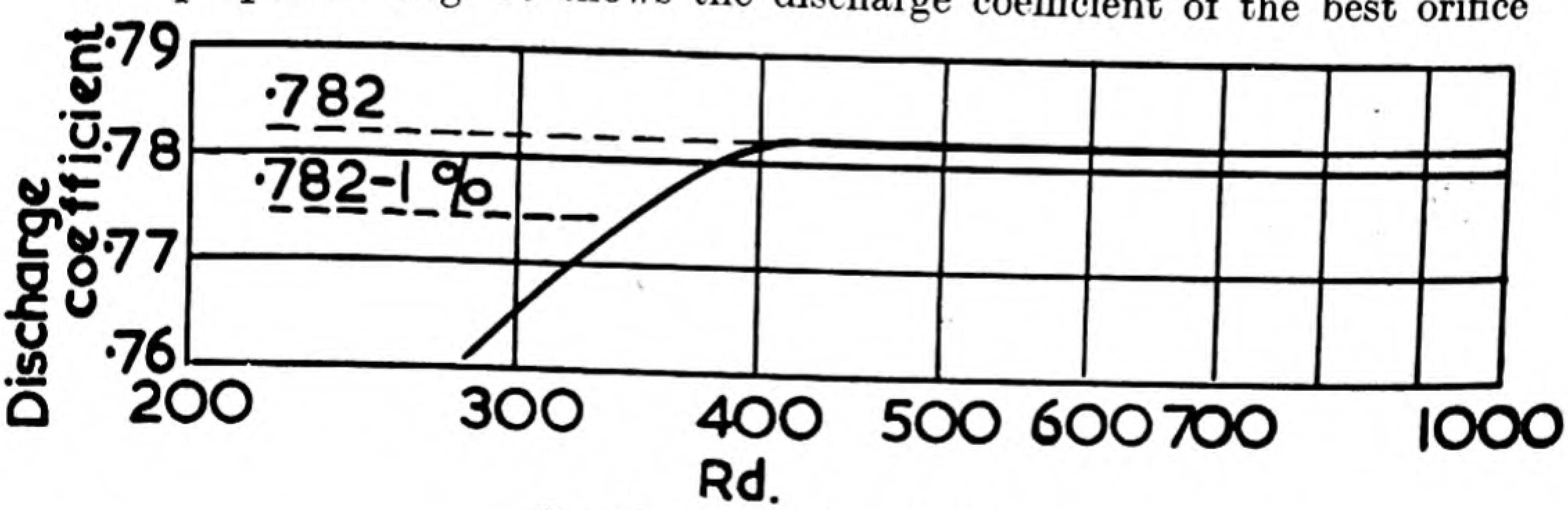


Fig. 10.—Orifice characteristic.

plotted against the Reynolds' number in the orifice. It is constant at 0·78 from a Reynolds' number of 10,000 down to 400. The orifice diameter is 0·2 of the pipe diameter and the plate thickness 0·25 of the orifice diameter.

Work on metering pumps for use with oil supply has shown that these also have an accuracy sufficient for use on open hearth furnaces.(18) While dealing with the open hearth furnace it is convenient to mention two other problems on which so far no work appears to have been done.

5. WATER COOLING

The first of these problems is in connexion with the water cooling of various parts of the furnace such as the gas port and doors. Little attention has been paid to the efficiency of the cooling. Judging from the low temperature of the cooling water when it leaves the furnace a considerably greater temperature rise might be permitted if local boiling can be avoided. An investigation into the design of cooling chambers generally with a view to eliminating dead spaces where boiling might occur might lead to a considerable reduction of the amount of cooling water pumped round the system.

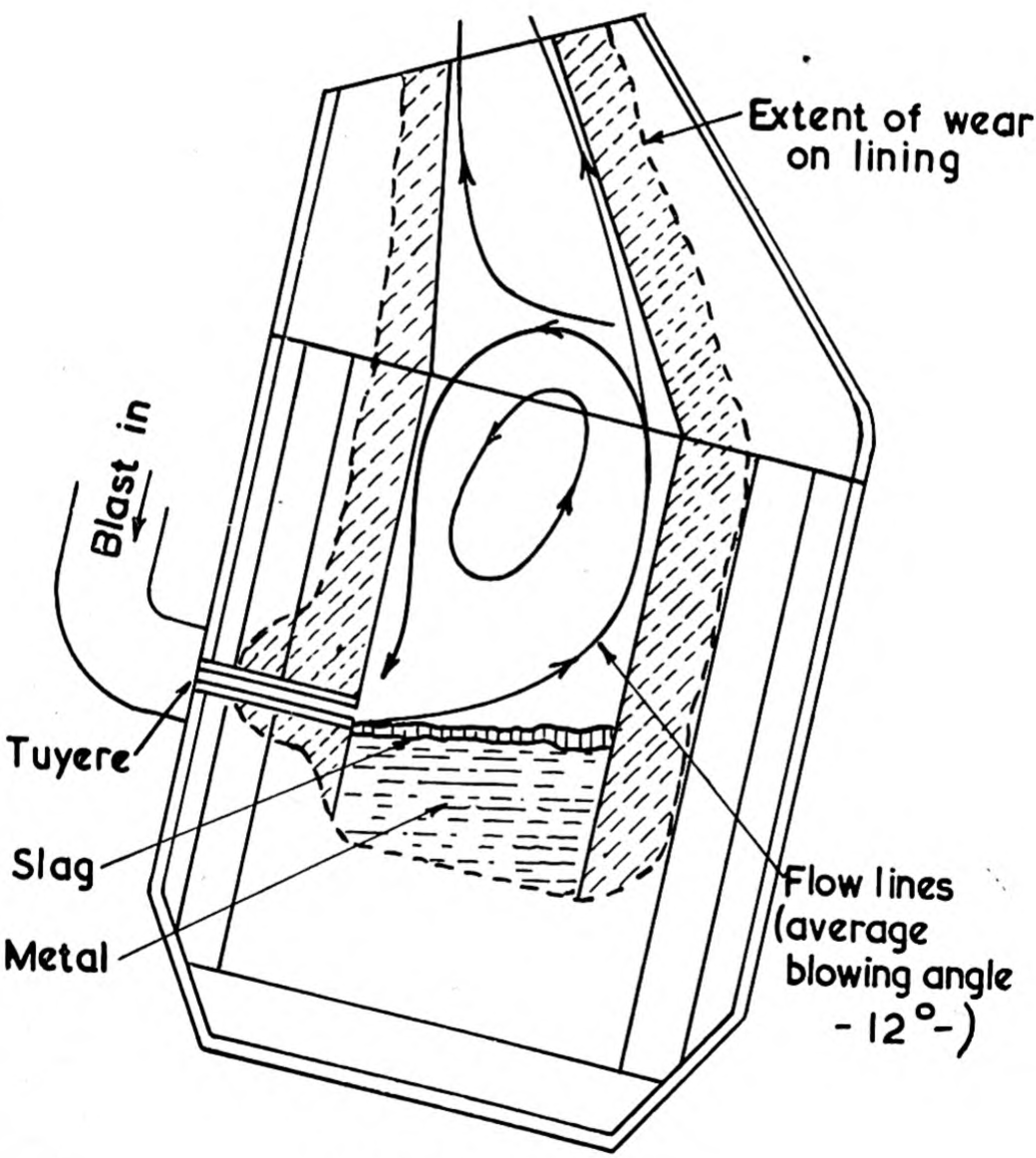


Fig. 11.—Side-blown converter.

6. MELTING SHOP VENTILATION

The second problem is the ventilation of open hearth melting shops. A large amount of heat is liberated from steelmaking furnaces and this, by causing large scale convection currents, enables good ventilation to be attained. The rising hot air leaves the roof of the melting shop through ventilators of various designs and fresh air flows in at the open ends of the melting shop and through holes in the wall sometimes made deliberately for this purpose when the furnace operators feel they have insufficient fresh air. No study has yet been made of the details of air circulation in melting shops nor of possible means of control to ensure the best working conditions for the furnacemen.

7. CONVERTERS—GENERAL

In converters the steel is produced by a blast of air being blown through or on to a bath of molten iron. The metal either obtained from the blast furnace or from a cupola (or sometimes partly from scrap steel, melted in the converter) is refined by the passage of the air which oxidises the impurities and in particular the excess carbon present.

In the Bessemer type converter, the air blast enters below the metal through tuyeres in the bottom of the chamber. In the converter of the Tropenas or Side Blown type, the air is blown into the chamber so as to impinge on to the surface of the charged metal and slag (Fig. 11).

The difficulties of investigating the fluid flow are greater for converters than for open hearth furnaces as access to the working chamber by instruments is much more difficult and the flow processes are much more vigorous.

7.1. *Bottom Blown Converter.*

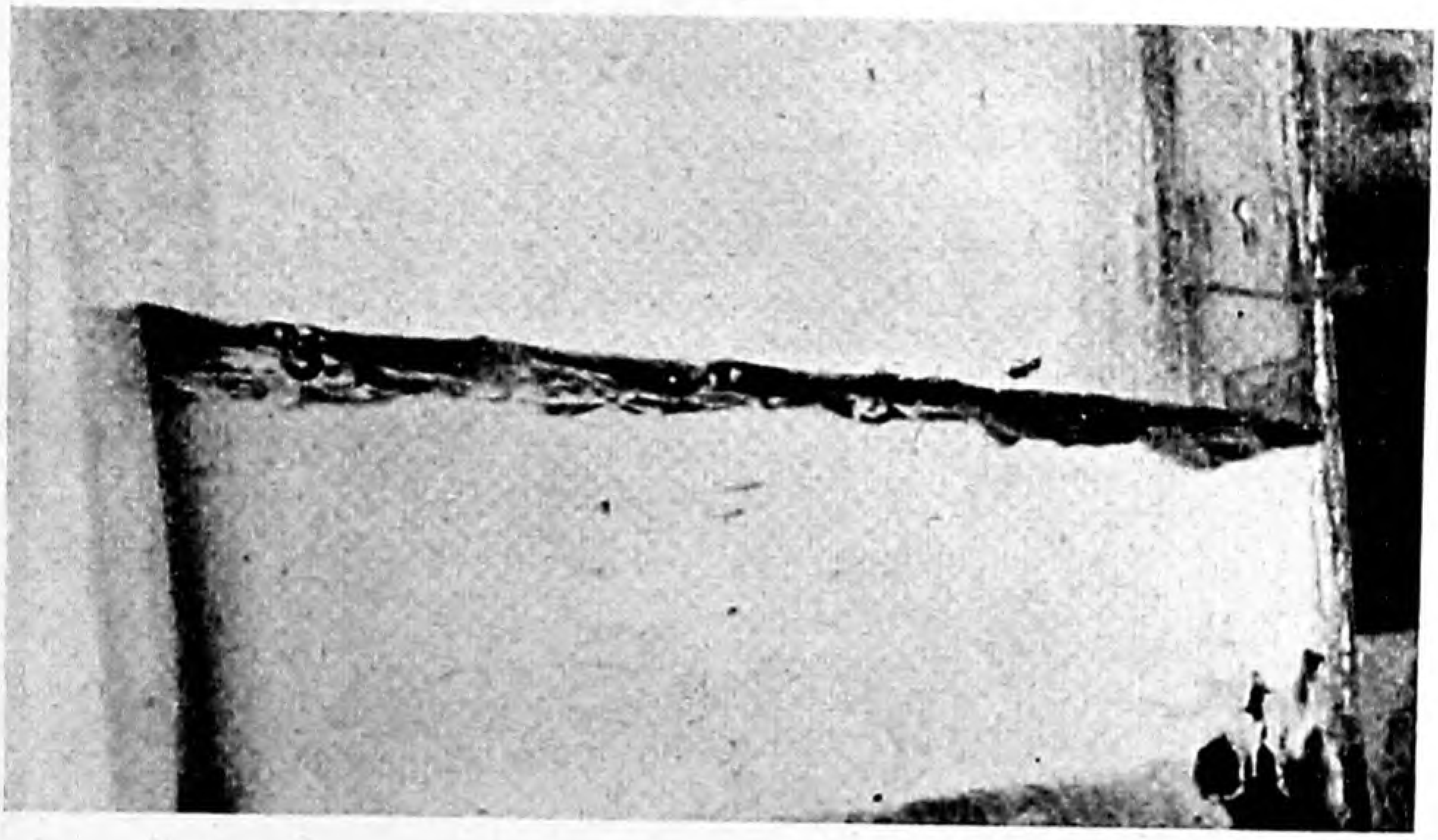
In the bottom blown converter a limit is set to the rate of blowing by the appearance of ejections of metal and slag when the rate becomes too high. The permissible rate at any period of refining depends upon the chemical composition of the bath, and thus on the chemical reactions proceeding at the given time. For any initial composition of the bath the subsequent composition at any stage during the blow will depend upon the total quantity of air which has passed at that stage. Recent research in France has been directed to constructing a meter which will indicate the rate of blowing at any time during the blow and thus allow the maximum amount of air to be blown at that time.[19]

A little work has been done on converters of this sort in Germany mainly to establish the form of converter and position of tuyeres which would eliminate oscillations of the metal during the blow.[20]

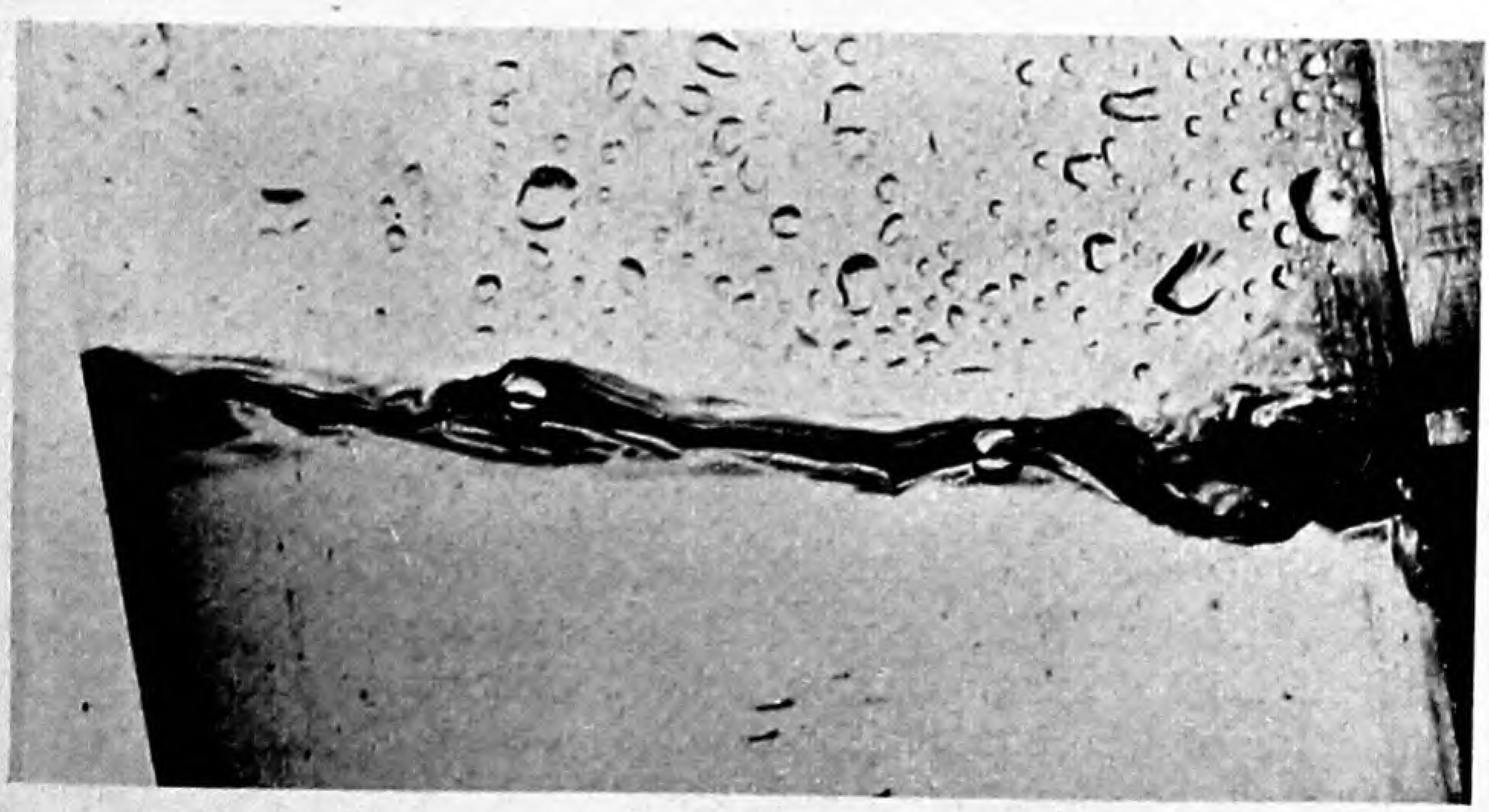
As a result of these experiments it was shown that oscillations of the bath of cylindrical converters could be prevented only by having the tuyeres slightly submerged. Deep submersion gave rise to large oscillations. The application of this discovery in practice enabled blowing to be obtained with a minimum of agitation of the bath and consequent small loss of metal by ejection. Similar results were obtained for a pear-shaped converter which was blown in a tilted position so that the tuyeres were not submerged very deeply.

7.2. *Side Blown Converter.*

Research on the fluid flow in converters in this country has been directed to the problems of the side-blown converter where the blast is introduced above the metal surface. Here the erosion of the refractory lining is of special importance and work on models has been used to throw some light on this.[21] A 1/8 scale model was constructed and the velocity of the air in the jets was adjusted so that the depressions of the metal surface were 1/8 of the depth of those occurring in the full scale plant (Fig. 12). To obtain this condition, a Froude criterion must be obeyed.



(*a*)



(*b*)

Fig. 12.—Depression of liquid surface in a side-blown converter (*a*) tuyeres just above liquid (*b*) tuyeres just below liquid.

Operation of the model under Froude similarity shows that the air jets through the tuyeres cause only a small disturbance and that any agitation of the metal during the refining period must be largely due to the chemical reactions proceeding when the impurities in the metal are oxidised.

During the main reaction a considerable amount of metal is probably thrown up from the bath. Fig. 13 shows the circulation of gases which occurs above the bath providing the tuyeres are not submerged by the

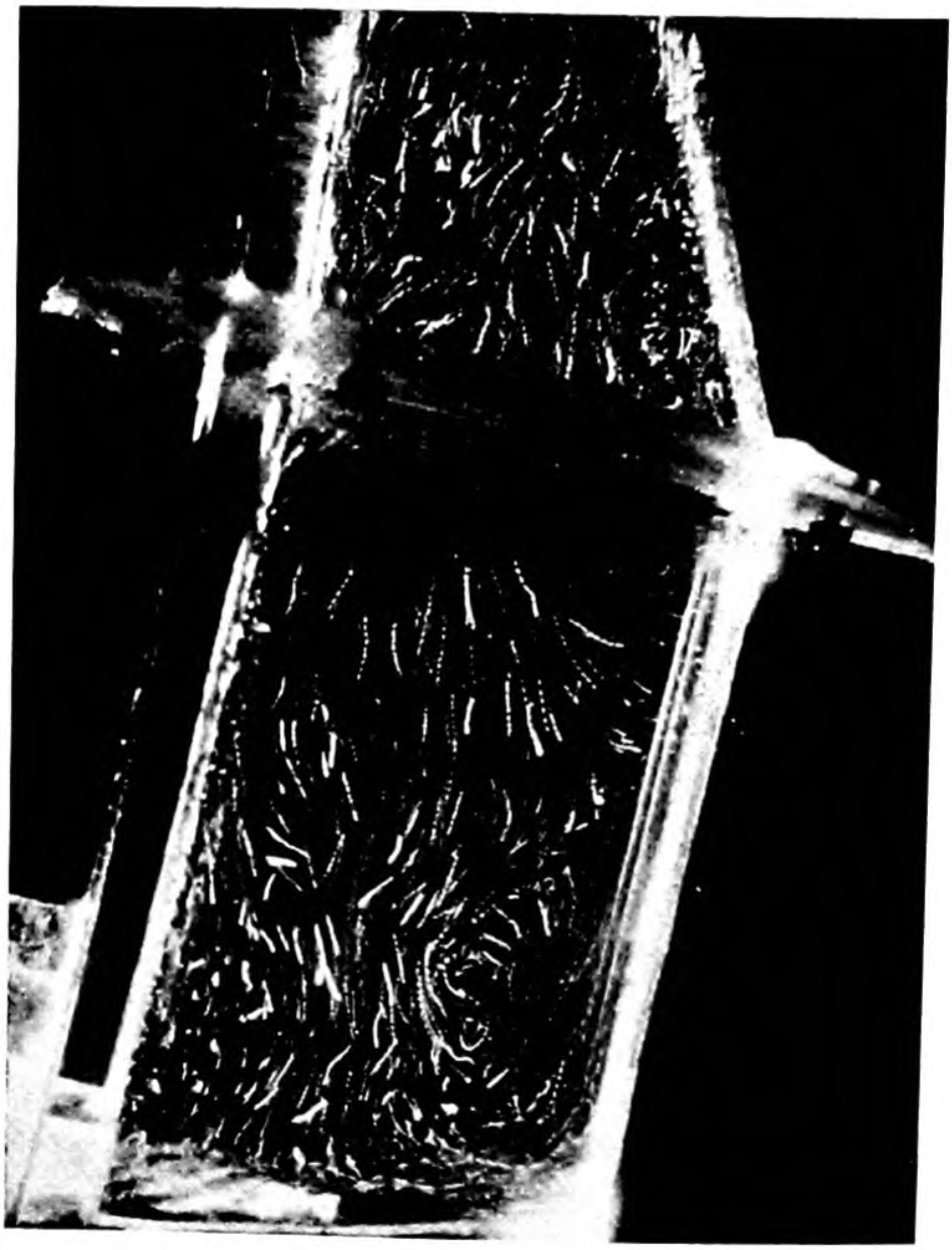


Fig. 13.—Flow above liquid in side-blown converter model.

intensity of the boil. The whole of the chamber is seen to be filled with an eddy of hot recirculating gas which flows back towards the tuyeres and which will assist deposition of ferrous oxide on the back wall. In practice it is found that the greatest erosion does take place in the region of the tuyeres and may be connected with this recirculation. Research has shown that the back eddy cannot be eliminated but its form and position may be controlled by a suitable placing of the tuyeres.

8. CASTING AND POURING

When the steel has been manufactured either by one of the processes so far discussed or by others, a series of problems arises in connexion with the casting of the metal. In the large scale manufacture of steel this casting is usually into the form of ingots.

Generally metal for large scale casting is carried to the casting pit in a ladle and the metal is poured into the moulds through a nozzle in the ladle bottom. The nozzle is plugged by a stopper on the inside of the ladle operated by an arm covered with refractory material and passing down through the molten metal. The nozzle erodes considerably during the pouring of the metal and uneven wear causes bad seating of the stopper on to the nozzle entrance. The relation between the erosion of the stopper and nozzle and the flow of the fluid is still awaiting investigation.

When the steel stream leaves the nozzle it falls into moulds and the initial distance of fall is of the order of eight feet when ingots are being produced. A considerable degree of splash occurs when the steel first strikes the bottom of the mould and a lesser degree during the subsequent filling. As the splashes of steel solidify on the mould wall and produce blemishes, elimination of the splashing would be very desirable. There is very little possibility of full elimination but various methods are employed for minimising it. In one method cans are inserted in the bottom of the mould to catch the initial splashes, the can subsequently melting. In another method steel wool is placed in the mould bottom or a depression is made in the mould bottom which helps to prevent the steel splashing sideways.

A steady stream of steel from the ladle is also very desirable and the study of the causes of irregularities in the stream due to the conditions of the nozzle and stopper would form a useful field of research.

9. SOAKING PITS AND REHEATING FURNACES

Before steel ingots are put through the rolling mill they have to be raised uniformly to a high temperature. This is done in furnaces into which the ingots are inserted and withdrawn by a crane. These furnaces are called soaking pits. In others the ingots are placed on trolleys and passed along a tunnel at high temperature. Owing to the lower temperatures the problems of erosion which occur in open hearth furnaces do not occur although considerable deposits of scale from the ingots accumulate on the floors of the furnace and may cause considerable damage.

The fluid flow problems mainly associated with these furnaces concern the flow of the gases around the ingots so that efficient heat transfer may take place. As the gases flow relatively slowly through parts of the pit they will be controlled to some extent by forces of buoyancy.

So far no investigations have been carried out on the flow in furnaces of this type. Any work done on models will probably have to take account of the buoyancy forces and the models will thus be more difficult to operate than other models so far used in investigations of flow in the steel industry.

10. CONCLUSIONS

One of the greatest needs which must be met before further significant advances can be made is the development of new instruments for measuring the flow of fluids at high temperatures. In the steel industry this is particularly true of the determinations of low gas velocities and flow directions. In order to study the flow of gases in steelmaking furnaces it is necessary for more accurate instruments to be developed for measuring gas velocities at temperatures of the order 1500°C. especially when the velocities are less than about 15 ft./sec.

The low dynamic head and high kinematic viscosity of such gases makes the use of pitot tubes inaccurate even when special designs are used which give an increased pressure differential for a given dynamic head and constant coefficients at the low Reynolds numbers encountered. Instruments working on some other principle will have to be developed.

The measurement of mixing between fuel gas and air at these temperatures is also one which needs further instrumental development. A development of the infra-red gas analyser technique described earlier will probably be possible in which carbon dioxide and carbon monoxide concentrations are measured against a controlled sample.

In the oil fired furnace new instruments are also required for sampling the materials in the flames under working conditions. For example, means will have to be developed for sampling particles of all kinds from all parts of the flame. This is a difficult task as the velocities of the flame may be about 1,000 ft./sec. initially and may drop to 1/100 of this velocity at its extremity. The oil particles as they burn will change in structure and different sampling techniques will have to be devised according to whether or not they are liquid or solid.

The result of the difficulty of carrying out measurements on the full scale has resulted in the use of models as described elsewhere in this paper. However, even when instruments are developed which are capable of tackling flow problems at high temperatures, models will still be used owing to their convenience and we may expect that comparison of the results of work on models with full scale systems will give information on the deviations between the model and archetype. At the present time only scanty information exists for correcting the results obtained from models for temperature and combustion effects.

So little work has been done on the properties of fluid flow at high temperatures in industrial processes that with the several groups of research workers now tackling the subject important advances are now being made and may be expected to continue in the near future.

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12

A New Aerodynamic Technique employing Radon for tracing Gas Flow in Hot Systems

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ABSTRACT. There is a great need for a technique to measure mixing and flow quantities of gases at elevated temperatures. The use of an indicator or tracer gas is a valuable method if the very small proportions necessarily used can be accurately determined. Radon, detected by its radio-activity, enables this to be done and is inert in combustion reactions. The development of such a technique is described, together with a summary of the experimental difficulties encountered and the precautions necessary for consistent results. With these precautions this technique can be readily applied to industrial problems.

1. INTRODUCTION

Two basic problems in hot aerodynamics are in urgent need of solution for improved furnace design. The first is to know how to control and how to measure the quantities of air and gas which will flow through a system under given conditions.

The second concerns a knowledge of the inner structure of the fluid flow, that is to say, the laws governing the velocity distribution in channels, the mixing of two or more streams, the formation of stagnant pockets, of large eddies, the persistence of jets and the subdivision of the flow between channels connected in parallel. Especially in open hearth and glass tank furnaces this second problem is of great importance in order to obtain high convection heat transfer from the gases to the charge; to give good utilisation of the whole of a distributed heat transfer surface such as the checker work of a regenerator and to control the rate of combustion of gas-air diffusion flames.

Many investigators have done valuable work in this field, but an accurate picture of the mixing pattern in a hot gas system has never been given because suitable experimental techniques have been lacking. That radon can provide a technique capable of giving this picture, and also of measuring accurately the rates of flow of gases, whatever their temperature, is suggested in this paper.

2. PREVIOUS WORK

2.1. *Metering Methods*

A method of metering which has been used for liquid flow is to introduce a measured quantity of soluble material into the stream and to measure the resulting concentration at a point sufficiently downstream for it to be perfectly mixed. In the case of water, salt has been used as the indicator material. A similar method is, in theory, applicable to gas flow measurement if a second gas, of which very low concentrations can be measured, is introduced. The possibilities of this method for furnace measurements have been fully discussed in a paper by Rummel[1]. It is of particular value when orifices cannot be used owing to the absence of a straight stretch of channel, or because the flow is pulsating, or because the temperatures are too high.

The main difficulty is that considerations of cost restrict the quantity of indicator gas to a very small fraction of the main gas flow and it is very difficult to measure such low concentrations with sufficient accuracy. Rummel concludes that the method requires great care and is only possible for research purposes, and even then it tends to be very expensive.

2.2. *Mixing Patterns in Models and Cold Flow Systems*

Previously models have constituted the greater part of the engineer's equipment for solving problems of flow distribution in hot systems. These models and cold flow techniques[2, 3, 4] must, however, necessarily be supplemented by experiments in which the flow in the full scale working system is made visible in order to answer such questions as " why does one particular furnace work better than another."

Eddy diffusion has been studied for many years by Prandtl,[5], von Kármán[6], Dryden[7], Taylor[8] and their colleagues. Their techniques included hot wire anemometry and shadow photography (Gawthrop[9]) but none of these methods can be applied directly to the study of hot flow systems including cases where chemical reaction is taking place. Combustion itself produces a modification of the mixing pattern due to expansion of the reacting gas streams. It is clear that although many lines of research have been followed, and that these in themselves have given some guidance on the two problems mentioned earlier in the paper, no method has yet withstood the test of translation from the laboratory to actual furnace practice.

3. OBJECTIVES OF EXPERIMENTS

Having shown the need for a technique for tracing the mixing pattern in full scale systems, especially where chemical reactions are taking place, it was concluded that it would be of the greatest value to develop instruments which could probe the furnace and enable one to " see " what is going on inside. The heat flow meter[10], narrow angle radiometer[11], and multiple sampling probe[12] are all watercooled instruments which can traverse the flame in, for example, an open hearth furnace to investigate its characteristics. To supplement these instruments a further one, enabling the construction of an aerodynamic picture of the flame was necessary.

It seemed that the dilution method was the most promising providing that the objections raised by Rummel were overcome satisfactorily. The tracer material must therefore satisfy the conditions that : (*a*) its detectable properties are not destroyed by chemical combination with any of the gases met in industry ; and (*b*) it can be detected accurately in such small quantities that its cost does not become prohibitive.

A gas is more or less essential because then the physical behaviour of the tracer and main gas is identical. An inert gas automatically satisfies condition (*a*) and a choice may, therefore be made from the group of so-called rare gases subject to modification by condition (*b*). Rummel has shown that to use the effect of the indicator gas on the thermal conductivity of the mixture is expensive for hydrogen ; a comparison of the thermal conductivities of hydrogen with those of the gases in the inert group shows hydrogen (40×10^{-5} c.g.s. units) in the most favourable position and hence

the use of any of the other gases would be even more prohibitive. Methods dependent on chemical analysis are obviously excluded. A property of the last gas in the inert group gives the clue to the means of detection.

Radon is the radioactive emanation from radium and this property of disintegration allows detection of single atoms. The actual volume concentration of radon which can be measured to an accuracy of 1% using a simple gold leaf electroscope is $1{\cdot}2 \times 10^{-15}$ c.c. per c.c. of air. It is possible then, to meter air rates of 300,000 ft.^{3} per hour continuously for 1 hr. with about 15 mc. of radon*. By reducing the duration of metering or using more sensitive apparatus such as the electrometer triode, this quantity can be reduced still further.

The choice of radon satisfied, in theory, the conditions set earlier in the paper ; the gas is chemically inert and accurate results can be obtained with only small quantities of it. Preliminary experiments were designed to show whether in practice the technique would prove valuable or not. Not only the resultant mixture but also the distribution downstream was to be analysed thus enabling the full pattern of mixing to be calculated. Further, the distribution from an approximately point source should agree with the normal Gaussian error curve, as shown by Goldstein(13). If this agreement was found then the technique must be sound theoretically, leaving only experimental difficulties to solve.

These experiments, reported in Nature(14) and described later in this paper, led to the development of a satisfactory technique for dealing with the first problem enunciated in the introduction ; the second problem forms part of a future programme of work.

4. DEVELOPMENT OF THE TECHNIQUE

4.1. *Apparatus and Experimental Lay-out*

4.1.1. *Choice of Ionization Measuring Instrument*

The golf leaf electroscope is the simplest and most robust type of ionization measuring instrument and it was therefore used in the laboratory tests even though smaller concentrations could be detected with apparatus such as the electrometer triode.

4.1.2. *Design of Electroscope*

The electroscope designed for our purpose was essentially similar to that used by Rutherford for measuring the decay rates of radio-active substances. However, in the technique to be developed where samples would have to be analysed at the rate of about 6—8 every hour the residual radio-activity of the solid disintegration products would render the chamber unusable even when the original sample had been removed by evacuation.

The recognised procedure is to substitute an exactly similar uncontaminated chamber and this was made possible by a modification of the basic electroscope design. The positions of the gold leaf and ionization chambers were reversed so that easy access was available to the latter, thus enabling a clean tin to be inserted for each sample, the bulk of the deposit being removed on the tin each time.

* 1 curie is the amount of radon in equilibrium with 1 gram of radium. It occupies 0.6×10^{-3} c.c. at N.T.P. 1 gram of radium produces 70—80 millicuries per day.

The electroscope is shown in Fig. 1.

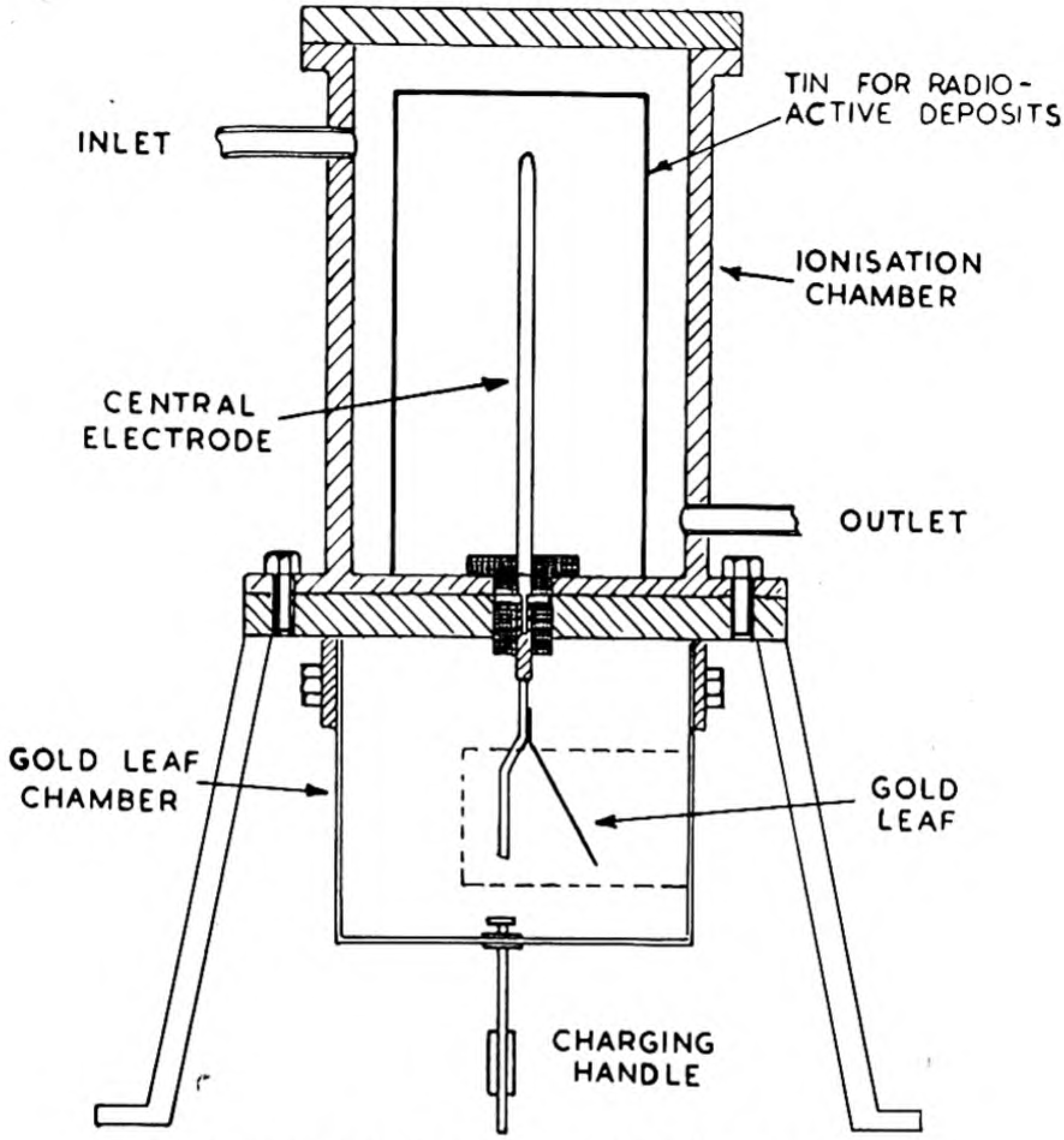

Fig. 1.—Gold leaf electroscope used in the experiments.

It was necessary to take several precautions in the operation of this instrument in order to obtain consistent results. As the activity of radon increases for the first few minutes after admission into the ionization chamber while coming into equilibrium with its disintegration products, the ionization has to be measured a set time after admission. Also the usual precautions had to be taken to ensure that the " natural leak " of the electroscope was kept as low as possible.

4.1.3. *Radon Supply and Preparation of Stock Solution*

The radon was supplied by The Radon Centre of The Medical Research Council in small glass capillaries about 3 cm. long and with a strength determined for a given date and time, from which the strength for any other time could be calculated from the normal decay curve. This radon was, for the first tests, diluted in a cylindrical water enclosed gas holder of about 25 cu. ft. capacity.

The ensuing dilution was the stock solution for passing into the air streams.

4.2. *Results of Preliminary Experiments to Test the Theory*

The first experimental runs were made in a 1 ft. square section air duct about 10 ft. long, a small electric blower providing air at rates up to about 150 cu. ft./min.

Owing to the distance of the electroscope from the air duct, about 20 ft., it was impracticable to evacuate the connecting tube right back to the sampling point and therefore the final sample was continuously drawn through the tube past the electroscope by a pump. When required, a sample was let into the evacuated electroscope through a T-piece connexion controlled by a vacuum tight valve. In the experiments to test the theory the samples from the air duct were drawn off through probe tubes placed at various distances and positions downstream.

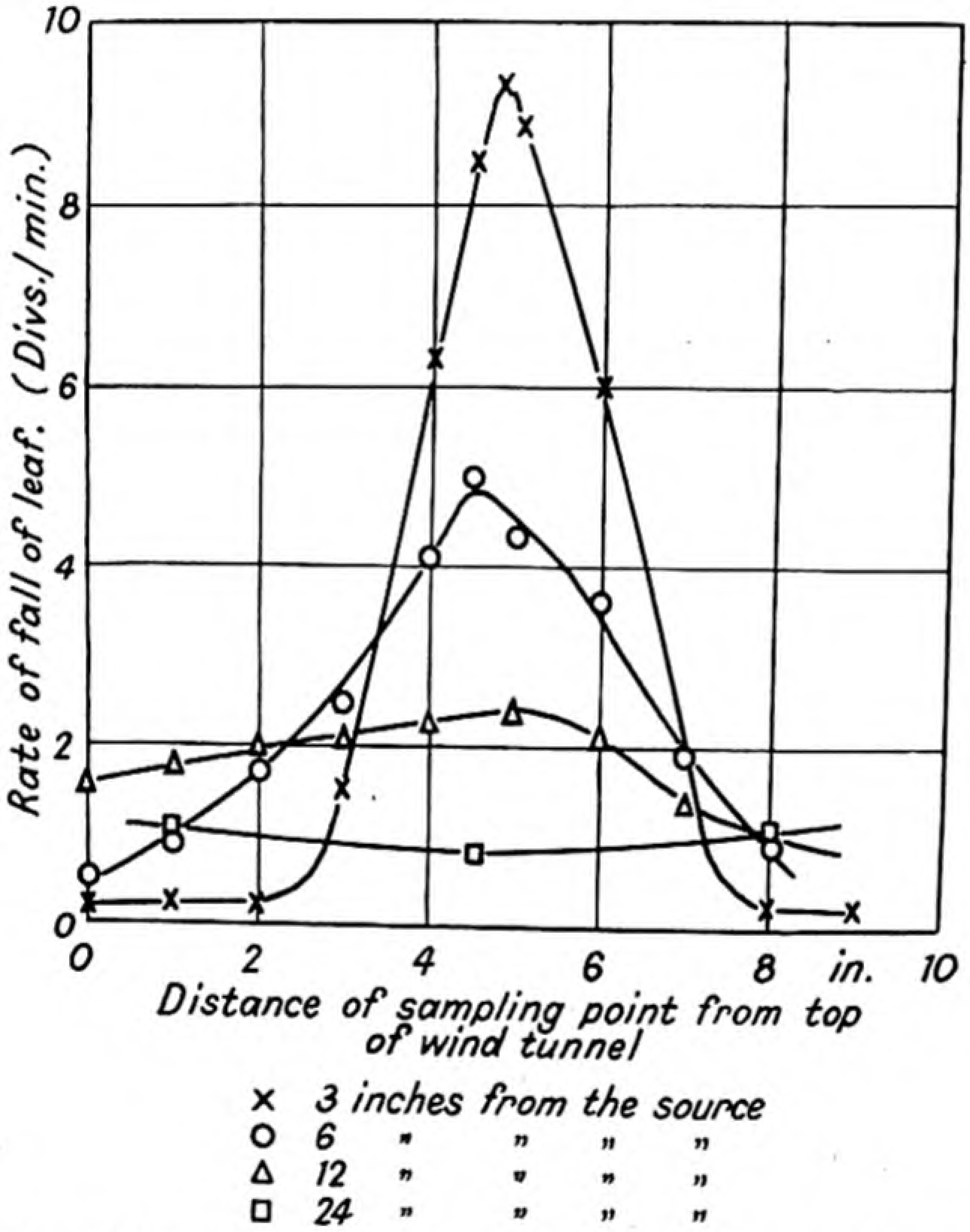

Fig. 2.—Distribution of radon in the wake of a point source.

Curves obtained in this manner are reproduced in Fig. 2, in which the mean relative activity of the samples (scale divisions per minute) taken at one point are plotted against the vertical height of the probe point in the duct. It will be seen that the curves corresponding to sampling planes near the inlet point, where the wall effect is small, obey the Gaussian error law sufficiently well to be used for calculation of the mean mixing length. The relative spread ($\frac{1}{2}$ value width) and relative maxima of the curves also show the expected agreement with theory.

These results were considered to justify the adoption and development of the technique and a series of trials was therefore begun in order to determine and solve any experimental difficulties.

4.3. *Beginning of Trials to determine Experimental Difficulties*

In the first trials a wide scatter of $\pm 8\%$ in the readings of any one run was encountered together with a similar spread between the means of successive runs. It was thought that the adhesion of radon atoms to the walls of the container, air line or connecting pipes (Hirst and Harrison[15]) might account for part at least of this variation, but little attempt at a systematic investigation into the source of these errors was possible before the apparatus was taken for a works trial to the United Glass Bottle Co. Ltd. at Charlton. Here it was attempted to use the technique to obtain the mixing pattern of the air and gas in a traverse across the off-take.

Out of the samples taken, three gave a gas distribution curve similar to that expected but the number of rejects and the wide scatter in the readings obtained from the other samples made it impossible to draw any conclusive results.

4.4. *Conclusions and Decisions for Future Work*

This trial showed that the technique in its present form was clearly not ready for works application and that much more laboratory work was required.

A necessity was a calibrated and controlled air duct together with such compression of the apparatus as to obviate the need for long connecting pipes between the duct, gas holder and electroscope. Access was available to such an air duct and round it a new series of experiments were planned.

5. A METHOD OF CALIBRATION

5.1. *Improved Layout*

The air duct on which it was decided to work was supplied by a fan capable of delivering up to 800 ft.^{3}/min. through a cylindrical 1 ft. diameter pipe and metered through a calibrated orifice, direct reading on to a meter. The orifice was 8 ft. up stream of the fan, the line then passing after a further 20 ft. through a gas fired air preheater and thence could be diverted either through a furnace or out to atmosphere.

For ease of operation a trolley was designed upon which the electroscope and all further apparatus required by the operator was within easy reach. Gauges showing the vacuum attained in the electroscope and the pressure of radon in the electroscope whilst the ionization was being measured were situated upon the back panel where they could be easily observed. A switch for the vacuum pump and a set of vacuum tight glass cocks controlled the evacuation and subsequent filling with radon of the electroscope, and with this trolley (shown in Fig. 3) placed as near to the sampling point in the air duct as possible, all operations during a run could be carried out from one position. All relevant data were recorded as the observations were made, the results for each sample admitted to the electroscope being calculated while the instrument was being evacuated and cleared of radio-active contamination. In this way any sudden change could be detected and checked upon before the run was completed.

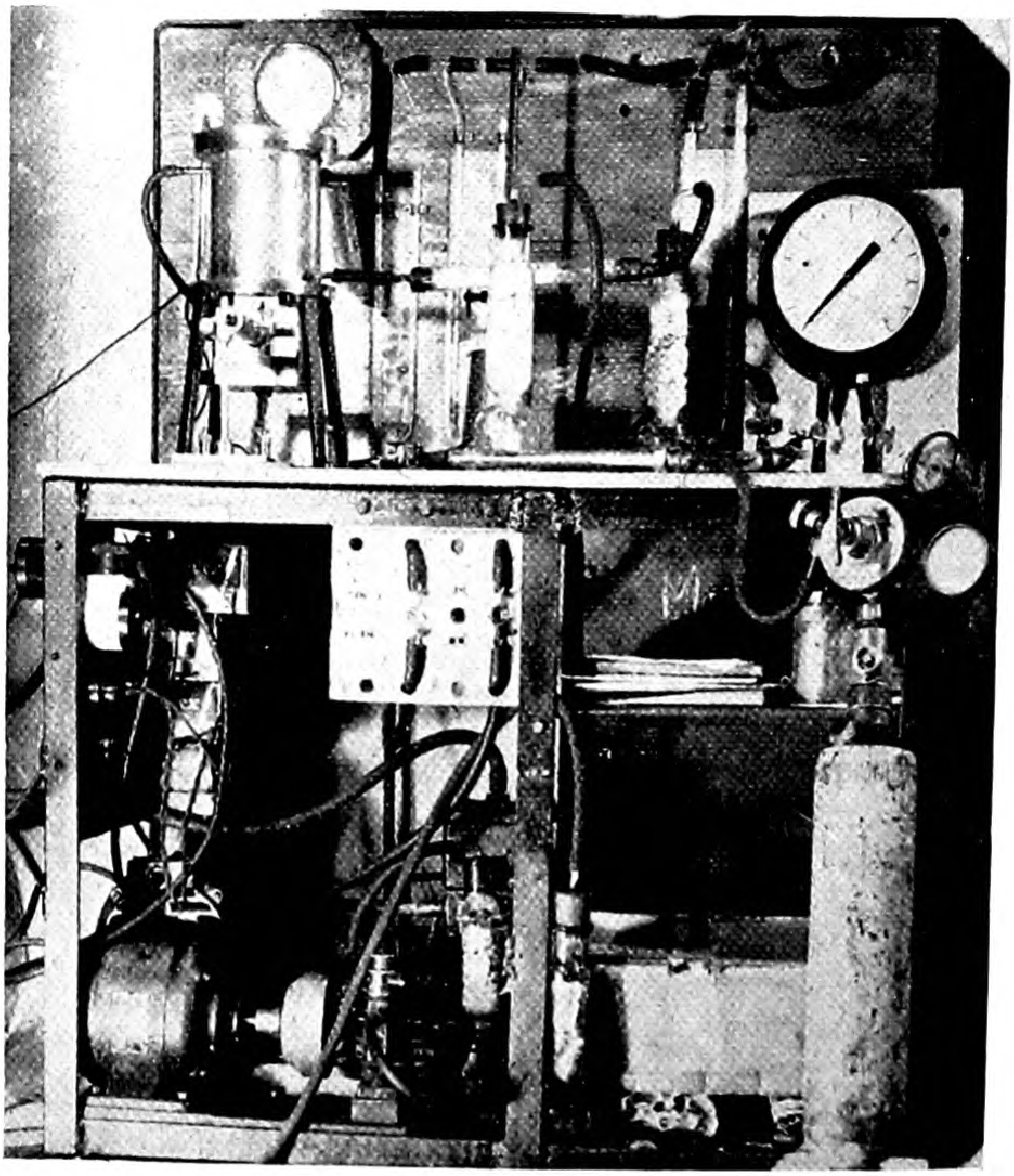

Fig. 3.—General view of trolley with apparatus.

5.2. *Summary of Experimental Difficulties and Sources of Error Encountered*

It was soon found that any water displacement method for passing the radon stock solution into the air stream was unsatisfactory as it failed to give the necessary control, due to varying pressure, over the radon inlet. In addition, Hirst and Harrison(15) have shown that the adhesion of radon atoms is greater the more irregular and pitted the walls of the container and in any water displacement method corrosion occurred on the inside surface of the storage vessel. It also proved more difficult to clear such a container of residual radio-activity and a short series of experiments showed that the only reliable method of ensuring this was by evacuation.

These objections demanded that the radon be diluted into a dry rigid container of accurately known capacity and the choice obviously fell on a high pressure cylinder of the British Oxygen Company's type. The diluent

air required for diluting the radon was obtained from a full 120 atm. air cylinder throttled down to a suitable pressure, the radon capillaries being crushed in rubber tubing between the cylinders.

When the stock radon solution was required for passing into the air line, a normal regulator head was fitted to the cylinder, the contents pressure gauge giving the data for dilution calculations while the outlet pressure gauge and needle valve controlled the radon flow. It was found that in order to completely clear this storage cylinder of residual radio-activity after use, six evacuations followed by a day's " rest " were quite sufficient.

The accurate control over the radon flow possible by these means reduced the spread of the readings within one set, that is, from one stock solution, to $\pm 2\%$. However, the scatter between the means of successive dilutions showed little improvement, i.e., it dropped from $\pm 8\%$ to about $\pm 6\%$ and this might well be accounted for by increasing accuracy in calculating the dilution owing to the use of the cylinder.

It was decided that the scatter remaining must be due to one of two causes : either the radon in the glass capillaries was not always being fully transferred to the storage cylinder or the amount of radon adhering to the walls of the storage cylinder varied widely between one dilution and another.

A series of tests in which a suitably increased radon dilution was passed directly from the cylinder into the electroscope showed that the radon storage cylinder introduced no part of the error encountered. Similarly there was no improvement in the results upon the introduction of a new and more efficient design of radon capillary crushing apparatus in which the presence of metal connections throughout permitted the use of a high pressure air blast. Having eliminated these two most probable sources of error, it was finally decided to check up the incoming batches of radon capillaries.

At the time, no radium was available so that a quantitative check on the millicurie contents was impossible ; it was therefore decided to take one batch of radon capillaries at its given value and correct all future batches by reference to this. It was found that a variation of nearly $\pm 5\%$ existed between the stated radon values and those determined by ourselves by reference to the standard radon batch. When using these radon capillaries and their corrected values in dilutions the variation between the means of successive runs fell to $\pm 2\%$, the same order of magnitude as the variation of readings within one run.

5.3. Final Experiments on Air Duct

5.3.1. *Check on Meter Scale.*

A series of runs was made on the air duct comparing the readings obtained at air rates of 150, 300, 450, and 600 ft.^{3}/min. It was found that consistent results were obtained in all cases and except for the 150 ft.^{3}/min. rate the readings were in the correct ratio. This showed that once the reading obtained from a known air rate was available the radon technique was capable of giving quantitative results in any unknown flow system. The

fact that the 150 ft.³/min. reading was in all cases 10% below that expected was fairly conclusive in view of the agreement of the other air rate readings that the meter calibration at this point was incorrect. The results obtained are shown in the table.

Air Rate ft.³/min.	Mean activity measured Corrected to Radon concentration of 1 mc/ft³ Divs/min x 10^4
150	1·49
300	1·68
450	1·65
600	1·64

5.3.2. *Check on loss by adhesion to the walls of the Air Duct.*

In order to determine whether there was any serious loss of radon *en route* through the air duct due to adhesion to the walls, samples were taken to the electroscope from three positions in the line : (*a*) just before the measuring orifice, i.e., 6 ft. up stream of the fan ; (*b*) at the normal sampling point before the preheaters, i.e., 20 ft up stream of the fan ; and (*c*) on the opposite side of the preheaters, i.e., about 50 ft. up stream of the fan. In all cases the samples from all three positions fell within the $\pm 2\%$ spread encountered at the normal sampling point. Furthermore a series of samples taken in a traverse across the air duct at the point before the orifice showed that the radon was completely mixed even at that stage.

5.3.3. *Preheat Run.*

With the gas fired preheater lit it was possible to raise the temperature of the air in the duct (running at 300 ft.³/min) to about 100° C. This was the nearest approach to a hot gas system that could be attempted in the controlled air duct but it sufficed to show that there was no change in the reading. It was necessary to cool the gases before passing them into the electroscope so that the reading obtained could be directly compared with that from the air duct running cold.

5.3.4. *Additions to technique required when used in Works.*

It was realised that the ionization reading obtained from a radon sample passed into the electroscope would depend upon the density and chemical composition of the diluent gas. In works where the radon technique would be used this gas might be producer gas, town's gas or a mixture of both and a quick method was required of calibrating the electroscope for these conditions.

For this purpose a small air line was attached to the trolley through which a small calibrated flow of the diluent gas (about 30 ft³/min.) could be diverted.

A small flow of radon from the storage cylinder was passed into this air line and a sample taken to the electroscope from a point where the radon was fully mixed, the air conditions in the line being turbulent.

Tests using air as the diluent gas were carried out and the results obtained from the small air line were in all cases well within the $\pm 2\%$, spread encountered in the readings from the large air duct showing that this method of calibrating the electroscope was therefore practicable.

6. CONCLUSIONS

In the successful use of this technique for the metering of gas systems where absolute gas rate figures are required three conditions must be rigorously controlled. Firstly, the millicurie strength of the radon must be accurately known for dilution calculations and secondly all this radon must be transferred to the storage vessel in making the stock solution. Lastly, the radon input to the gas system must be capable of accurate control as a small change in radon concentration will make a large error apparent in the resultant ionization current and in the final calculation.

The electroscope must be calibrated for one small known rate of the gas to be metered and where this is air, the calibration can be carried out in the laboratory. In other cases it will probably be most convenient to by-pass a small flow of the gas to be metered through a portable air line with a calibrated orifice which can be carried to the scene of the trial.

The technique has been developed to a condition where it could, with little modification, be used in industrial works to meter gas flow or determine the mixing pattern of the gas and air streams in combustion systems.

This method can easily be adapted where the use of a more sensitive detector such as the electrometer triode or Geiger counter is desirable in order to reduce the amount of radon to be used.

7. ACKNOWLEDGMENTS

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GROUP IV

APPLICATIONS OF PRESENT KNOWLEDGE AND TECHNIQUES

13

Theory and Design of Simple Ejectors

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ABSTRACT. A formula has been derived for determining the pressure rise in an ejector in which there are negligible changes in the densities of the driving and entrained fluids on mixing. Some experimental results are given, showing that this formula (Equation 13) may be used, with certain reservations, to give the pressure rise in an air-ejector. A similar formula (Equation 20) has been derived and checked experimentally, for determining the pressure rise in a vapour-driven liquid-ejector, provided that the liquid is sufficiently cool to cause the vapour to condense completely on mixing.

The capital and maintenance costs of an ejector are negligible in comparison with the running costs. Hence the economic design of an ejector for a given duty, is the one for which the amount of driving fluid required is a minimum. With the two types of ejector considered here, the economic design is obtained when the proportions of the ejector are such that :—

$$\frac{\text{Cross-sectional area of nozzle at outlet}}{\text{Cross-sectional area of diffuser throat}} = \frac{\text{Overall pressure rise required}}{\text{Kinetic pressure of driving fluid at nozzle outlet}}$$

provided that the value of this ratio is less than about 0·2 in the case of the second type.

From this and the general equations for pressure rise, two formulæ (Equations 17 and 24) have been derived for the design of the two types of ejector considered; these formulæ are of sufficient accuracy for engineering design. The first may be applied to the design of liquid-ejectors in which the driving fluid is either a gas or a liquid, provided that no change in volume takes place on mixing; it may also be applied to the design of gas-driven gas-ejectors in which the pressure rise of the entrained fluid is small compared with the absolute pressure. The second equation may be applied to the design of steam-driven water-ejectors. Vacuum-ejectors cannot be dealt with by simple theory.

INTRODUCTION

There is a frequent demand for ejectors in industrial chemical processes, as cheap installations for boosting gases and, to a lesser extent, for pumping liquids. Ejectors may be preferred to fans or pumps because the fluid to be handled is dusty or corrosive, or because the installation is only put into commission for short periods and hence the cheaper but less efficient ejector is more economical. Occasionally a fluid at a high pressure must be added to a fluid at a lower pressure; if this is done in an ejector, the low pressure fluid can be boosted. Furthermore, an ejector can be designed, constructed and installed very quickly.

To meet such requirements as these, a simplified theory of ejectors has been developed and checked experimentally. It is assumed in this theory that the fluids mix at constant volume and thus, when applied to gas-ejectors, it is not so accurate as the method of Keenan and Neumann(1), and it is limited to small pressure ratios. However, it has the advantage that lengthy trial-and-error calculations are avoided and a single equation is produced for the pressure rise. This equation can be differentiated to

give the economic design, in which the quantity of driving fluid required is a minimum, for a given duty. When higher pressure ratios are required, the equation for the economic design is still applicable and the equation for the pressure rise can be used as a first approximation when finding the quantity driving fluid required. A similar theory has been developed by Silver[2] for the determination of the entrainment rate of air in a gas burner.

Early work on vacuum-ejectors has shown that diffusers with parallel throats (as illustrated in Fig. 1) produce a higher vacuum than diffusers

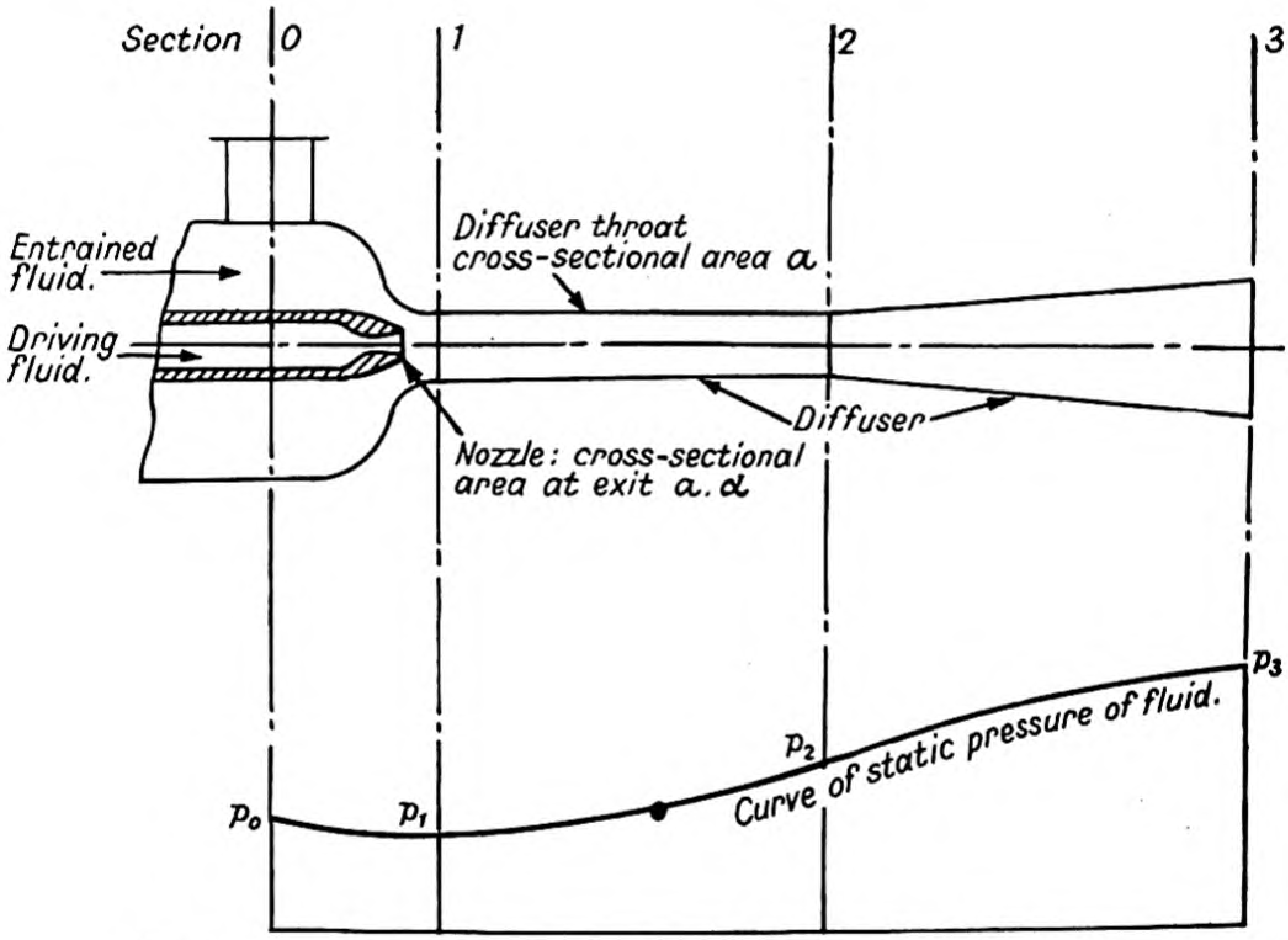


Fig. 1.—General form of ejector and typical curve of pressure distribution.

consisting only of a convergent section followed immediately by a divergent section[3, 4, 5]. It is assumed here that the throat of the diffuser is sufficiently long for the complete mixing of the two fluids to take place in it, and that the flow through it is at constant cross-sectional area ; the validity of this second assumption has been shown by Keenan and Neumann[1]. The pressure rise that takes place in the parallel-sided throat can be calculated from the momentum equation, while the pressure rise in the divergent section can be calculated from the energy equation. The theory developed here has been found not to apply to ejectors in which a jet of liquid entrains a gas, the achieved entrainment rates being considerably less than the predicted. It is thought that this is due to the difficulty of ensuring complete mixing and that the velocity of the liquid at the end of the throat is still somewhat greater than that of the gas.

If the driving fluid is a vapour and the entrained fluid is a liquid sufficiently cool to cause the driving fluid to condense, the pressure rise is many times greater than that obtained with constant-volume mixing. The explanation of this is that condensation reduces the velocity of the mixed fluids at the end of the throat. Thus their momentum is reduced and consequently the pressure at this point is increased. A modified theory has been developed to cover this case and can be applied to ejectors in which steam is used to pump water. The theory does not hold if the water rate is very low, since complete condensation does not take place ; whereas at very high water rates cavitation conditions are liable to occur at the inlet to the ejector.

The designer of an ejector must first determine the minimum quantity of the driving (or high pressure) fluid that is necessary to give the required boost to the entrained (or low pressure) fluid. Before he can use the formulæ given below, he must calculate the density and velocity of the driving fluid as it leaves the nozzle, using the normal equations for flow through nozzles. The minimum rate of the driving fluid can then be calculated from the appropriate formula and hence the nozzle can be designed. The diameter of the parallel portion or " throat " of the diffuser is fixed in relation to the diameter of the outlet of the nozzle by the formulæ for the economic design [Equations (16) and (22)]. Some details of design are discussed later in this paper.

DERIVATION OF FORMULÆ

Symbols.

a	=	cross-sectional area of diffuser throat $= \left(\frac{\pi}{4}D^2\right)$	cm^2
d	=	diameter of nozzle at outlet	cm.
D	=	diameter of throat of diffuser	cm.
D_3	=	diameter of outlet of diffuser	cm.
K	=	wall friction losses, expressed as a fraction of the kinetic pressure of the mixed fluids at the end of the diffuser throat (found by experiment to equal 0·5)	—
m	=	mass rate of flow of driving or driven fluid, as denoted by suffix	g./sec.
p	=	absolute pressure at any point, as denoted by suffix	dynes/cm^2
v	=	mean velocity at any point, as denoted by suffix	cm./sec.
α	=	$\dfrac{\text{cross-sectional area of nozzle at outlet}}{\text{cross-sectional area of throat of diffuser}} = \left(\dfrac{d}{D}\right)^2$	—
β	=	$\dfrac{\text{Overall pressure rise}}{\text{kinetic pressure of driving fluid}} = \left(\dfrac{p_3 - p_o}{\frac{1}{2}\rho_d v_d^2}\right)$	—
γ	=	ratio of specific heats of a gas	
Δp	=	overall pressure rise	milliatmospheres
ρ	=	density of fluid at any point, as denoted by suffix	g./cm^3
o	=	suffix referring to initial conditions of entrained fluid	

1	=	suffix referring to conditions at entrance to diffuser
d	=	suffix referring to driving fluid after expansion through the nozzle, i.e. at entrance to diffuser.
e	=	suffix referring to entrained fluid at entrance to diffuser
2	=	suffix referring to mixed fluid at end of diffuser throat
3	=	suffix referring to mixed fluid at outlet from ejector

Overall Pressure Rise. The conditions of the driving fluid as it leaves the nozzle and enters the diffuser can be determined from the usual equations for flow in a properly shaped nozzle. The changes in the pressure of the entrained fluid as it flows through the ejector are first calculated neglecting friction at the walls of the diffuser; a term, to be determined empirically, is then added to allow for this.

The pressure drop of the entrained fluid entering the ejector is obtained from the energy equation, neglecting changes in density and neglecting the initial kinetic pressure :—

$$p_o - p_1 = \tfrac{1}{2}\,\rho_e\, v_e^2 \tag{1}$$

The pressure rise in the throat of the diffuser is obtained by applying the momentum equation to the cross sections (1) and (2) (Fig. 1).

$$p_1\, a + m_e\, v_e + m_d\, v_d = p_2\, a + (m_e + m_d)\, v_2 \tag{2}$$

The pressure rise in the divergent section of the diffuser is obtained from the energy equation, neglecting changes in density and neglecting the final kinetic pressure :—

$$p_3 - p_2 = \tfrac{1}{2}\,\rho_2\, v_2^2 \tag{3}$$

It is assumed that the loss of pressure due to wall friction can be expressed in the form

$$K\,.\,\tfrac{1}{2}\,\rho_2\, v_2^2 \tag{4}$$

Thus the overall pressure rise is given by

$$p_3 - p_o = -\tfrac{1}{2}\,\rho_e\, v_e^2 + \left\{m_e\, v_e + m_d\, v_d - (m_e + m_d)\, v_2\right\}/a + (1-K)\,.\,\tfrac{1}{2}\,\rho_2\, v_2^2 \tag{5}$$

Neglecting any mixing that takes place between the outlet of the nozzle and the inlet of the diffuser, it is assumed that the entrained fluid occupies an area $a(1-\alpha)$ at the entrance to the diffuser; where α is the ratio of the cross-sectional area of the nozzle at exit to the cross-sectional area of the diffuser throat. The velocities at the various places are thus given by :—

$$v_d = m_d/\rho_d\, \alpha a \tag{6}$$

$$v_e = m_e/\rho_e\, (1-\alpha) a \tag{7}$$

$$v_2 = (m_e + m_d)/\rho_2 a \tag{8}$$

Substituting these in equation (5) and putting $\beta = (p_3 - p_o)/\frac{1}{2}\rho_d v_d^2$:—

$$\beta = \frac{m_e^2}{m_d^2}\frac{\rho_d}{\rho_e}\left\{-\frac{\alpha^2}{(1-\alpha)^2} + \frac{2\alpha^2}{1-\alpha}\right\} + 2\alpha - \left(\frac{m_e + m_d}{m_d}\right)^2 \frac{\rho_d \alpha^2}{\rho_2}(1+K) \tag{9}$$

This can be rearranged in the form of a quadratic equation in m_d :—

$$m_d^2 \left\{ \frac{\beta}{\alpha^2} - \frac{2}{\alpha} + \frac{\rho_d}{\rho_2}(1+K) \right\} + 2m_d m_e \frac{\rho_d}{\rho_2}(1+K) - m_e^2 \left\{ \frac{\rho_d}{\rho_e} \frac{(1-2\alpha)}{(1-\alpha)^2} - \frac{\rho_d}{\rho_2}(1+K) \right\} = 0 \quad (10)$$

A solution of this equation, however, involves a knowledge of the density of the mixed fluids (ρ_2), which depends on the fluids involved and on the pressures and temperatures in the ejector. Two cases are considered below, and for each of them an equation is derived for the economic value of α. The most economic design of an ejector is the one in which the value of α is such that m_d is a minimum, the quantity of entrained fluid m_e, the, ratio β and the fluid densities at inlet (ρ_d and ρ_e) being fixed.

Constant-volume Mixing of the Fluids in the Ejector. The first type of simple ejector to be considered is one in which the volumes of the fluids do not change appreciably in the diffuser. The density of the mixed fluids at cross-section (2) (Fig. 1) is then given by :—

$$\rho_2 = \frac{m_e + m_d}{m_e/\rho_e + m_d/\rho_d} \quad (11)$$

Substituting this in equation (9) gives

$$\frac{\beta}{\alpha^2} = \frac{m_e^2}{m_d^2} \frac{\rho_d}{\rho_e} \frac{(1-2\alpha)}{(1-\alpha)^2} + \frac{2}{\alpha} - \frac{(m_e + m_d)(m_e \rho_d/\rho_e + m_d)}{m_d^2}(1+K) \quad (12)$$

Therefore :—

$$m_d^2 \left\{ \frac{2}{\alpha} - \frac{\beta}{\alpha^2} - (1+K) \right\} - m_d m_e \left(1 + \frac{\rho_d}{\rho_e}\right)(1+K) - m_e^2 \frac{\rho_d}{\rho_e} \left\{ K + \left(\frac{\alpha}{1-\alpha}\right)^2 \right\} = 0 \quad (13)$$

The economic design is obtained by differentiating this equation treating m_d and α as variables, and putting $dm_d/d\alpha = 0$. Thus

$$m_d^2 \left(-\frac{2}{\alpha^2} + \frac{2\beta}{\alpha^3} \right) - m_e^2 \frac{\rho_d}{\rho_e} \frac{2\alpha}{(1-\alpha)^3} = 0 \quad (14)$$

i.e.

$$\beta = \alpha \left\{ 1 + \frac{m_e^2}{m_d^2} \frac{\rho_d}{\rho_e} \left(\frac{\alpha}{1-\alpha} \right)^3 \right\} \quad (15)$$

In this type of ejector α is usually less than 0·3, while m_e and m_d are of the same order, and ρ_e and ρ_d are of the same order ; thus the second expression on the right hand side of equation (15) may be neglected. The economic design is then obtained when

$$\alpha = \beta \quad (16)$$

Substituting this in equation (13), the minimum amount of driving fluid required for a given duty in an ejector in which the fluids mix at constant volume, is given by the positive root of m_d, in the following equation

$$m_d^2 \left\{ \frac{1}{\beta} - (1+K) \right\} - m_d m_e \left(1 + \frac{\rho_d}{\rho_e}\right)(1+K) - m_e^2 \frac{\rho_d}{\rho_e} \left\{ K + \left(\frac{\beta}{1-\beta}\right)^2 \right\} = 0 \quad (17)$$

Vapour-driven Ejector. The other type of simple ejector to be considered is one in which the driving fluid is a vapour which condenses completely as it mixes with the entrained fluid, which is a liquid. In such cases, the density of the mixed fluids is given by :—

$$\rho_2 = \rho_e \tag{18}$$

Substituting this in equation (10) gives :—

$$m_d^2\left\{\frac{2}{\alpha} - \frac{\beta}{\alpha^2} - \frac{\rho_d}{\rho_e}(1+K)\right\} - 2m_d m_e \frac{\rho_d}{\rho_e}(1+K) - m_e^2 \frac{\rho_d}{\rho_e}\left\{K + \left(\frac{\alpha}{1-\alpha}\right)^2\right\} = 0 \tag{19}$$

In this type of ejector, ρ_d/ρ_e is of the order 10^{-3}. Thus equation (19) can be simplified to :—

$$m_d^2\left(\frac{2}{\alpha} - \frac{\beta}{\alpha^2}\right) - 2m_d m_e \frac{\rho_d}{\rho_e}(1 + K) - m_e^2 \frac{\rho_d}{\rho_e}\left\{K + \left(\frac{\alpha}{1 - \alpha}\right)^2\right\} = 0 \tag{20}$$

As before, the economic design is obtained by differentiating this equation, treating m_d and α as variables and putting $dm_d/d\alpha = 0$. Thus :—

$$m_d^2\left(-\frac{2}{\alpha^2} + \frac{2\beta}{\alpha^3}\right) = m_e^2 \frac{\rho_d}{\rho_e} \frac{2\alpha}{(1 - \alpha)^3} \tag{21}$$

$$\text{i.e. } \beta = \alpha\left\{1 + \frac{m_e^2}{m_d^2}\frac{\rho_d}{\rho_e}\left(\frac{\alpha}{1 - \alpha}\right)^3\right\} \tag{22}$$

This is identical with equation (15) and provided $\beta < 0{\cdot}2$, can likewise be reduced by approximation to

$$\alpha = \beta \tag{23}$$

When $\beta \geqslant 0{\cdot}2$, the economic value of α must be obtained from equations (20) and (22) by successive approximation (see Table 2).

Eliminating α from equations (20) and (23), the minimum amount of driving fluid required for a given duty in an ejector in which the driving fluid is a vapour which condenses completely, is given by :—

$$m_d = m_e\left[\frac{\rho_d}{\rho_e}(1 + K)\,\beta + \sqrt{\frac{\rho_d}{\rho_e}\left\{K + \left(\frac{\beta}{1 - \beta}\right)^2\right\}\beta}\,\right] \tag{24}$$

neglecting the term $\beta^2\left(\frac{\rho_d}{\rho_e}\right)^2(1+K)^2$ under the square root sign.

Design Charts. It is of assistance in the design of ejectors, to have charts available, giving the mass ratio of driving to entrained fluid (m_d/m_e), for different values of the density ratio (ρ_d/ρ_e) and the ratio of overall pressure rise to kinetic pressure of driving fluid (β). To enable such graphs to be drawn, Tables 1 and 2 have been included giving numerical values of the solutions of equations (17) and (20). Economic values of α are given in Table 2, since they are not always equal to β. It has been assumed that the wall friction factor K is equal to $0{\cdot}5$, for the reasons given below.

Table 1. *Economic Design of Ejectors in which Fluids mix at Constant Volume.*

Solution of equation (17) assuming $K = 0.5$.

Economic Value of m_d/m_e

$\downarrow p_d/p_e$	$\beta=0{\cdot}05$	$\beta=0{\cdot}12$	$\beta=0{\cdot}2$	$\beta=0{\cdot}3$
<0·06	0·102	0·252	0·47	0·894
0·1	0·113	0·271	0·503	0·94
0·5	0·192	0·423	0·75	1·36
1·0	0·264	0·578	1·02	1·84
2·0	0·385	0·85	1·5	2·72
5·0	0·694	1·57	2·85	5·27

Table 2. *Economic Design of Ejectors in which the Driving Fluid condenses on mixing.*

Solution of equation (20) assuming $K = 0{\cdot}5$.

Economic Value of m_d/m_e

$\downarrow(p_d/p_e)10^3$	$\beta=0{\cdot}05$	$\beta=0{\cdot}10$	$\beta=0{\cdot}20$	$\beta=0{\cdot}30$
	$\alpha=0{\cdot}05$	$\alpha=0{\cdot}10$	$\alpha=0{\cdot}18$	$\alpha=0{\cdot}25$
0 2	0·0023	0·0032	0·0048	0·0063
0·4	0·0032	0·0046	0·0068	0·0089
0·6	0·0039	0·0056	0·0084	0·0110
1·0	0·0051	0·0073	0·0109	0·0143
1·5	0·0063	0·0090	0·0134	0·0176

The economic values of α have been found from equations (20) and (22) by successive approximation. The values obtained for m_d/m_e when $\beta=0{\cdot}3$ and $\alpha=0{\cdot}25$ are only 3% less than the values obtained by using the simplified equation $\alpha=\beta$ for the economic value of α.

DETAILS OF DESIGN

Several experimenters have made careful studies of the effects on the performance of ejectors of small variations in the general shape of the diffuser and in the position of the nozzle relative to the diffuser [1,3,4,5,6]. The recommendations given below are based on these studies and were used when making the ejectors for checking the formulæ in this paper. Fig. 2 shows the general shape of ejector used and expresses the leading dimensions in terms of D, the diameter of the throat, and D_3, the diameter of the exit of the diffuser. The value of D_3 is normally specified but it should not be less than $2D$; the recommended value, if circumstances permit, is $3D$.

Diffuser. The entrance to the diffuser should be rounded as shown in Fig. 2, the radius being approximately equal to the diameter of the throat. With very large ejectors, however, the cost of fabrication can be considerably reduced by using a conical entrance. The total angle of the cone should be about 25°, the entrainment rate with this arrangement being only 2% less than with a round entrance[4]. The optimum length of the throat

of the diffuser has been found to be between 6 and 7 diameters(1). As there is very little variation in performance within these limits, the lower value is recommended. The divergent section of the diffuser should have a total angle of 10—15°.

Position of Nozzle. The nozzle should be a convergent-divergent nozzle when the driving fluid is a gas and the pressure ratio is greater than the critical. The centre line of the nozzle must be carefully aligned with the centre line of the diffuser. The distance between the end of the nozzle and the beginning of the parallel section of the diffuser should be between 0·5 and 1·0 times the diameter of the throat of the diffuser(1). Care must be taken to avoid any constriction to the flow of the entrained fluid as it passes between the outside of the nozzle and the inside of the rounded entrance. The throat of the diffuser should be machined.

TEST RESULTS

An account is given in this section of some experiments that have been carried out at the Billingham factory of Imperial Chemical Industries Ltd., to check the accuracy of the formulæ given above, when applied to ejectors of the general shape shown in Fig. 2.

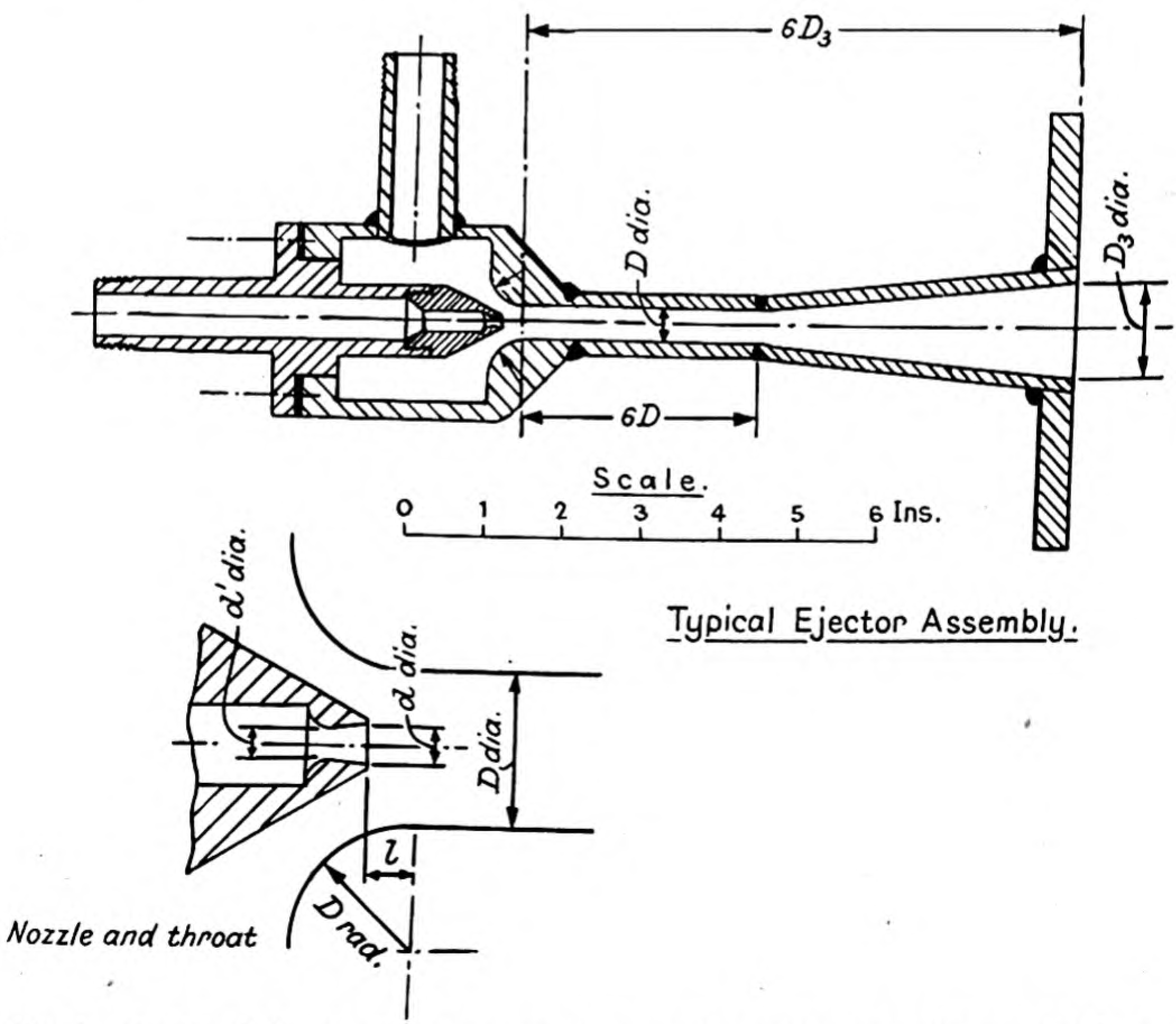


Fig. 2.—Details of ejector. The diameters D, d and d^1 should be accurately machined to the calculated dimensions. The other dimensions should be approximately to the values shown. The distance l should be between $0{\cdot}75D$ and $1{\cdot}0D$.

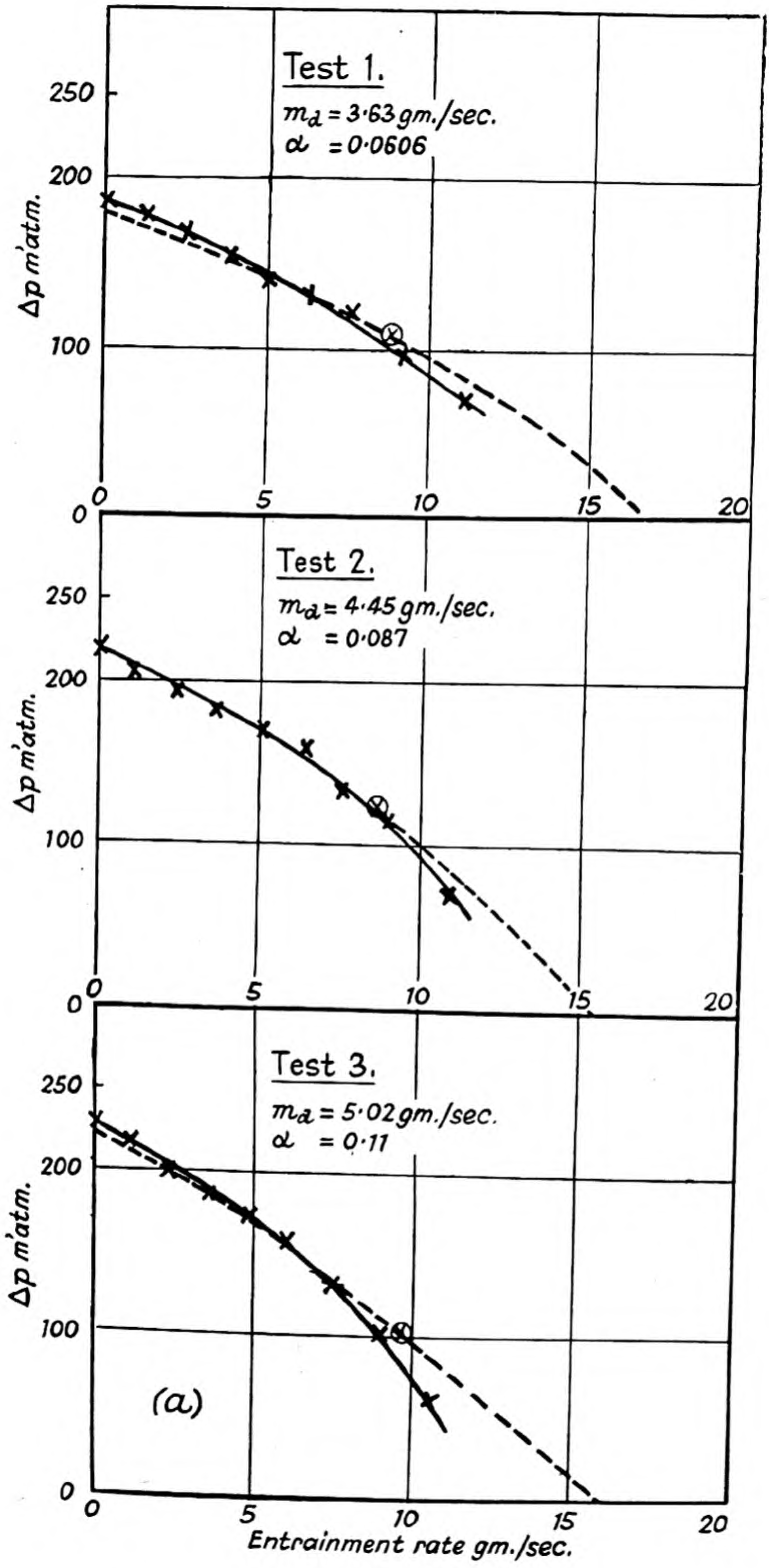
In tests 1—3, compressed air was used as the driving fluid, the entrained fluid being atmospheric air which was drawn through a control valve before entering the ejector; the ejector discharged direct to atmosphere. The rates of driving and entrained fluids were measured by orifice-plate flowmeters and a water or mercury manometer was used to measure the suction at the inlet of the ejector. A small correction was made for the loss of pressure in the inlet branch. The final velocity and density of the air leaving the nozzle were calculated assuming that $\gamma=1{\cdot}4$. The body of the ejector was the same in all three tests but three different nozzles were used.

In tests 4 and 5, superheated steam was used as the driving fluid and the entrained fluid was cold water. The rate of flow of steam was calculated from the measured diameter of the throat of the nozzle, and the temperature and pressure of the steam immediately before it; in these calculations, a Mollier diagram, based on Callendar's 1939 *Steam Tables*, was used, and it was assumed that the efficiency of the nozzle was 90%. The final velocity and density of the steam leaving the nozzle were calculated in a similar manner. The rate of entrained water was measured by an orifice-plate flowmeter. The suction at the inlet and the pressure at the outlet of the ejector were measured by mercury manometer or pressure gauges.

Table 3 below summarises the dimensions of the ejectors and the conditions of the fluids. The symbols are as defined in the list of symbols at the beginning of the paper and as shown on Fig. 2. In Fig. 3 the experimental measurements of pressure rise are plotted against the rate of flow of the entrained fluid.

Table 3. *Ejector Tests*

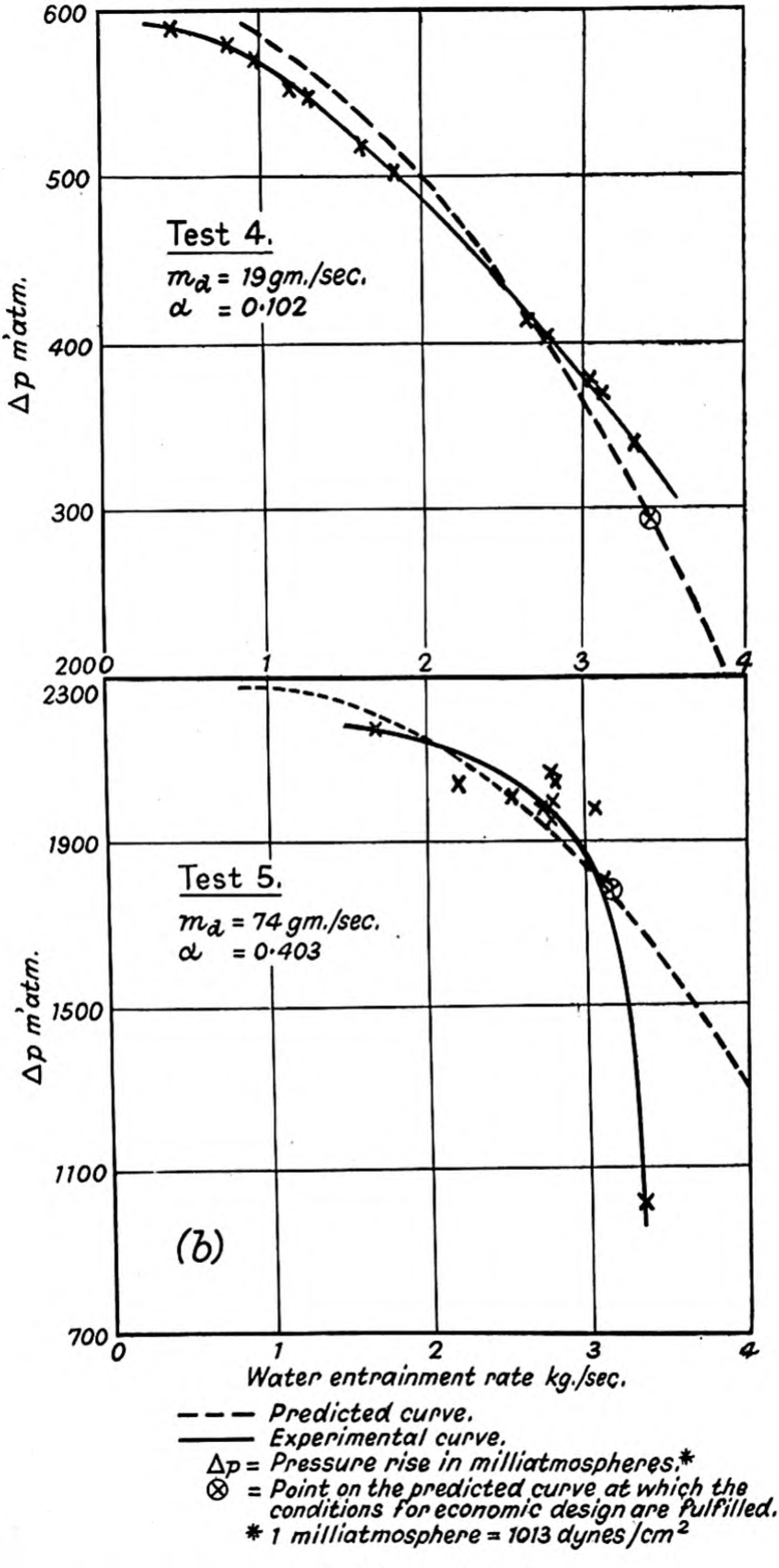
		Test No. 1	2	3	4	5
Driving fluid			Air			Steam
Pressure	lb/in² gauge	45	35	25	270	250
Temperature	°C	Atmospheric			270	300
Rate	g/sec	3·63	4·45	5·02	19	74
Entrained fluid			Air			Water
Temperature	°C		Atmospheric		45	21
Diameter of diffuser throat (D)	in		$\frac{13}{32}$			0·82
Diameter of diffuser outlet (D_3)	in		$1\frac{1}{4}$			$1\frac{1}{2}$
Distance of nozzle from diffuser (l)	in		$\frac{5}{16}$			$\frac{3}{4}$
Diameter of nozzle throat	in	0·085	0·104	0·126	0·123	0·252
Diameter of nozzle outlet (d)	in	0·10	0·120	0·135	0·262	0·52



AIR EJECTOR.

Predicted curves calculated from equation 13. (K = 0·5).

Fig. 3a.—Experiment results.



STEAM/WATER EJECTOR.

Predicted curves calculated from equation 20. (K = 0·5).

Fig. 3b.—Experiment results.

The method adopted for the analysis of the experimental results has been to calculate for each result, the value of the friction factor K, which is defined as the loss of pressure due to wall friction expressed as a fraction of the kinetic pressure of the mixed fluids at the end of the throat of the diffuser. Calculations were made from equation (13) for tests 1 to 3 and from equation (20) for tests 4 and 5.

From considerations of the friction in straight pipes and Venturis, the value of the factor K should be of the order of 0·3 to 0·5 depending to a small extent on the entrainment rate. From each experimental result a value of K was calculated, which would make that result fit the appropriate theoretical formula (equation (13) or (20)). These values of K (excepting that obtained from the last point in test 5) were found to vary between 0·3 and 0·8. The mean value was 0·5, and the observed value was found to equal the mean at roughly the point of the economic design, as given by equations (16) and (22). In practice the friction factor should be approximately constant for all ejectors and for all entrainment rates; a fixed value of $K = 0{\cdot}5$ is therefore recommended for design purposes. The design data in Tables 1 and 2 and the theoretical curves in Fig. 3 have been calculated using this figure.

DISCUSSION OF EXPERIMENTAL RESULTS

While it is realised that there is scope for a considerable amount of further experimental work on the subject of ejectors, it is felt that the few experiments described here substantiate the approximate theory derived for two types of simple ejector. It is seen from Fig. 3 that some of the experimental points fall below the theoretical line and these discrepancies are discussed below.

Air-Ejectors (tests 1 to 3). These tests were carried out to check equation (13) for the performance of ejectors in which the two fluids mix at constant volume, and it is this assumption of constant-volume mixing that leads to the discrepancies at the highest entrainment rates. Normally in air-ejectors the two fluids are initially at atmospheric temperature; the driving fluid is cooled appreciably by expansion through the nozzle but the mixed fluids leave the ejector at atmospheric temperature because there has been a rise in temperature due to the loss of energy on mixing. Hence the actual density of the mixed fluids is less than that calculated on the assumption of constant-volume mixing. This is partially offset by the fact that there is also an increase in pressure on mixing, but the effect of the increase in pressure is comparatively small. It can be seen from equation (9) that if the value of the final density ρ_2 is overestimated, the value of β and hence of the pressure rise is overestimated. This error is only appreciable at high entrainment rates; as the entrainment rate is reduced, the momentum of the mixed fluids is reduced and slight errors in calculating this momentum become negligible.

Steam-Driven Water-Ejectors (tests 4 and 5). Tests 4 and 5 were carried out to check equation (20) for the performance of ejectors in which the driving fluid is a vapour which condenses completely on mixing with the

entrained fluid. In test 5 the pressure rise was less than the predicted value at rates less than 1·6 kg/sec. The temperature of the water at the outlet was always greater than 70° C at these low rates and so it is probable that the falling off in performance was due to incomplete condensation of the steam or the liberation of dissolved air. As the entrainment rate was further reduced, the pressure rise fell down to the value given by equation (13) for constant-volume mixing. The very rapid drop in pressure rise at rates above 3·1 kg/sec was due to the occurrence of cavitation in the water at the inlet to the diffuser. Thus when designing an ejector of this type it is essential to check (*a*) that the driving fluid will condense completely, and (*b*) that the absolute pressure at the inlet to the diffuser (p_1 in equation 1) is sufficiently great to prevent cavitation.

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14

The Laws of Motion of Particles in Fluids and their Application to the Resistance of Beds of Solids to the Passage of Fluid

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ABSTRACT. The paper deals with the following items. The mathematical statement of the resistance coefficient of beds of particles and the differences in various national conventions for "resistance"; the Reynolds' number for beds of spheres are the same as those for pipes if the resistance of the side walls of the container are included, but lower values than for pipes are obtained with beds of irregular particles even when using the projected area of the particles as the single linear dimension; values for voidage and "wall effect" for beds of spheres and of coke particles—similarities and differences and the misleading nature of Furnas' calculations; Ladenburg's wall effect formula for single spheres and laminar flow—calculation of wall effect for single spheres and turbulent flow from Lunnon's data and application to beds of spheres; standard expression of resistance coefficient/Reynolds' number for beds of spheres and the relationship to the resistance coefficient/Reynolds' number curves for single spheres and single discs; possible application to beds of irregular particles.

Much experimental work has been carried out by different workers since Newton enunciated, in the 17th century, his law of Quadratic Resistance, namely, that the resistance of a fast-falling particle in a fluid is proportional to the square of the ultimate velocity. It has been shown that the law applies whether the particle is falling in a fluid, or the particle is stationary and the fluid flowing past the solid, the conditions being described as turbulent flow. Sir George Stokes, in 1851, showed that, when a sphere moves slowly through a fluid, the resistance to motion is directly proportional to the product of the velocity, the diameter of the sphere and the viscosity of the fluid. These conditions are known as streamline, or laminar, flow. In between the ranges of velocity covered by these two laws the resistance to the motion of a sphere is proportional to a power of the velocity intermediate between the quadratic of Newton's Law and the first power of Stokes' Law, Allen's Law (1900) being a useful approximation, with a power of 1·5 for the velocity and terms for the density and the viscosity of the fluid. The complete relationship of the resistance to the motion of a sphere in a fluid can be expressed by Rayleigh's graphical method relating two dimensionless coefficients, namely, the resistance coefficient and the Reynolds number, usually plotted on logarithmic co-ordinates.

The fact that, for single spheres, it is immaterial whether the particle is falling and the fluid at rest, or the fluid in motion and the particle at rest, has led several experimenters to believe that the resistance to the passage of fluids through static beds of irregular particles can be related to the laws of motion of a fluid past a single sphere, and that even the laws of resistance of suspensions of particles will be capable of expression in terms of the resistance to motion of single spheres. Such a simplification has not yet met with general acceptance, and it is in the hope that the ground can be prepared for a proper appreciation of these problems that the present paper has been prepared.

Some of the misconceptions which have hindered a simplification of ideas on this subject are due to a difference in British and Continental practice in the use of the symbol C in aerodynamics. In the Continental (and American) practice, this coefficient expresses the relation between the force producing motion and the product of the stagnation pressure for incompressible flow $\frac{1}{2}\rho v^2$ and a measure of area or volume. In British practice, ρv^2 is used and $K = M/\rho v^2 L^3 = \frac{1}{2}C$.

Thus when a sphere falling in a fluid attains its ultimate velocity, its effective weight is equal to the resistance of the fluid and :—

$$wg = \psi \rho d^2 v^2 \text{ (British convention)}$$

or $$wg = \psi \rho \frac{\pi}{4} d^2 \frac{v^2}{2} \text{ (Continental and American convention)}$$

where w = the mass
g = acceleration due to gravity
ψ = the resistance coefficient
ρ = the density of the fluid
d = the diameter of the sphere and $\frac{\pi d^2}{4}$ its projected area.
v = the ultimate velocity

The effective weight $wg = V(\sigma - \rho)g = \pi d^3 (\sigma - \rho)g/6$
where V = the volume
σ = the density of the sphere

$$\therefore \frac{\pi d^3}{6}(\sigma - \rho)g = \psi \rho \frac{\pi}{4} d^2 \frac{v^2}{2} = \frac{\pi}{8} \psi \rho d^2 v^2 \text{ (Continental and American convention)} \quad (1)$$

or $$= \psi \rho\, d^2 v^2 \text{ (British convention)} \quad (2)$$

Thus the values given in most British publications (e.g. by Lunnon[1], need multiplying by $\pi/8$ to agree with values used by American and Continental workers, though Needham and Hill[2] have followed the Continental practice. To avoid confusion, since much of the data is from American sources, the American and Continental practice is adopted in this paper.

Equation (1) gives $$\psi = \frac{4d}{3} \frac{(\sigma - \rho)g}{\rho v^2} \text{ for spheres} \quad (3)$$

If the projected area of irregular particles is expressed, following Heywood[3] as fd_p^2 where d_p is the diameter of the circle of equivalent projected area, and the volume of the irregular particles as kd^3_p then, since $k = \pi/6$ for spheres, $$\psi = \frac{8kd_p(\sigma - \rho)g}{\pi \rho\ v^2} \text{ for irregular particles} \quad (4)$$

Just as misconception has arisen with regard to the difference in convention in using the stagnation pressure (ρv^2) or ($\frac{1}{2}\rho v^2$) and with the inclusion or omission of the term $\pi/4$ in the resistance coefficient, giving two possibilities of error on this account, similar differences occur with regard to the use of the Reynolds' number, R_e. There is little chance of misunderstanding when this is used in connexion with the flow of fluids through pipes when

$$R_e = \frac{vd\rho}{\eta} \quad (5)$$

where v = velocity of the fluid in the pipe
d = diameter of the pipe
ρ = density of the fluid
η = coefficient of absolute viscosity

Rayleigh's curves relating the resistance coefficient and the Reynolds' number became familiar to engineers in the classical work of Stanton and his co-workers at the National Physical Laboratory, and are sometimes known as Stanton curves. Schiller[4] has shown that, for non-circular ducts, the mean hydraulic radius $m = \dfrac{\text{the cross-sectional area normal to flow}}{\text{the perimeter presented to the fluid}}$ can be used instead of d in the Stanton plot of the dimensionless groups $R/\int v_e^2$ and $\int v_e d_e/\eta$

where R = frictional force per unit of area
v_e = effective velocity
d_e = diameter of the equivalent channel.

For circular and square ducts, $m = \frac{1}{4}d_e$ or $d_e = 4m$; for a diamond shape duct $d_e = 4{\cdot}63m$, and for a duct of triangular section of equal sides $d = 7m$. For a duct formed by a bed of spheres in a cylindrical container m is the ratio of the integral of the cross-section between the spheres and the perimeter containing this area, per unit volume of the bed. These integrals are, in fact, the voidage and the surface area per unit volume of bed, as Carman[5] has already suggested. For a bed of particles, therefore, an alternative expression is

$$m = \frac{\text{volume of fluid in the container}}{\text{surface presented to the fluid}} = \frac{\epsilon}{s}$$

where ϵ = the fractional voidage.
s = the surface area per unit volume of packed bed, the dimensions being cm^2/cm^3. Carman[6] has given an alternative expression :

$$s = (1-\epsilon)s_o$$

where s_o is the specific surface of the particle or surface area per unit volume of packing material $= 6/d$ for spheres, so that m, for spheres $d/(1-\epsilon)6$. This expression of Carman's can be put in more general terms to include irregular particles, and also the surface area of the container, which is not always negligible. For spheres in a cylindrical container

$$\begin{aligned} m &= \frac{\text{volume of fluid in container}}{\text{surface presented to the fluid}} \\ &= \frac{\text{voidage of bed}}{\text{surface area of spheres + unit surface of container}} = \frac{\epsilon}{s + s_i} \\ &= \frac{\epsilon}{(1-\epsilon)\ (6/d) + (4/D)} = \frac{\epsilon d}{6(1-\epsilon) + 4d/D} \qquad (6) \end{aligned}$$

Equation (6) it should be noted, includes the surface area of the container where the ratio of the height of the container h is large in relation to its diameter D, so that the surface per unit volume is $\dfrac{\pi Dh}{\pi D^2 h} = \dfrac{4.}{D}$ When using irregular particles of large size, h/D often approximates to 1 and the surface area of the base of the container has to be added.

Similarly $R_e = (v_e x m \rho)/\eta$ and the effective velocity $v_e = v/\epsilon$ where v is the velocity above or below the packed bed and x is for the numeral in the equation $d_e = xm$, x having a value of 4 for circular and square ducts ; it appears to be desirable to put $x = 4$, that is, for the container of the bed and :

$$R_e = \frac{v}{\epsilon}\frac{\rho}{\eta}\frac{4}{(1-\epsilon)(6/d) + 4/D} = \frac{4vd\rho}{\eta(1-\epsilon)6 + 4d/D} \tag{7}$$

the proviso that $4/D$ is large being understood as for equation (6).

THE VOIDAGE OF PACKED BEDS OF SPHERES—WALL EFFECT

The resistance of beds of particles to the passage of fluids is intimately associated with the voidage, but, although most experimenters take a value of the order of 0·38—0·40 for small spheres in a large container, some experimenters have used large spheres where the wall effect is considerable.

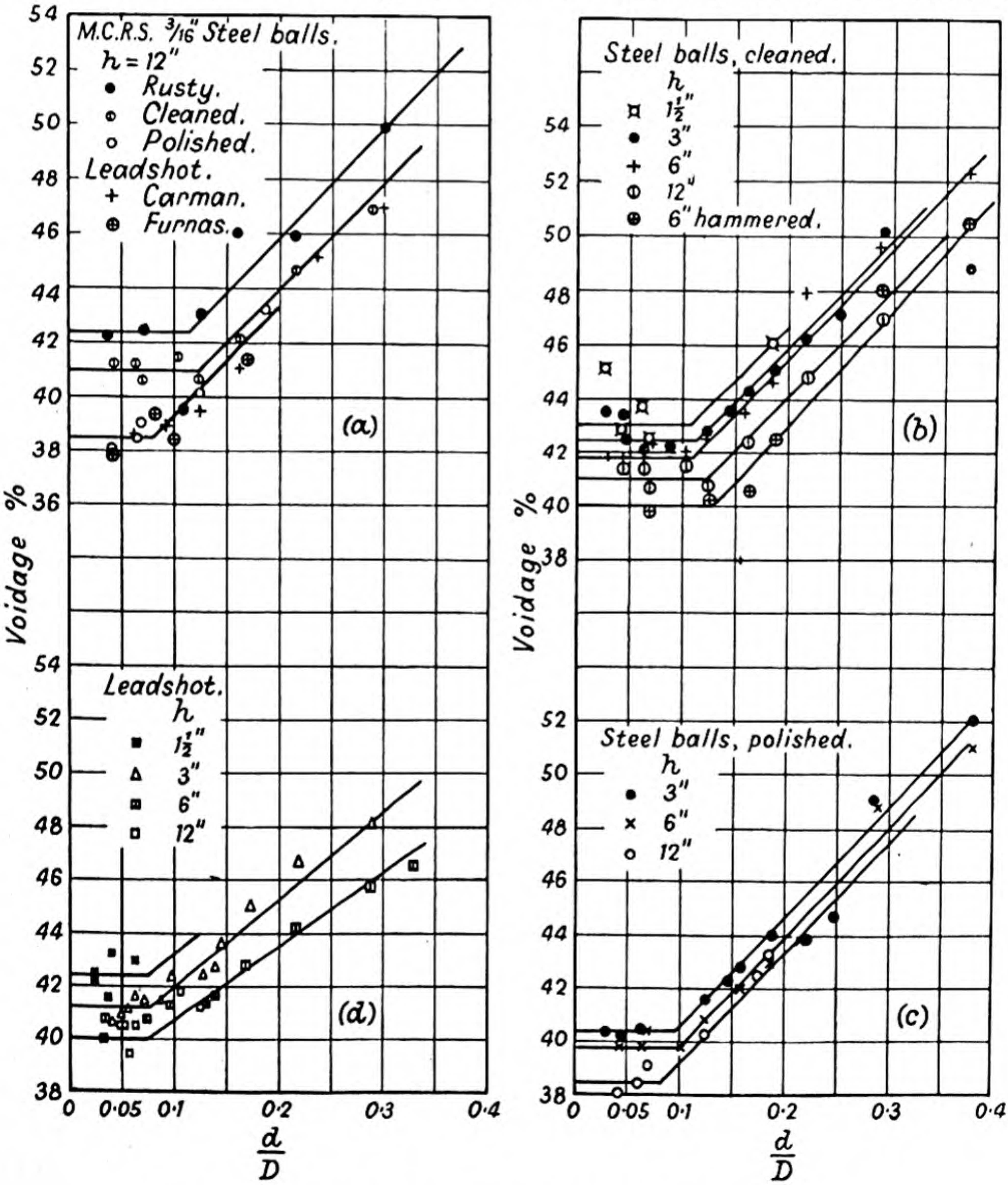

Fig. 1.—The wall effect and voidage of beds of spheres.

Others hammer the outside of steel containers containing the spheres, and some shake the container so that varying voidages are to be expected. Furnas[7] has given a complex solution to the "wall effect" based on an assumption that only layers of spheres or irregular particles adjacent to the wall are affected by the "wall effect". Carman[5] has recorded the results of tests for lead shot shaken in glass tubes. He does not record the size of lead shot used, but assuming it to be the commonest size No. 8 ($d = 0{\cdot}227$ cm), his results can be plotted in terms of d/D. We have carried out tests for the voidage of lead shot, and of steel $\frac{3}{16}$ in. ball bearings poured into containers of 12, 6, 3 and $1\frac{1}{2}$ in. height, and varying diameter, the voidage being calculated from the measured volume of the containers and the weight and density of the balls used. The results are recorded in Fig. 1 and include the effect of the condition of the surface of the steel balls which are (1) rusty, (2) cleaned (by treatment with powdered coke in a ball mill) and (3) polished (by treatment with boiled distilled water containing powdered coke and ammonia, thus giving a brilliant polish which was retained during the tests by storage in a dry atmosphere). The effect of hammering the containers was also found.

It will be seen that the height of fall of the balls in the container influences the minimum voidage appreciably for cleaned balls, but has much less effect with polished balls when the forces of friction would be low. The outstanding effect is, however, the "wall effect" which is more than a small effect on the layer of spheres contiguous with the surface, as Furnas assumed, and amounts to a partial support or bridging of the whole bed, a difference of d/D from 0·1 to 0·3 for cleaned balls reducing the amount of balls by one-sixth. The slope of all the lines for steel balls for a ratio of d/D over 0·125 or 0·1 is $= 0{\cdot}4d/D$. Although the corresponding slope for the lead shot in these tests was only $0{\cdot}3d/D$, the results of Carman and Furnas for lead shot support the use of a slope of $0{\cdot}4d/D$ for all spheres. Hammering reduces the voidage of cleaned balls by about 1 per cent. for all tests. The similarity of the tests for polished balls, and those of Furnas (who used vibration to consolidate his bed) and of Carman (who shook his tubes) suggests that a minimum voidage of 0·385 is a good value for well-polished balls dropped through a good height (in this case $h/d = 64$ for 12 in. tubes). The equation for polished steel balls dropped 12 in. into 12 in. high tubes is

$$\epsilon = 0{\cdot}4d/D + 0{\cdot}350 \text{ or } = 0{\cdot}4\,(1 + d/D) - 0{\cdot}035 \qquad (8)$$

The values for polished steel balls in tubes of different height can be expressed as

$$\epsilon = 0{\cdot}4\,(1 + d/D) - 0{\cdot}0022\,(h/d)^{0{\cdot}2} \qquad (9)$$

but the significance of this need not be pursued. The significant fact is that for values of d/D under 0·125 and certainly below 0·10 the wall-effect is negligible. It is not desirable to reduce all data of resistance of packed beds to a standard condition of $d/D = 0{\cdot}10$ or 0·075 when the wall-effect is negligible, but departure from the "normal voidage" (as indicated in equation (8)) should be taken into account in comparing the results of different experimenters.

It may be noted here that Ladenburg[8] has introduced a " wall-effect " formula for single spheres slowly falling in tubes in which (for stream-line flow)

$$\psi_1/\psi = 1 + 2{\cdot}4d/D \tag{10}$$

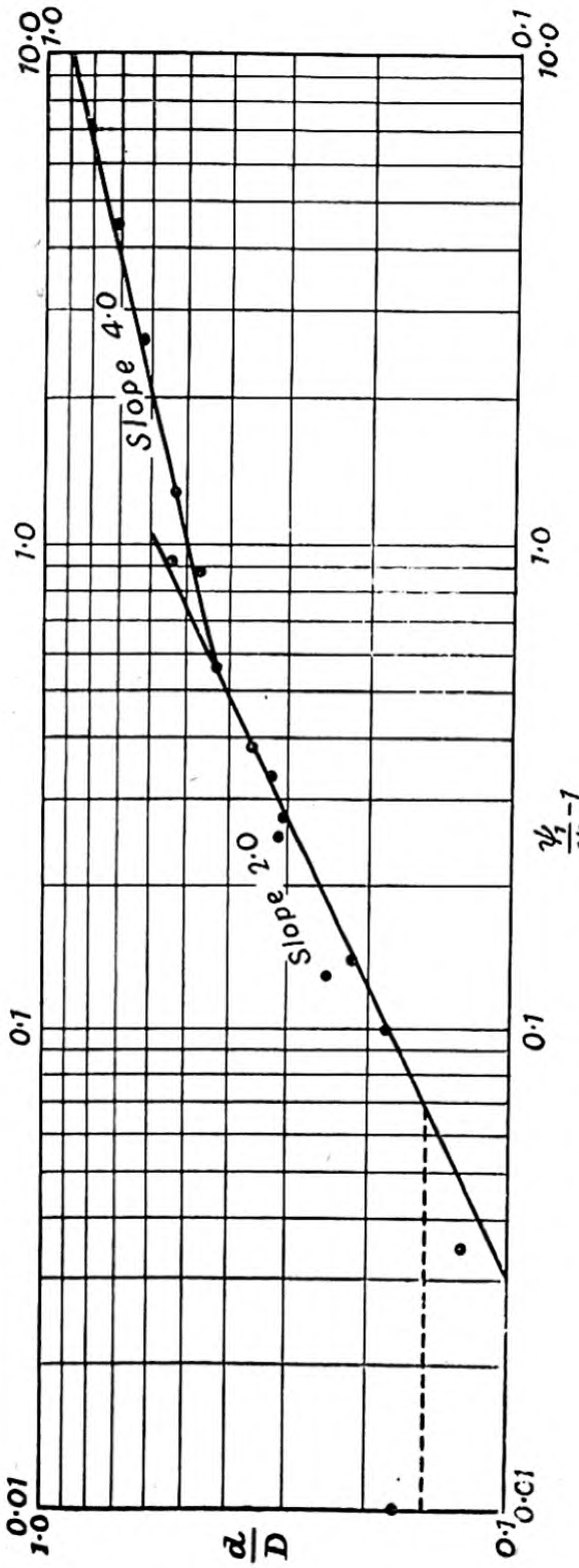

Fig. 2.—Wall effect for single spheres calculated from Lunnon's results.

where ψ_1, ψ are the relative resistances with and without wall effect. Lunnon[1] has given data for the fall of single spheres in tubes and in a tank which covers a range of R_e of 2,200 to 24,000, that is, turbulent conditions. His data, plotted on log. log. paper in Fig. 2, show that for values of d/D of 0·2 to 0·5 his "wall-effect" can be expressed as

$$\psi_1/\psi = 1 + 3{\cdot}2(d/D)^2 \text{ or } 1 + (1{\cdot}8\, d/D)^2 \tag{11}$$

an equation of a form similar to that of Ladenburg with the appropriate quadratic in place of the single power of equation (10) for streamline flow. The data are not sufficiently numerous to show with precision the lower ratio of d/D at which the wall-effect becomes negligible. It is less than 0·2 and can be put at 0·15 as an approximation only. For ratios of d/D over 0·5 the power of the wall effect becomes 4 and for d/D, 0·5—0·7

$$\psi_1/\psi = 1 + 16(d/D)^4 \tag{12}$$

From the similarity of form of equations (10) and (11) with the appropriate change in power for streamline and turbulent flow it might be expected (though this at the moment is an hypothesis not a fact) that for the transition zone between streamline and turbulent flow (which is the range of most practical importance in so many problems) might be of the form

$$\psi_1/\psi = \tfrac{1}{2}\left\{1 + 2{\cdot}4d/D + 1 + (1{\cdot}8d/D)^2\right\} = 1 + 1{\cdot}2d/D + 1{\cdot}6(d/D)^2$$

an equation which for the range d/D is 0·1—0·4 can be approximated by

$$\psi_1/\psi = 1{\cdot}86d/D + 0{\cdot}90 \tag{13}$$

THE WALL EFFECT FOR LARGE IRREGULAR PARTICLES

We have carried out a careful series of tests using coke of different sizes (hand-placed through square mesh screens, the pieces being counted as undersize if they will pass the screen in any position), namely 4-3, 3-2½, 2½-2, 2-1½, 1½-1, 1-2, 2-¼ and ¼-⅛ in. The bulk density of most of these sizes was determined in boxes of 4, 2, 1, 0·6 and 0·3 ft³ capacity, the mean of three tests for being taken for each box. The apparent density was determined for the same size by means of the soaking test of Reif[9] which reduces the error in determining the volume of a vesicular material like coke. For these two values the voidage was calculated. It was found that the results fall on a series of straight lines with different slopes but with a focus at 0·480 (Fig. 3).

An analysis of the curves showed that the factor which was influencing the voidage for the different containers was the ratio of the surface of the box in contact with the coke to the volume of the box or $A/V = 1/D(4+D/h)$ where D is the vertical dimension of the side and h its height. By trial and error it was found that the different equations could be reduced to

$$\epsilon = 0{\cdot}480 + Ad_p/10V$$

$$\epsilon = 0{\cdot}480 + (4 + D/h)d_p/10D \tag{14}$$

In the containers used the value of D/h varied from 2 to 0·5. To obtain a condition corresponding with that of equation (8) in which the base of the tube is not counted as effective surface, all the results were corrected to a value in which the effect of the base was excluded, that is where the surface is $4/D$ and

$$\epsilon = 0{\cdot}480 + 0{\cdot}4d_p/D \tag{15}$$

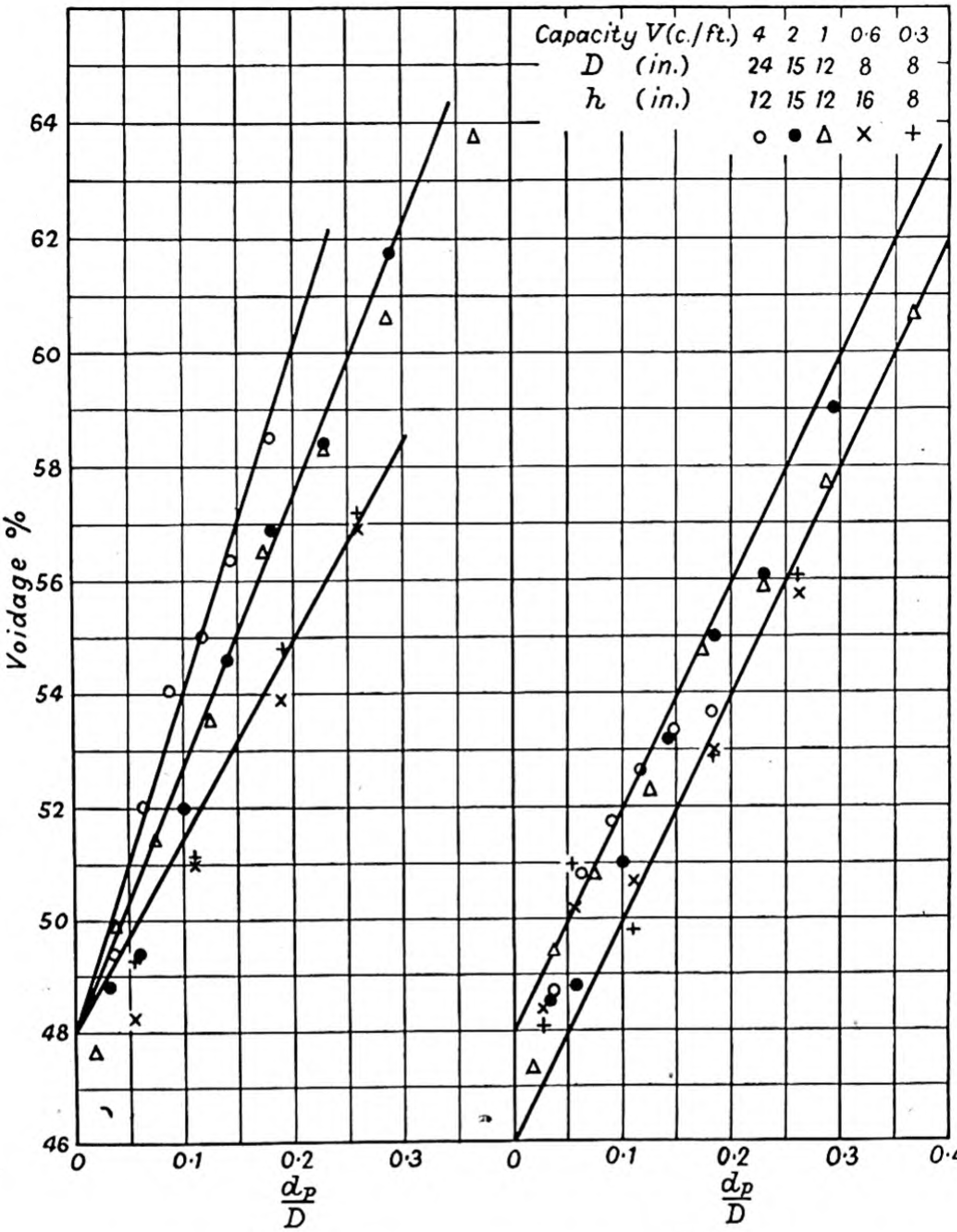

Fig. 3. Fig. 4.

Fig. 3.—The wall effect and voidage of beds of coke in containers of different size.

Fig. 4.—The voidage of beds of coke corrected to a standard wall effect.

The corrected results are given in Fig. 4 and show that they fall into a band the centre line of which is represented by $\epsilon=0{\cdot}47+0{\cdot}4\ d_p/D$. Thus when the conditions are comparable with those of the tests in spheres, the slope of the line revealing "wall-effect" is the same for spheres and for irregular particles such as coke. There is, however, no indication of a complete absence of wall-effect when d/D is under 0·1 and the wall-effect is shown for all ratios of d/D. This is rather surprising for a good working rule for the maximum size of coke to use in a coke appliance is to limit d/D to $1/6=$

0·166, for it is known that bridging occurs for larger ratios. It may be that the absence of wall effect with spheres for d/D under 0·1 is not revealed in a minimum voidage in coke, though it is obviously shown in a more freely falling bed undergoing combustion when d/D is under 1/6. Possibly the rough surface of coke gives such a resistance to consolidation of a bed that the voidage is increased because of the high friction, and that when this is reduced with low ratios of d/D, the bed tends to consolidate naturally when the friction forces can no longer act to the same extent.

Whatever the reasons for the behaviour, these tests with coke show how important is the effect of surface in contact with the particles on the voidage which is known to influence the resistance of a bed considerably.

THE REYNOLDS NUMBER OF BEDS OF SPHERES

In comparing different data for the resistance of beds, it is obviously desirable to have the same "wall effect". Equation (8) gives values for d/D of 0·075 and 0·10 and voidage of 0·380 and 0·390. A value of 0·385 has been taken as "normal" voidage for steel balls whose d/D is under 0·10.

The value of ϵ in equation (8) can be substituted in equation (7) and, for packed beds of spheres,

$$R_e = \frac{vd\rho}{\eta}\,\frac{4}{3{\cdot}9 + 1{\cdot}6d/D}$$

when $d/D = 0{\cdot}0875$, then

$$R_e = \frac{vd\rho}{\eta}\,\frac{4}{4{\cdot}04} \tag{16}$$

and even when $d/D = 0{\cdot}3$ the denominator only becomes 4·36, that is, for low values of d/D, R_e is approximately $vd\rho/\eta$ and the Reynolds number for a packed bed of spheres is approximately the same as the normal values of R_e for pipes ; or for single spheres falling in fluids.

THE REYNOLDS' NUMBER OF BEDS OF IRREGULAR PARTICLES

It was shown in equation (6) that the value of m, the mean hydraulic radius, for packed beds of spheres was $\dfrac{\epsilon d}{6(1-\epsilon) + 4d/D}$ where the wall surface of the container (excluding the base) is taken into consideration By substituting, for the irregular particles, Heywood's ratio f/k for the term d ($f/k = 6$ for spheres), equation (7) can be applied to irregular particles, where allowance is made only for the wall surface (excluding the base).

For all particles $R_e = \dfrac{vd_p\rho}{\eta}\cdot\dfrac{4}{(f/k)(1-\epsilon) + (4d_p/D)}$ (17)

The term $(f/k)(1-\epsilon)+(4d_p/D)$ may be evaluated for coke using equation (15) in which wall surface, excluding the effect of the base, only is considered, and the equation becomes

$$(f/k)\left\{1-(0{\cdot}480 + 0{\cdot}4d_p/D\right\} + 4d_p/D$$
$$= (f/k)(0{\cdot}520 - 0{\cdot}4d_p/D) + 4d_p/D$$

Taking a value of $f/k = 8$ for coke,

$$= 4{\cdot}16$$

Therefore R_e, for coke beds $= vd\rho/\eta{\cdot}4/4{\cdot}16$

Thus for an irregular particle like coke (or other particles which give a high voidage), the Reynolds number is approximately the same as that taken for pipes.

THE RESISTANCE OF BEDS OF SPHERES TO THE PASSAGE OF FLUIDS

Many experimenters have followed Blake[10] in plotting $v\rho/\eta s$ instead of the Reynolds' number for granular solids, the term $1/s$ being a method of expressing the mean hydraulic radius. Comparison with the Rayleigh or Stanton plot can only be obtained by using $4m$ for circular pipes, e.g.

$R_e = v_e d_e \rho/\eta = v\rho 4m/\eta$ since, for circular pipes $m = d_e/4$ when v_e, $d_e=$ the effective velocities and diameters. For beds of spheres, since $v_e = v/\epsilon$ and $m = \epsilon/s$

$R_e = \frac{v}{\epsilon} \frac{\rho}{\eta} \frac{4\epsilon}{s} = \frac{4v\rho}{\eta s} = \frac{vd\rho}{\eta}$, the form obtained in equation (16), when the surface of the wall is taken into consideration and d/D is fairly low. In many American publications an "effective" Reynolds' number is used by converting v, the velocity above or below the bed, to $v_e = v/\epsilon$ and using this with d the linear dimension of the sphere, thus almost doubling the Reynolds number.

The fundamental equation of permeability given by D'Arcy in 1856, based on the flow of water through sand, is

$v = k\Delta P/L$ where $k=$ the coefficient of permeability, and $\Delta P/L$ the drop in pressure per unit of length.

This is analogous to Poiseuille's law for the flow of a viscous fluid through a capillary,

$$v = \frac{d_e^2}{32\eta} \times \frac{\Delta Pg}{L}$$

In place of Stanton's dimensionless group $R/\rho v_e{}^2$ where R can be replaced by $\frac{\Delta\rho 2g}{L} \cdot \frac{\epsilon}{s}$ and $v^2{}_e$ by $\left(\frac{v}{\epsilon}\right)^2$ and the dimensionless term becomes

$$\frac{\Delta\rho 2gt}{L\eta s} \cdot \frac{\epsilon^2}{u^2} = \frac{\Delta\rho 2g\epsilon^3}{LS\rho v^2} = \psi \tag{18}$$

substituting for s as in equation (6),

$$\psi = \frac{\Delta P}{L} \cdot \frac{2g}{\rho v^2} \cdot \frac{\epsilon^3 d}{(1-\epsilon)\ 6 + 4/D} \tag{19}$$

Equation (18) is essentially the term recommended by Blake and used by most who have studied the resistance of beds of granular solids and is known as the resistance coefficient ψ, but equation (19), by taking in the resistance of the walls, brings the data into the same conditions as equation (7).

The data for beds of spheres of Burke and Plummer[11], Chilton and Colburn[12], Chalmers, Taliaferro and Rawlins[13], Bakhmeteff and Feodoroff [14], and Saunders and Ford [15], corrected to normal voidage are assembled in Fig. 5. The corrections allow for the wall effect (*a*) by introducing the wall surface of the container into the Reynolds' number as in equation (7), (*b*) by taking note of the wall-effect on the voidage and taking the "normal voidage" as in equation (8); if by hammering or other effect the voidage was not normal as indicated by equation (8) by a method

to be described later. It should be emphasized that the results are not corrected to a single standard voidage (except when d/D is under 0·10), but to the normal voidage of beds of balls of good clean surface dropped through a height of about 10 in, so that results for excessive low voidage obtained by hammering can be compared with the others.

The correction for the resistance from an abnormal to a normal voidage was based on results of Bakhmeteff and Feodoroff[14]. These workers in two series obtained voidages of 0·402 and 0·459 in horizontal packings for which the resistance coefficients naturally differed. They found that they fell on the same curve taking the cross sectional area of the voids as 0·67 (the effective velocity being increased accordingly) and the effective void diameter as being 0·33, thus giving a reduced Reynolds' number. This method is not very satisfactory, and it was found that the two series of the results fall on the same line if the corrected resistance was multiplied by $(\epsilon/\epsilon_n)^2$ where ϵ is the observed voidage and ϵ_n the " normal voidage " calculated from equation (8). Equation (19) although it has a third power of the voidage in the numerator has a single power in the denominator and so a square law effect seems reasonable. Since ϵ is linearly related to d/D (equation 8) and a square law for d/D occurs in equation (11), this gives further support for a square law correction.

The data in Fig. 5 lie close to a curve, which is not drawn, which becomes horizontal, i.e. is independent of the Reynolds number at $R_e=2000$. It will be recalled that the " friction factor " in the Stanton curves show streamline flow for Reynolds' numbers up to approximately 2000 and between $R_e=2000$—3000 shows a transition to turbulent flow with a sudden increase in resistance, which gradually falls with further increase in value of R_e. The curve for Fig. 5 is reproduced in Fig. 6 and compared with values for single spheres based on the work of Allen[16], Liebster[17], Schmiedel[18], and Lunnon[1]. This curve lies close to that for beds and spheres but joins it at $R_e=2000$ after which it is substantially horizontal to $R_e=8000$, the limit of the data. A line for single disks based on the work of Squires and Squires[19], Schmiedel[18], Simmons and Dewey[20] and Wieselberger[21] is also given and falls slightly below that for single spheres until $R_e=100$ when the resistance becomes independent of R_e.

Thus the curves for single spheres or beds of spheres in Fig. 6 only have one feature in common with the Stanton curves for fluid flow through pipes, that resistance attains a minimum value at $R_e=2000$, but the sudden onset of turbulent conditions is not revealed between R_e, 2000 and 3000, and the resistance is substantially unaffected by further increase in R_e. True streamline conditions are only found for single spheres and beds of spheres for values of R_e under 2 to 3 (when the slope of curve is unity), which is not surprising when the tortuous nature of the path of the fluid is considered. It is not possible in this paper to consider beds of irregular particles, but it may be noted that the plot of R_e for single particles of the free falling of single mineral particles plotted by Needham and Hill[2] (to which certain corrections have been applied) falls almost exactly on the line for disks in Fig. 6 for R_e, 0·1 to 20,000.

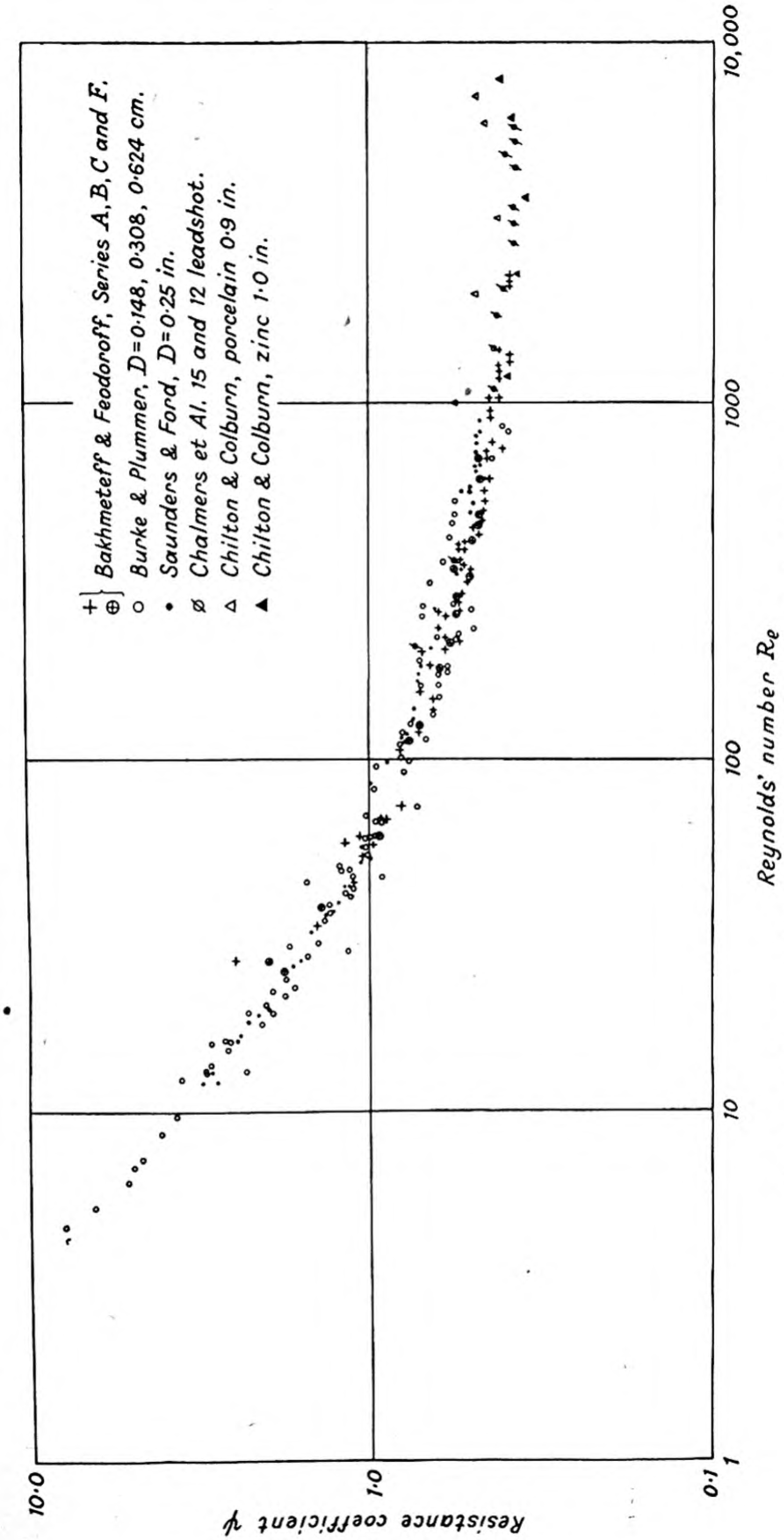

Fig. 5.—Resistance coefficient ψ and Reynolds number R_e for beds of spheres.

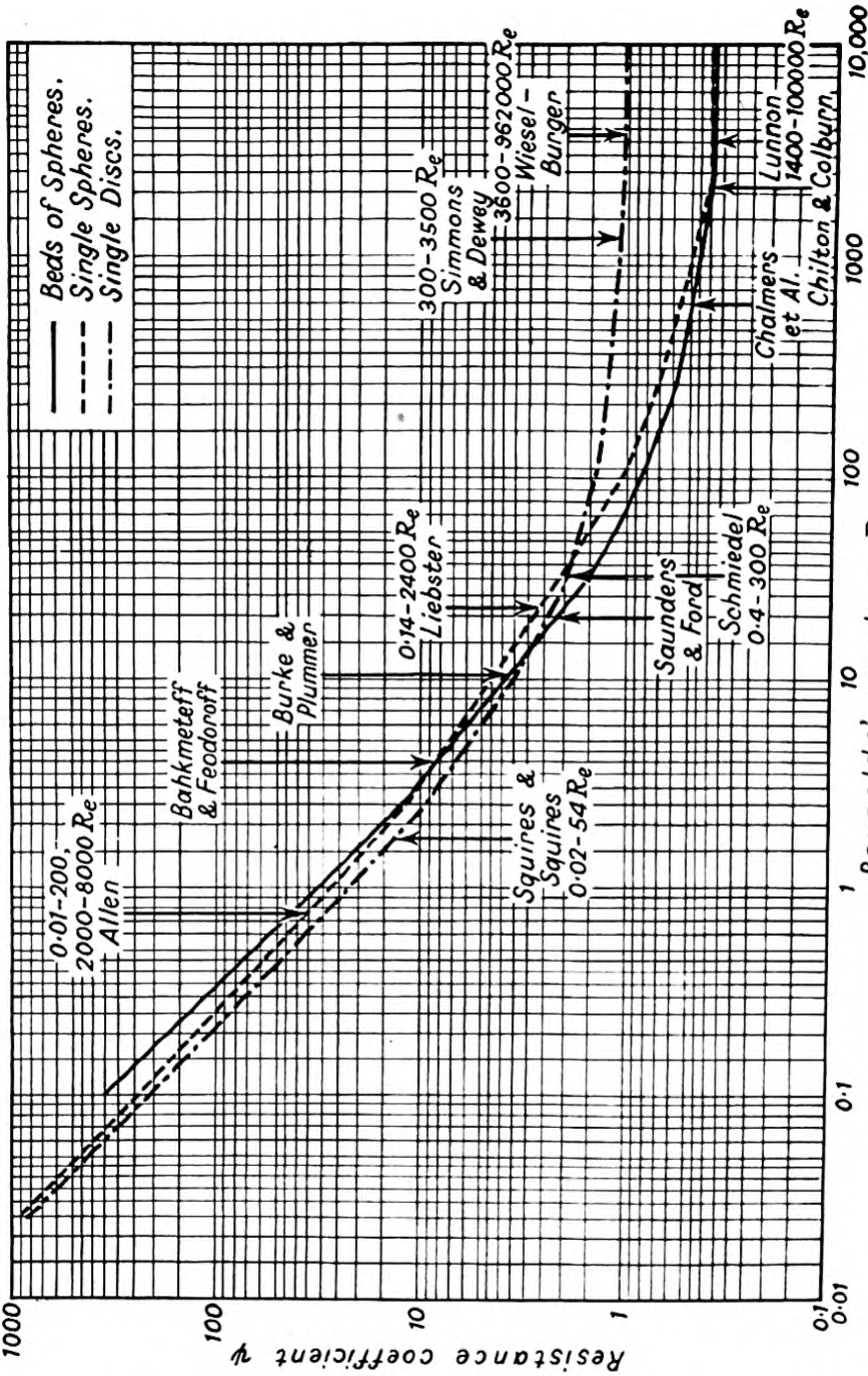

Fig. 6.—ψ/R_e curves for single spheres, single disks, and beds of spheres.

Thus the conclusion of Fehling that the resistance of a bed of spheres can be expressed as a simple function of the relationship ψ/R_e for single spheres, in the region R_e 20 to 300 is confirmed and extended to a range of values of R_e 10 to 1000 in which the ratio of the resistances is only 0·81. Fehling applied this relationship also to irregular particles by adding a shape factor (which Rose transposed to Heywood's ratios of f/k) but it appears that for R_e 10 to 10,000 the resistance of single irregular particles is better

related to the resistance of single disks, and therefore the resistance of beds of irregular particles (when they can be considered) are better compared with the resistance of single disks. The values of ψ for different values of R_e in Fig. 6 are given in the table.

Values of Resistance Coefficient (ψ) and Reynolds' number (R_e).

R_e	0·025	0·1	0·25	1·0	2·5	10	25	100	250	1,000	2,500	10,000
ψ Single Spheres	910	245	103	28	12·0	4·55	2·45	1·0	0·72	0·48	0·40	0·40
ψ_1 Single Disks	840	210	86	22·5	9·3	3·5	2·1	1·4	1·3	1·2	1·1	1·1
ψ_2 Beds of Spheres	—	350	140	36	2·5	3·8	1·8	0·8	0·58	0·42	0·38	0·40
ψ_3 Single Irregular particles	—	220	90	22·5	9·0	3·5	2·5	1·6	1·35	1·2	1·2	1·2
Ratio ψ_3/ψ		0·95	0·95	1·0	1·0	1·0	0·85	0·88	0·96	1·0	0·92	0·92
Ratio ψ_2/ψ_1		1·43	1·36	1·29	1·04	0·84	0·74	0·80	0·80	0·87	0·95	1·0

CONCLUSION

The resistance of beds of particles to the passage of fluids has been studied by many workers who report their results in different ways which are difficult to compare. Real progress will only be made when agreed conventions are made and it is suggested that the Continental or American conception of the resistance coefficient is necessary to give mathematical exactness, which was not called for in the Stanton curves. Moreover the wall effect and the effect of voidage must be established beyond doubt, and this paper is intended to clarify this concept. Further experimental work is required to establish beyond question that the misleading trends from Furnas's analysis of " wall-effect " can be overcome by some comparatively simple equations which can be simple fractions of the ratio d/D, as in equations (8), (14), and (15). By adopting a standard wall effect, namely, the side wall surface in contact with the bed (and noting that the effect of base must be included in shallow beds), the wall effect can be included, for small ratios of d/D in the Reynolds number as in equations (16) and (17). The conception of normal voidage is useful and in comparing different data the results should be reduced to this for which a square law seems reasonable with the limited data available, as well as by analogy with Lunnon's data analysed in equation (11), though further experimental results are desirable. The resistance of beds of spheres of normal voidage is proportional to the third power of the voidage, though the range of voidage is only 0·35—0·51. This requires testing for wider ranges of voidage.

The problems for beds of irregular particles are more complex though from the limited references possible in this paper it appears that Heywood's projected area d_p is a satisfactory basis, and that comparisons of resistance should be made with those of single disks. On these lines it is believed that fundamental data for the resistance of single simple geometrical bodies can be applied to the more complex problems of the resistance of beds of irregular particles and that eventually the resistance of a fuel bed in small domestic boilers, a foundry cupola or an iron blast furnace can be evolved in terms used in classical physics.

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See Note added in proof, p. 163.

15

Turbulence Excitation on Ship Models

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ABSTRACT. This note gives a general description of recent work directed towards improving the technique of ship model experiments. The latter are essential for evolving the optimum form of hull for the specified conditions, and it has been found that measures must be taken to avoid laminar flow on the wax models used in experimental tanks.

INTRODUCTION

The naval architect has many problems in fluid flow with which he must contend. Rather than supply a list of these various problems, this paper considers one particular problem only.

For many years it has been known to experimenters that the presence of a wire or string around the bow of a ship model (or the nose of a torpedo) could have a marked effect upon the measured resistance. In general, the latter was increased when a wire was fitted. It was generally assumed that the increased resistance so obtained proved the existence of laminar flow over the naked model forebody. In those cases where the increases in model resistance due to a trip wire were proved greater than the resistance of the wire itself, it was assumed that the wire had stimulated completely turbulent flow in place of the partial laminar flow which had obtained before fitting the wire.

Certain continental tanks adopted the practice of fitting a trip wire near the bow in all models, for both resistance and propulsion tests. It was believed that this practice obviated possible errors in the estimates of full-scale ship resistance by ensuring turbulent flow over the whole model surface. Model results obtained when a trip wire (or other turbulence-stimulating device) was fitted were therefore considered more reliable guides to ship estimates, since full-scale conditions were certain to be fully turbulent.

Tests have recently been carried out in each of the British experiment tanks for the British Shipbuilding Research Association on a standard wooden model of 0·75 block coefficient* having a raked stem and on wax replicas of this model. It was found in every case that the fitting of a turbulence stimulator, either a trip wire or a strut ahead of the model, increased the model resistance by a large amount over a long range of speed. Further experiments at the National Physical Laboratory and elsewhere on a variety of models have given increases in model resistance of varying amounts, ranging from 2% to about 10%. The evidence thus accumulated has tended to show that these increases in resistance were due to the excitation of turbulent flow over the forebody in place of the laminar flow which prevailed in the absence of a turbulence-stimulating device.

*Block coefficient = volume of displacement/volume of the circumscribing rectangular solid.

It was appreciated that the measured increases in model resistances made it probable that the results of resistance tests on full models must be regarded with suspicion, particularly over the lower parts of the speed range. For example, form variations may have caused changes in the location of transition from laminar to turbulent flow, and as a result an erroneous indication of the effect of change of form may have been obtained. Investigations have been made in an effort to explore the extent of laminar flow in ship models, to study the physical causes underlying it and so to be in a position to control transition and to interpret the results correctly.

PHYSICS OF THE BOUNDARY LAYER

Consider a thin, straight plank moving at constant velocity through

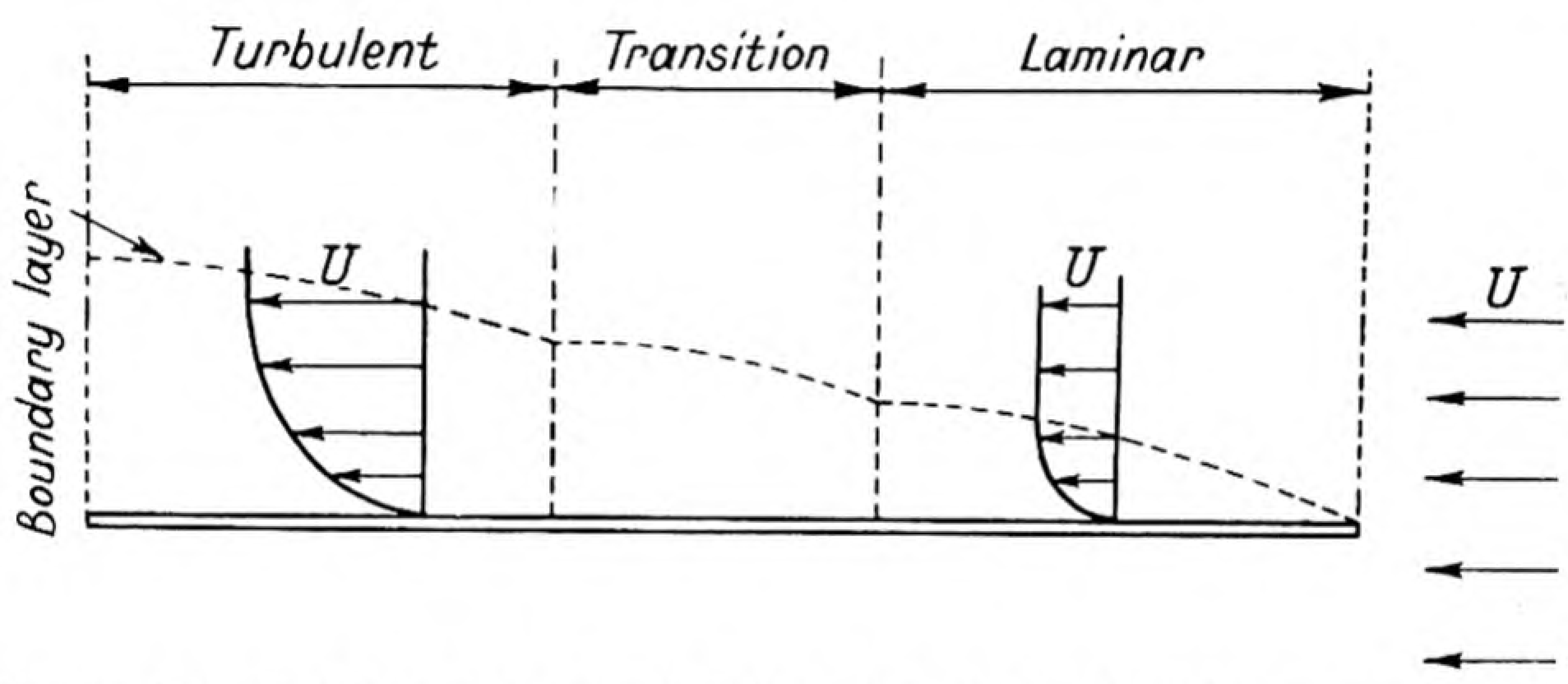

Fig. 1.—Boundary layer and velocity distribution along a plank (diagrammatic only).

undisturbed fluid (Fig. 1). According to Prandtl's theory of fluid friction, the fluid in contact clings to the solid surface without slipping, but slipping occurs between successive fluid layers. In any plane normal to the plank the velocity varies from zero at the plank surface to a maximum in the free stream. The layer of fluid in which most of this velocity variation occurs is found by measurement to be relatively thin and is called the boundary layer.

The velocity distribution within the boundary layer depends on the type of flow, whether laminar or turbulent. The thickness of the layer is zero at the leading edge and increases with length along the plank from the leading edge. For some distance the layer is laminar, with viscous forces predominating. The boundary layer flow changes gradually from laminar to turbulent in a region of transition. In the turbulent layer, beyond the transition, the velocity distribution differs from that in the laminar layer. Within the turbulent boundary layer itself there is a very thin layer next to the plank in which the flow is laminar ; this region is called the laminar sub-layer.

No general rules can be laid down as to the nature of the boundary layer for all cases. The transition from laminar to turbulent flow is not sharp. The extent of the laminar boundary layer in the direction of motion

depends on the initial turbulence in the stream ahead of the plank, on the shape of the leading edge and on the surface roughness.

If μ = local velocity component
ν = kinematic viscosity
L = total length of vessel
v = free stream velocity

then under given conditions transition will take place at a critical value of $\mu L/\nu = vx_c/\nu$ where x_c is the distance from the leading edge to the point where transition starts. Results in air show that this value may vary from $0{\cdot}5 \times 10^6$ to $2{\cdot}5 \times 10^6$ depending on the factors mentioned above. A turbulent boundary layer can be obtained at low Reynolds numbers, for example, by using a relatively blunt nose at the leading edge. The important point is that resistance in the turbulent regime is appreciably higher than in the laminar region. (See Fig. 2).

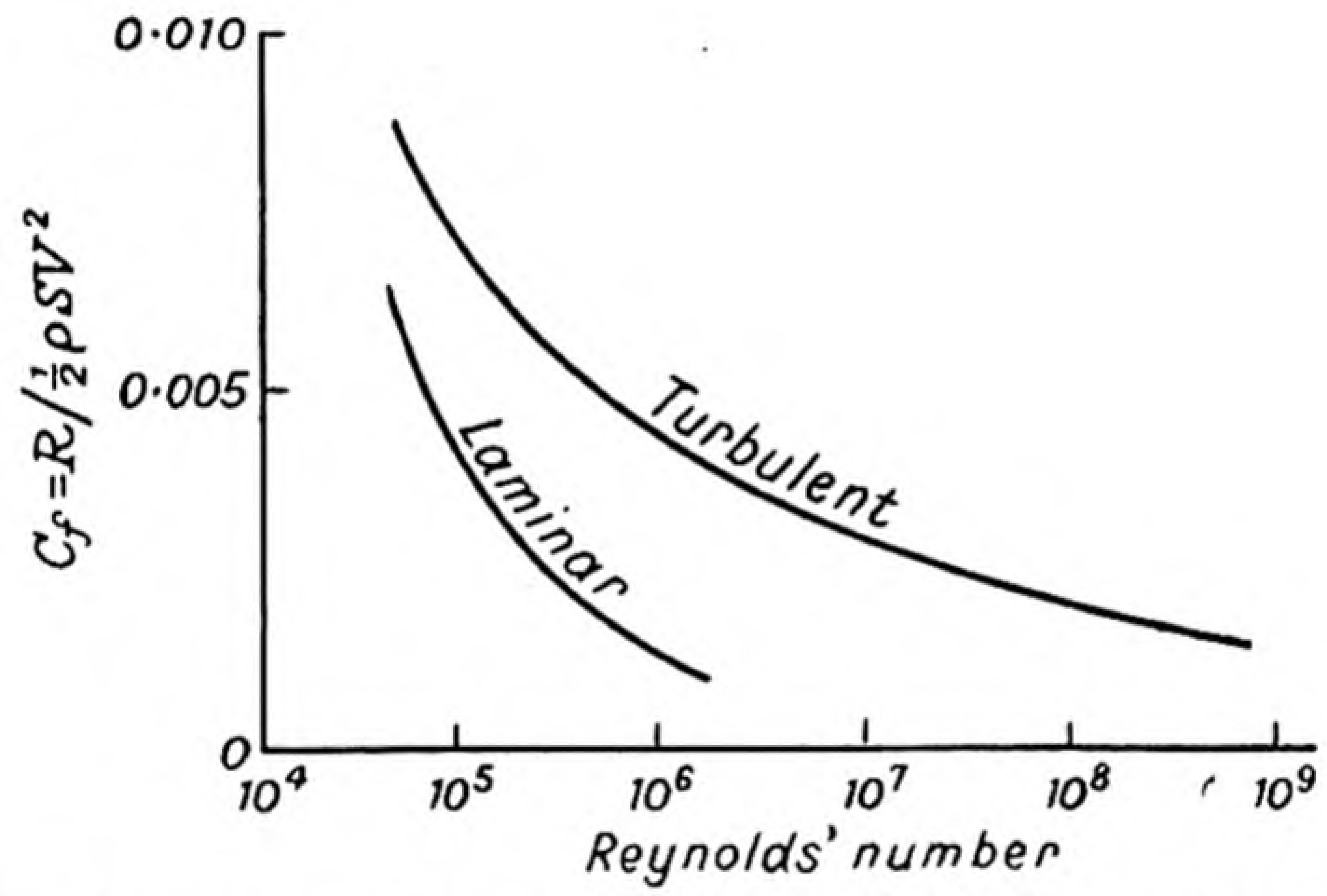

Fig. 2.—Frictional resistance coefficient as effected by Reynolds number and flow regime.

So far we have considered the simple case of the plank where the superimposed pressure on the boundary layer is independent of length. For a ship or wing shape the conditions are complicated by the addition of curvature of surface and pressure gradients along the streamline.

Effect of Pressure Gradient. This pressure gradient may have two distinct effects (1) on the point of transition from laminar to turbulent flow in the boundary layer, and (2) on the point of complete separation of the boundary layer from the body.

When the intensity of pressure decreases in the direction of motion, i.e., when the potential flow is accelerating, the conditions tend to accelerate the boundary layer and therefore conduce to making it thin, and delay transition. With an adverse pressure gradient (i.e., increasing in the direction of motion) the conditions combine with the boundary shear to retard the layer, thus causing it to thicken, and promote transition.

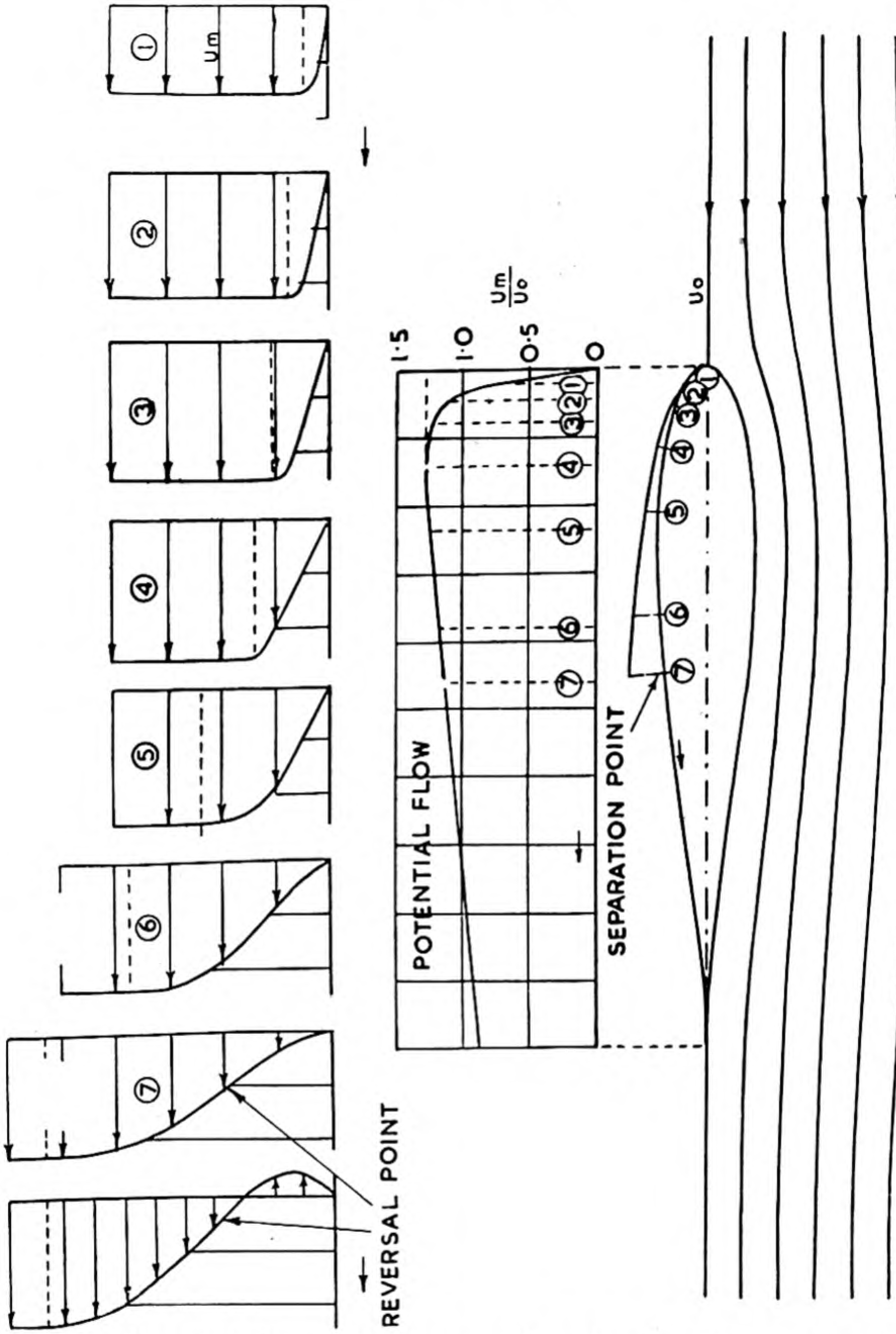

Fig. 3.—Typical velocity distribution around a streamlined body.

In Fig. 3, within the critical range of vx_c/ν, transition would probably be delayed for the above reasons to a point between positions (4) and (5), as shown by the rapid thickening of the boundary layer in this region. In other words, the critical value of vx_c/ν is increased in a zone of negative pressure gradient and decreased in one of positive pressure gradient.

Moving towards positions (6) and (7) in Fig. 3, if there is not sufficient kinetic energy in the boundary layer to carry it through the region of increasing pressure, the fluid particles near the wall lose their forward velocity and the streamline will be diverted from the boundary. This phenomenon is called separation. Back or reversed flow occurs downstream from the point of separation and eddies develop.

In Fig. 3 the slope of the velocity distribution curve grows more steep until it becomes vertical, at which point separation occurs. There the streamline leaves the boundary and reversal of flow takes place, causing heavy eddy shedding with resultant large energy loss.

We now summarise briefly as follows :—

(i) The resistance of a completely submerged body moving in a viscous fluid may be regarded as made up of frictional resistance and pressure resistance. When the body moves on the free surface of a liquid the pressure system creates a gravity wave system on the free surface. Frictional and pressure resistances are expended in vorticity, the generation of eddies, and the formation of waves ; this kinetic energy is subsequently transformed into heat.

(ii) Both tangential stresses and form resistances are influenced by the nature of the boundary layer.

(iii) A typical pressure distribution is shown in Fig. 4. A region of high pressure at or near the stem is followed by a falling pressure gradient to the forward shoulder. Minor fluctuations occur over the mid-length depending chiefly on the speed and wave formation. Towards the after end a rising pressure gradient is established, but this may break down due to separation of the boundary layer taking place.

(iv) The boundary layer on the model is laminar at the stem and may remain laminar over the length of the falling pressure gradient. If it has not become turbulent for other reasons before reaching the point where the pressure gradient reverses at the forward shoulder or near midships, transition will generally take place at this position.

(v) Froude's classic assumptions were :—

(*a*) Skin friction resistance is independent of hull shape and depends only on the length, nature, and extent of surface.

(*b*) Residuary resistance (total resistance minus frictional) is a function of Froude number v^2/Lg only.

BRITISH SHIPBUILDING RESEARCH ASSOCIATION EXPERIMENTS

As stated earlier, tests were made in all British tanks on a 0·75 block model. Typical results from one tank are given in Fig. 5. There was reasonable agreement between the various tanks both with and without turbulence-promoting devices. It should be noted that the results are given corresponding to the load draught of 26 ft. for a 400 ft. ship. The effect of turbulence promoting devices was less at a draught corresponding to 21 ft. and had disappeared at 16 ft. Two further points of importance are illustrated in the results shown in Fig. 5, namely, the effect of a strut and a trip wire is not additive, and moving a trip wire aft reduces the addition, as might be expected. It should also be noted that some increase was measured up to the highest speeds tested.

Methodical tests have been made to determine the optimum diameter and position of trip wire. The results may be summarised briefly as follows : For models of the size usually employed, i.e., about 18 ft. in length, the

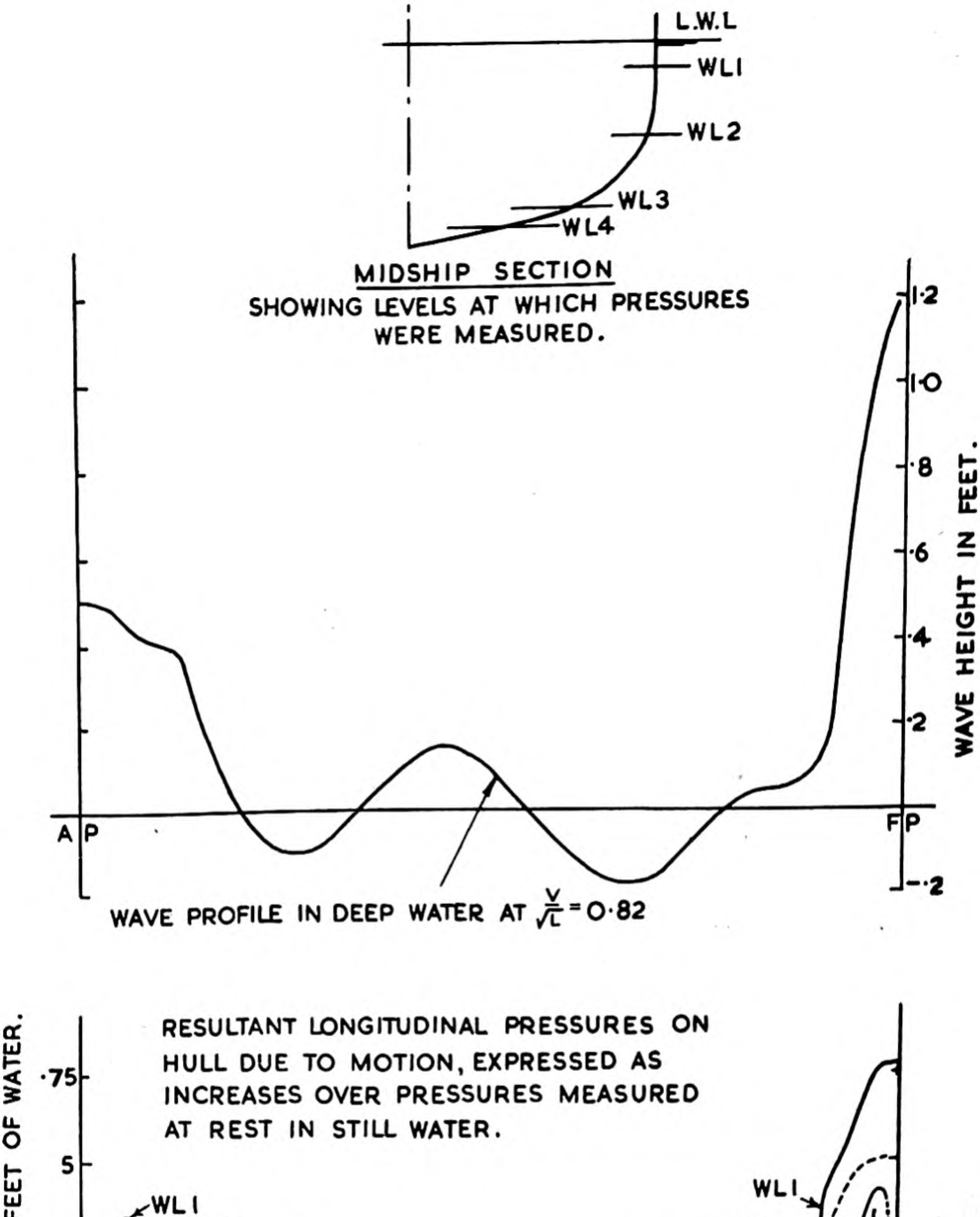

Fig. 4.—Wave contour and longitudinal pressure distribution on hull.
35.9 ft. pinnace at 5.14 knots.

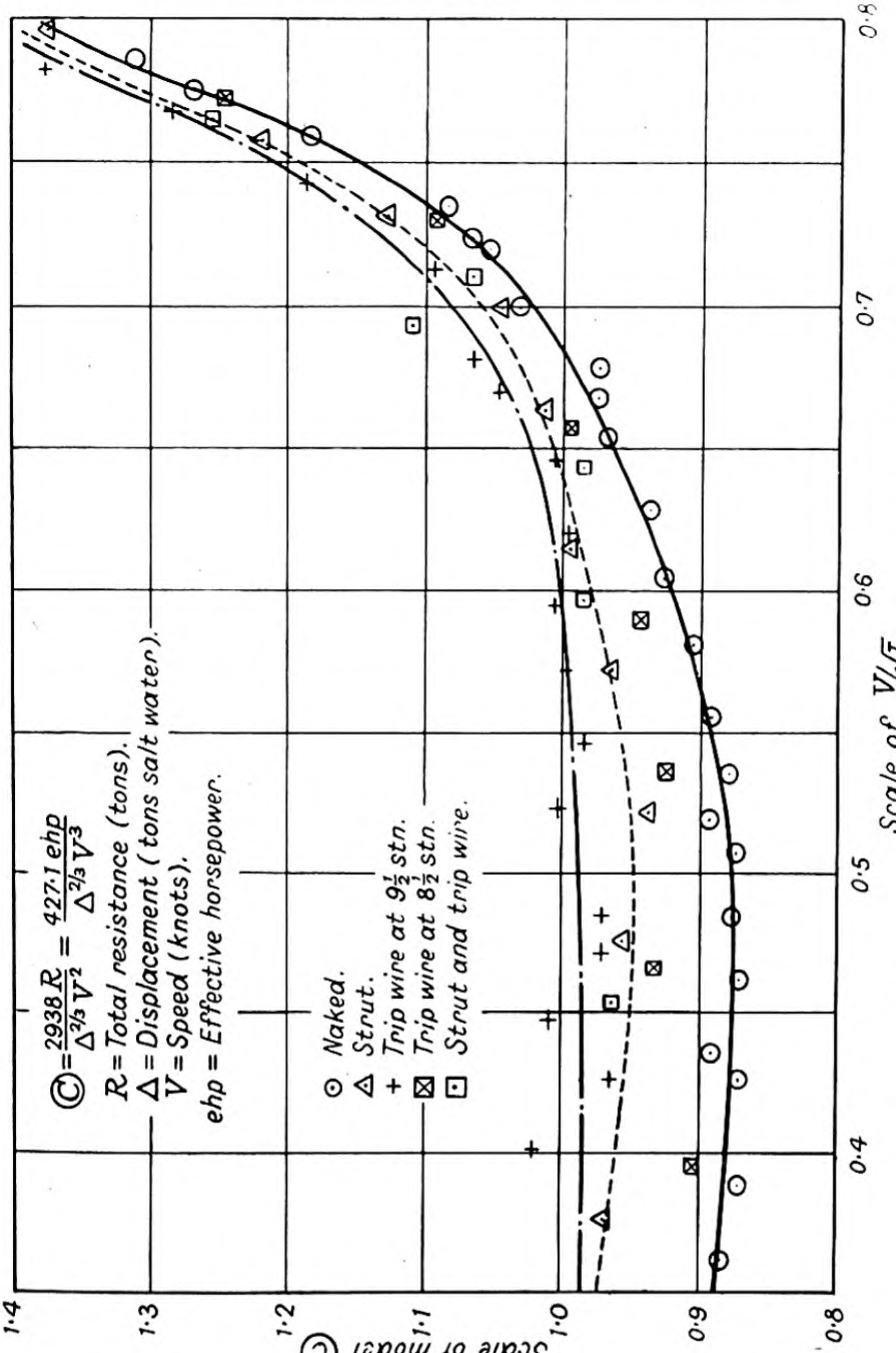

Fig. 5.—Effect of turbulence stimulation on B.S.R.A. 0·75 C_B model.

trip wire should be located 5% of the length abaft the bow and its diameter should be about 0·03 in.

It appeared desirable to study the extent of the area of laminar flow on various forms and the development of one method of doing this has been given recently by Walker.* In this paper it is shown for the particular case of the 0·75 block model referred to above, that the difference in measured resistance with and without a trip wire on station $9\frac{1}{2}$ is substantially account ted for by the calculated difference between turbulent and laminar flow over

*W. P. Walker, *Trans. Inst. Naval Arch*, **91**, p. 220 (1949).

the area indicated by the chemical spray tests. The method, however, is tedious and involves a large amount of handling of the model, which is a drawback when dealing with wax models. In certain circumstances also the interpretation of the chemical spray pictures has presented some difficulty.

Some attempts were made at the National Physical Laboratory to apply a hot-wire anemometer technique to the problem, but this was unsuccessful. It is believed, however, that such a method could be developed.

As an alternative, the National Physical Laboratory tried out two simple methods which have proved effective and are easily applied. These methods are, firstly, the release of ink streams into the boundary layer at a number of points and the observation of the behaviour of these streams as they flow aft, and, secondly, the exploration of the boundary layer by a small total head tube.

A full account of the whole boundary layer problem has been given by Allan and Conn*.

Investigation of Pressure Change along Streamlines. The general problem was investigated further in a number of typical cases by tracing a suitable streamline on the forebody of a model and measuring the pressure distribution on the model surface along this line. This was done by fitting a number of copper pipes at intervals along the chosen line and with their outlets flush with the outer surface of the model in a similar manner to that described for the ink jet tubes. These pipes were connected to a group of glass tubes mounted on a board in the model and arranged so that change in water level in the tube could be observed. The water readings were noted in each tube stationary and at the chosen steady speed of advance. In most cases the model was held in a fixed trim but in one or two cases the model was free to trim and correction was made to the readings for the measured change of trim. These streamlines were chosen along the path giving maximum length of laminar flow and the region of transition was noted by the jet method.

CONCLUSION

It is essential that fully turbulent flow be stimulated on ship models, since laminar flow may falsify the results. Cases are known where the resistance or drag of a hull *A* was worse than that of a hull *B* when tested without trip wire, and materially better when tested with a trip wire. It has therefore been decided to use a trip wire on all models undergoing test.

*J. F. Allan and J. F. C. Conn. *Trans. Inst. Naval Arch.*, 92, p. 107 (1950).

REPORTS FROM DISCUSSION GROUPS AND FINAL SUMMING UP OF THE CONFERENCE

The Chairmen, or in their absence the Recorders, of the five discussion groups made the following reports on the work of their groups to the final session of the Conference, and they included the groups' conclusions.

The Conference ended with a statement and summary by Mr. M. W. Thring.

DISCUSSION GROUP 1

Subject : Combined flow of fluids and solids

CHAIRMAN : Mr. R. L. Brown
RECORDER : Mr. F. J. Hiorns

In addition to a number of detailed amendments suggested to some of the papers, there were discussions on some major factors, mainly relating to flow through fixed beds.

The discussion was opened by Mr. W. H. Denton (Atomic Energy Research Establishment, Harwell), who gave a brief account of work, as yet unpublished, on the resistance to fluid flow and the heat transfer from packed beds of spheres in cylindrical containers. The experiments covered a wide range of Reynolds numbers up to 100,000, the Reynolds numbers being based on particle diameter, and the velocity based on volume flow per unit free area. The curve for the resistance coefficient against the Reynolds number shows a slight residual hump, which Mr. Denton compared, following earlier workers, with the large hump and dip found at high critical Reynolds numbers for flow past a single cylinder or sphere. He explained that the well-known hump for the isolated spheres arose from sudden alterations in the point of boundary layer separation but was very sensitive to the degree of turbulence in the incident flow. He pointed out that consequently, in a packed bed, only the spheres near the wall where the flow is less disordered would contribute to this hump, which would thus disappear completely for sufficiently large ratios of bed to sphere diameters. This contribution by spheres adjacent to the wall also explains the large scatter in experimental results found for beds having different values of the above ratio over a narrow and critical range of Reynolds number.

The heat transfer (Stanton number) curve plotted against Reynolds' number for flow through a pipe is far below that for flow past a single sphere, which is, on the other hand only just below that for flow through a packed bed of spheres. From these two facts he therefore suggested that flow through packed beds should be considered as a modification of that past a single sphere, and not by an equivalent capillary analogy. This remark substantiates the approach developed by Mr. Hawksley in paper 7.

For flow past bodies such as spheres, the heat transfer coefficient (Stanton number) was shown to be nearly proportional to the skin friction drag. The corresponding heat transfer coefficient against Reynolds number in a packed bed of spheres has a slightly smaller negative exponent than that for a single sphere ; here the comparison with skin friction drag in the bed cannot be made easily since only the total pressure loss for the bed was measured, i.e. skin friction plus form drag loss.

There was an interesting discussion on the relevance of the mass and heat transfer analogy to packed beds. In spite of the difficulty arising from the variations of velocity in a bed of spheres, calculations by Mr. Denton, based on experimental results of both heat and mass transfer, suggested that the analogy holds good, but that it is important that the exponent of the Prandtl and Schmidt groups respectively should be accurately known.

Another discussion was also concerned with flow through packed beds. It was first pointed out that the voidage of spheres is very close to 37 per cent. in a large enough container. This makes it difficult to find a voidage function experimentally. For instance, Carman's function now appears to be a limiting case, failing at high voidages. It was suggested that an expert in theoretical hydrodynamics might calculate the voidage function from the fact that a packed bed contains only two basic shapes of void in a series-parallel arrangement. A question which was asked but remained unanswered was: why is it so difficult to get a close random packing of equal spheres?

Coming next to the general question of characterising particle size for irregularly shaped particles, Mr. B. Standing (University of Manchester) entered a plea for standardisation of definitions, with which there was general sympathy. The method adopted to measure a particle diameter does not necessarily give the diameter appropriate to the application in question. Mr. Hawksley referred to unpublished work relating the Stokes' diameter to volume and surface diameters. He suggested that where there is fluid flow, the Stokes' diameter is the best one to take. This was confirmed by Dr. A. Klinkenberg, (N.V. de Bataafsche Petroleum Maatschappij) who reviewed recent Dutch work by Verschoor and Reman. The latter used Stokes' diameter to relate measurements by different workers, and in spite of the high Reynolds numbers, obtained good results. Dr. Klinkenberg also drew attention to the different friction factors used by Dr. Rose in paper 8 and Dr. Mott in paper 14, and Dr. Rose undertook to clear up the apparent discrepancy.

In reference to Mr. Hawksley's paper, Mr. P. Mason (Building Research Station) queried the use of the factor $\frac{2}{3}$ introduced for packed beds and flocculated suspensions, and asked why it did not apply also to concentrated dispersed suspensions. Mr. Hawksley pointed out that the factor arose from a difference in particle arrangement and that in even the most concentrated dispersed system (voidage about 50 per cent.) the particles would be free to choose an equilibrium arrangement.

Mr. Green (paper 5) was asked whether the air core in the swirl atomiser was equal along its length in both a conical and cylindrical container, and he confirmed that this was the case.

It is heartening that the need for a fundamental physical approach to these questions has begun to be recognised. This is long overdue and especially important in view of the extensive industrial processes in which there is combined flow of fluids and solids. It can perhaps be said that but for the discovery of the electron, we should be much furtherforward today in having adequate design data. At first sight these questions of classical physics have not the glamour of the atom or the aeroplane, but we can feel sure that a physicist in the 1850's would have found them intensely interesting.

It was noteworthy that there was no discussion on three simple questions. Firstly, the trajectory of an irregularly shaped particle in a non-rectilinear

fluid flow, fundamental in designing dust collectors; secondly, on the conditions under which a dense suspension gives separation of particles by density and not by size, (in this connexion some of the papers presented to the International Conference on Coal Preparation in Paris earlier in the year contain the first steps towards a statistical mechanics of dense suspensions); lastly, on the effect on fluid flow of accidental configurations in a shallow packed bed.

In general, we need further careful experiments and more efforts at devising theoretical explanations.

R. L. BROWN.

DISCUSSION GROUP 2

Subject : Fundamentals of mixing and flow patterns

CHAIRMAN : Mr. D. A. Oliver

RECORDER : Dr. A. H. Leckie

The Group discussed two topics in detail—the first was concerned with the mixing of fuel and air, in glass and steel melting furnaces ; and the second with flow phenomena in ejectors, in particular the special problems arising when shock waves were generated.

Dr. E. Seddon (United Glass Bottle Manufacturers Ltd.) opened the discussion on furnaces by asking whether the present system of mixing fuel and air in open hearth-type furnaces was the best one, and whether those present could offer advice on this. He sketched the system at present used in his works and other glass-makers outlined theirs. This at once invited comparison between the burner systems used in the glass and steel industries. The former used several ports and generally some type of pre-mixing chamber, whilst the steel industry used a one port system at each end and no pre-mixing chamber, its representatives expressing some surprise that the glass industry found it possible to use a pre-mixing length without getting sufficient pre-combustion to destroy the refractories. Dr. Seddon said he felt that the flow in the pre-mixing length within the port was stratified to some extent which delayed mixing.

It was clear that the glass tank, being cross-fired, demanded a somewhat shorter flame than steel furnaces which were fired along the long axis, and this need was met by the use of multiple ports and some degree of pre-mixing. In any case the faithful application of fundamental principles was appreciably interfered with by process needs and properties ; for example, the required flame length in glass tanks was governed to some extent by the diathermancy of the glass, and the simple flame jet in steel furnaces could be greatly disturbed by the presence of the piled-up scrap which was being melted down. Practical considerations gave different solutions to the needed compromise between rapid mixing giving a short non-luminous flame, and slower mixing giving a longer luminous flame.

Dr. J. H. Chesters (The United Steel Companies Ltd.) then outlined how the fundamentals of mixing and flow patterns had been applied to a furnace in one of the works with which he was associated. He and his colleagues had first considered the natural "horseshoe" sweep of gases through the in-going, working, and out-going systems of a regenerative furnace, and how the conventional type of furnace could be modified to give much better conformity with the natural flow pattern. The fuel, in this case oil, was injected across the air stream in a way which gave rapid mixing and combustion, and the ends of the furnace were sloped instead of being vertical. This resulted in a port configuration which gave good out-going conditions, the gases at all times running roughly parallel to the walls with little of the impingement which normally causes wear. He outlined another of the important principles involved in this design. In a normal furnace the flame striking the surface of the molten charge in the

hearth splayed out causing transverse vortices which ran up the front and back walls and caused twin channels of wear along the roof. By adjusting the ingoing air velocity, the air stream which normally might have been expected to run along the roof and then curve downwards to oppose the recirculating flame gases, could be made to suppress, at least in part, these damaging vortices. A furnace which had been built according to these principles had now been working long enough to show promising results regarding performance and refractory wear.

Dr. A. H. Leckie (British Iron and Steel Research Association) pointed out that such a design could be used equally well with gas-fired furnaces.

He also pointed out that in addition to the successful industrial trial of a furnace based on model studies quoted by Dr. Chesters, mention might also be made of the changes made at the works of the Shelton Iron, Steel and Coal Co. Ltd.* The single-uptake furnace design which had been shown to give good results by B.I.S.R.A. on the Shelton hot model had been adopted on the production plant, and all the furnaces at this works were now modified accordingly with very beneficial results. This particular model was not concerned so much with the flow pattern as with the mixing and heat transfer.

Mr. Oliver asked whether mixing and combustion could be accelerated by directing the beam from an ultrasonic emitter transversely across the mixing gases. This led to a general discussion of unexpected results observed under supersonic conditions and in particular to some discussion of effects in ejectors.

Dr. R. S. Silver (Federated Foundries Ltd.) said that whilst the simple Bernouilli treatment, which was used by Mr. Smith in paper 13 and had also been used in his (Dr. Silver's) earlier paper to which Mr. Smith had referred, applied where the recovery ratio was low (e.g. the value of 1.2 which might be attained in a thermal compressor), it did not apply in a vacuum ejector where there might be high pressure recovery of the order of 5 : 1. In both cases the velocity of the energising steam was supersonic so that was not the answer. In a steel-furnace fuel-jet the pressure recovery was negligible so that the simple Bernouilli theory should be applicable for such furnaces.

Mr. N. H. Johannsen (University of Manchester) suggested that the abnormal behaviour in high recovery ejectors might be due to the presence of shock waves whose effects would be more pronounced in high pressure recovery systems. He showed static pressure measurements along the length of a diffuser which indicated the position of the shock wave by a sudden jump in static pressure. There was some discussion on the form of these shock waves and Mr. Halliday asked whether such shock waves would help or hinder intimate mixing. Professor A. D. Young (College of Aeronautics) said he thought mixing would be assisted, probably by the shedding of vortices, at least after the passing of the shock wave itself. This phenomenon was stated to show analogies with the initiation of detonation and Mr. D. G. T. Colebrooke (Ministry of Supply) supported this

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and quoted an example of a damaging detonation wave in an internal combustion engine.

Mr. P. O. Davies (University of Cambridge) thought that some of the shock effects in ejectors might be overcome by adding an additional convergent-divergent section, though some doubt was expressed as to whether this would reduce the overall efficiency. At this point the discussion had to be closed through lack of time, although it was clear that lively argument could have been continued for some time.

More discussion of the four papers under general review might have been obtained. During the discussion few ideas on the problems outlined by the various authors had been put forward. However, at least the steel and glass industries had a thorough discussion of their problems.

D. A. OLIVER.

DISCUSSION GROUP 3

Subject : Flow problems in industries employing high temperature furnaces (steel, glass etc.)

CHAIRMAN : Dr. E. Seddon

RECORDER : Mr. I. M. D. Halliday

All the members of group 3, whether representing Government Science, the Research Associations, or the industries which employ high temperature furnaces, played an important part in the sometimes controversial, yet always enthusiastic discussions, in which a great many topics and aspects were considered.

It was thought essential to define, at an early stage of the discussion, the primary requirements of the industries represented, so that a definite objective could be set. There was a close measure of agreement in the case of the steel and glass industries, and the requirements, arranged in order of importance, were agreed to be :(i) high productivity, i.e. good steel-making or glass-making efficiency ; (ii) good thermal efficiency ; (iii) long working lives for furnaces, i.e. good performance by refractories.

The group agreed that simultaneous improvement in all three directions, which were interdependent, should be sought. Pursuing the requirements separately might not give real success. Thus one might operate a furnace at high thermal efficiency, without necessarily producing the required amounts of good glass or good steel. Equally there would be little point in obtaining long furnace life, unless steel or glass with the desired quality, and in the necessary quantity, could be obtained. Overall improvement must be sought rather than progress only in one of the three directions.

Although steel-making was an intermittent process, while glass-making was continuous, both industries aimed at raising the ratio of steel to fuel or glass to fuel. In the steel industry, the aim was to melt a charge in the shortest possible time, so that the steel melted per week was made as large as possible ; every attempt was made, at the same time, to reduce fuel consumption. In the glass industry, the glass load was increased, so far as the glass-making machines arranged around the furnace could be increased in number (this increase was usually limited by space considerations), and by raising the load on each machine ; very strong efforts were made to reduce fuel consumption to the minimum. Thus both industries sought to increase the numerator, and to reduce the denominator, in the ratio product to fuel.

Figures were quoted for the thermal efficiencies of steel-making, and of glass-making furnaces. Although such efficiencies were low in an absolute sense, they compared favourably with such installations as modern power stations. Present thermal efficiencies were probably not too far removed from the best standards which can be achieved by conventional designs, i.e. the better existing furnaces possessed a reasonable relative efficiency. For greater increases in thermal efficiency, revolutionary designs would be necessary ; such designs must, of course, satisfy all the three requirements outlined above.

The group agreed that the furnace loading should be increased to the maximum which is attainable while still retaining high quality. For minimum fuel consumption at such high loading, close attention must be paid to the pressure within the combustion space, and to the overall combustion conditions ; optimum conditions for these should be sought. In so far as the conditions of flow govern the mixing of the fuel and air, and also influence combustion, heat transfer, length of furnace life and performance of refractories, better knowledge of and conditions of flow must be sought. A careful choice of refractory materials, appropriate to their locations in the furnaces, must be made.

One school of thought, within the discussion group, favoured instantaneous combustion ; at the same time it wanted luminosity. This school also asked for recirculation to spread the flame somewhat and give greater uniformity of heating. It was admitted that instantaneous combustion might result in local over-heating of the refractories with a deleterious effect on the latters' life. A furnace with an accentuated Venturi design might give the conditions desired.

The other school of thought was in favour of less rapid combustion, with a longer flame to give better distribution over the bath, i.e. they preferred to measure efficiency by the *average* rate of heat transfer. This group was less concerned with recirculation, and preferred an approach which seemed more likely to avoid local overheating of the refractories.

Long life for the refractories appeared to be one of the main considerations for each of the schools of thought.

In the glass industry, external forced cooling was used on the refractories which were most severely stressed. No forced cooling was applied to refractories subjected only to medium stresses ; these had some natural cooling. External thermal insulation was applied wherever the conditions, i.e. very lightly stressed refractories, permitted. In the steel industry, the use of air or steam cooling on the hot face was considered as a possible means of avoiding local overheating. The new design of steel furnace, incorporating a single uptake, and sloping end wall, largely offered protection to the walls and roof, by preventing the gas velocities becoming too high. At present, the most practical solution to local overheating, seemed to be to keep the gases away from the refractories.

The use of somewhat higher crowns in glass tank furnaces, had already been tested ; the slightly higher crowns gave a reasonably good thermal efficiency, longer crown life, and greater flexibility, in terms of the ability to melt larger glass loads, while still maintaining high glass quality. In a steel furnace, a somewhat higher crown might result in longer roof life, even if vortex impingement was known to occur. It was by no means certain that the height of crown, used in present-day steel furnaces represented an optimum in terms of radiation or in terms of life of refractories.

Mr. J. A. Leys (British Iron and Steel Research Association) described work done on models in which air jets carrying aluminium powder, were used to study the impingement of the powder (which simulated iron oxide in the

full-scale furnace), on the sidewalls and roof. (Each of the latter had been coated with an adhesive.) The zones of deposition of the aluminium powder seemed to correspond with the positions of severest wear on full-scale furnaces. The design used initially was of the oil-fuel, double air port type, with a sloping back wall, having a " bulge " at its centre. If the oil burner was deviated, to an appreciable extent, from the normal towards the back wall, the deposition on the roof and back wall was diminished materially. When a single air port, beneath the oil burner, was substituted for the original double air ports, and when a vertical back wall, instead of a sloping one, was adopted, there was a marked reduction in deposition. The deposition in large steel furnaces seemed to be due to the action of vortices (demonstrated in the flow pattern studies) causing local overheating, and to the impingement of iron oxide particles. There was probably a compound effect of chemical attack on the silica refractories (resulting in eutectic formation), and of local overheating, by convection, with some scouring action.

Besides the flow techniques already described in the papers, the group considered further aspects of the use of radon.

Scepticism was expressed about the possible future application of fundamental fluid flow data to the blast furnace ; the complicated and unpredictable variations in fluid flow parameters in this type of plant, made any application extremely difficult and probably impossible.

E. SEDDON.

DISCUSSION GROUP 4

Subject : Flow problems in the process industries (gas, oil, chemical etc.)

CHAIRMAN : Mr. W. A. Simmonds

The group started its discussion with a contribution by Mr. G. H. Bygrave (Anglo-Iranian Oil Co. Ltd.) dealing with experimental work on liquid—liquid ejectors or jet pumps which supplemented paper 13. Mr. Bygrave showed a diagram of the design of ejector that he used, which was quite standard, and said that he had measured the efficiency of an injector system in which a jet of water was used to entrain water. He showed a graph of his results which demonstrated that efficiencies of the order of 25 per cent. were obtained although this could be reduced to 2 per cent. under faulty operating conditions. He suggested that this phenomenon explained the discrepancies in the published values of efficiencies given by various authorities. Mr. Bygrave had also examined the efficiency of an ejector in which a kerosene jet entrained water and obtained results qualitatively similar to the water/water system with efficiencies up to about 22 per cent. In the case of a kerosene/soda system (a common refinery material) the results were again quantitatively similar with a maximum efficiency of 18.5 per cent. The value of the ratio of the quantity of entrained to entraining fluid was sensibly constant over a large range of rates of flow but decreased to zero as the rate of flow of the entraining liquid approached zero and became zero at a finite value of flow in the jet. He pointed out that this differed from the case of gas entraining gas mentioned by Mr. Simmonds in the discussion on paper 13 when it was stated that this ratio had a minimum value and then increased as the flow of motive gas approached zero. The form of the variation of maximum efficiency with the density ratio of the two fluids was a straight line of negative slope. These experiments were repeated on a plant scale using a 6 in. pipeline for the motive fluid with similar results.

A discussion followed on what was meant by the efficiency of an ejector and it was decided that the energy efficiency must be used, that is the ratio of the energy gained by the entrained fluid to the energy available in the motive fluid.

The use of the phrase " most economical ejector " in paper 13 was then examined and it was agreed that Mr. Smith had chosen a criterion which was usually applicable although it was not generally true and in fact cases were suggested when it could not be applied.

The question whether two-stage ejectors gave higher efficiencies than the single stage type was considered and it was concluded that, while no direct experience was available to us, on the whole no advantage was obtained except in special cases—in particular when used for steam. In the course of this discussion, the use of petticoats in ejector design was mentioned and Mr. H. E. Dall (George Kent & Co. Ltd.) explained the meaning of this term and gave it as his opinion that no advantage could

arise from their use and in fact there must be an energy loss due to the dissipation of energy by friction on the extra surfaces introduced.

Several points arising from paper 9 were next considered. In answer to a query Mr. Dall explained that the discharge coefficient could be greater than unity in high ratio orifices or nozzles as a consequence of the method of defining this coefficient. Since the energy in the stream approaching the orifice was greater than that due to the mean velocity over the whole stream, it was possible for the energy in the jet issuing from the orifice to be sufficient to give a velocity in the jet (which is nearly uniform over its cross-section) such that the discharge coefficient is greater than unity.

In answer to points raised by Mr. E. Ower, (British Shipbuilding Research Association) Mr. Dall explained that there was some advantage in a small area ratio of an orifice because, as could be seen from the curves that he had given in his paper, he had plotted the variation of the discharge coefficient C against the square root of the Reynolds number and it was possible to obtain a working range for a small ratio meter over which C was sensibly constant although this could not be done for a large ratio. He agreed with Mr. Ower that the peaks in the curves of C against $\sqrt{R}$ occurred in the transition region which must be unstable. Consequently the repeatability of the measured values of C was poor for high area ratios but he claimed that this could be good for low ratios. He did not regard this as important since this region was out of the working range of the meter.

The problem of the measurement of very small rates of flow of gas—of the order of 100 c.c./min. was next posed. It was agreed that this could be accomplished using a series of small orifices of the order of 1/32 in. diameter in thin material spaced about 1/16 in. apart. Mr. Dall said that he had used jewelled orifices for work of this nature down to flow rates of 100 l/h. Dr. G. Brown (Ministry of Supply) suggested that the quartz fibre anemometer might be suitable for such measurements. This instrument is essentially a very fine quartz fibre mounted as a cantilever. The deflector of this by a low velocity gas stream is measured and the instrument is calibrated in a stream of known velocity. The Chairman said that he was using a quartz fibre anemometer and obtained satisfactory measurements of the speed of flow down to the order of 1 cm./sec. Mr. Dall gave a warning that while this method could be used satisfactorily under steady temperature conditions it was only applied in the region of viscous flow and was very susceptible to changes in viscosity caused by temperature variations.

This subject led on to the problems of measurements of small pressure differences—in particular to measurements over a range of up to 2 in. water gauge to an accuracy of 0.001 in. water gauge without recourse to the Chattock Fry type of gauge with all of its attendant inconveniences. The discussion included the consideration of the advantages and disadvantages of the inclined manometer, a modification of this given in Ower's *The Measurement of Air Flow*, the two liquid differential gauge and finally Prof. A. D. Young (College of Aeronautics) gave an account and drawing

of the Beck manometer which is a robust instrument, convenient to use and free from capillarity errors. It has an optical magnification system giving it a range of 0—4,000 mm. with an accuracy of 0.01 mm.

The next problem examined was that of flow control under industrial conditions—an accuracy of $\frac{1}{2}$ per cent. being suggested. Several types of controller were discussed—the mechanical, servo-operated and electronic (in particular the capacity pick-up). It was agreed that while a variety of highly satisfactory controllers were available which were sensitive to very small variation in flow conditions, the problem of the design of the valve mechanism to alter the flow in response to the output of the controller was very difficult.

Dr. J. O. Hinze (Royal Dutch Shell Laboratory) then contributed a discussion of the velocity distribution in a free jet. He explained that a theory based on the assumption of constant viscosity in the jet had been suggested for the fully developed turbulent jet, and that there was available an empirical formula giving fair agreement with experimental measurements. It was known, however, that the flow in the boundary region of the jet alternates between the streamline and turbulent states and while the velocity distribution acrooss the jet approximates to a Gaussian form, the true velocity departs from the predicted values towards the edges of the jet. Some kind of intermittency factor was required and one had been developed from the flattening factor, which was similar to the Kurtosis of Statistics. Application of this factor gives improved results, a fact which is of importance in mixing processes.

The discussion concluded with a short consideration of fluidization and what was implied by this term. It was explained that it was by no means new and that patents on some aspects of the subject had been taken out 20 to 30 years ago. The name is relatively new and the process is becoming increasingly popular. Apart from its advantages in giving greatly increased mixing, which is important in reaction and combustion processes, it is widely used for moving materials—such as the transport of finely divided materials and in such operations as removing grain from silos.

W. A. Simmonds.

DISCUSSION GROUP 5

Subject : Flow problems in industries based on steam generation (power industries)

CHAIRMAN : Dr. R. S. Silver

RECORDER : Dr. W. B. Carlson

The group started with a discussion arising from paper 15. It was asked whether the ship hull design was in fact nearly final since there appeared to the layman to have been relatively little change in general hull form since Viking days. In reply it was pointed out that there had been considerable changes particularly those caused by the substitution of screw propulsion for sails. The flow phenomena in this problem differ fundamentally from those discussed elsewhere in the conference in the presence of an interface between two fluids of very different properties. A considerable proportion of the resistance arose from the wave drag of this interface—to which there was no analogy in the other flow problems. At high speeds the wave making resistance could equal the combined skin and displacement resistances.

The position of hydrofoil ship designs was queried. In these the displacement was to a large extent substituted by the drag associated with the lift forces of the hydrofoil, and the main body suffered resistance only from the air. Dr. Conn pointed out, however, that if the hydrofoil were applied to ships other than the light high-speed craft the main body had to be structurally connected by suitable struts and again the wave resistance of these struts would become serious.

To reduce skin friction the filling in of plate overlaps by suitable plastics seems to help. But aerodynamic design for hulls suffers considerably from fouling. Anti-foul paints offer hope of protection for reasonable periods.

The discussion then changed to formation of steam bubbles around propeller blades which causes the troublesome cavitation attack. The problem had also been referred to in paper 4 in relation to centrifugal pumps. It was pointed out that the hydrodynamic theory of cavitation given by Lord Rayleigh indicated pressures of order 100 t/in$^{(2)}$. which few materials would be expected to withstand. A modification by Silver of the Rayleigh theory showed that thermodynamic effects of latent heat release exerted a braking action on the bubble collapse and impact pressure on this theory came down to the order of 10 to 15 tons/in/$^{(2)}$. This result seemed reasonable as indicating that materials could withstand cavitation conditions for some time. It was suggested that perhaps bonded rubber on the propeller or impeller blades could preserve them by absorbing the cavitation shocks. It was reported however, that in trials the rubber bond had not held.

The effect of dissolved air in cavitation was discussed and it was said to promote bubble formation but to provide a cushioning effect when the bubbles collapsed. Some discussion centred around the possibility that super pure liquids might not cavitate.

Various opinions were expressed as to the ultimate action by which material of blades was removed during cavitation. There seemed to be considerable scope for physical research into the conditions of temperature and chemical activity and the collapse of a bubble.

The question was asked whether the methods of fluid flow study reported in the conference could equally well be employed in steam boiler furnace research. It was said that in some respects it was more difficult to apply to boilers owing to the large temperature variations occurring in the passage of gases through the furnace. Possibly a hot model technique would have the greatest hope of success, although some workers have used cold models for studying very limited portions of boilers wherein the gas stream could be regarded as having constant temperature.

When designing boiler furnaces the steam engineer has some advantage over the steelmaker in that he can to a large extent shape his furnace to suit his fuel and mode of firing. Having determined the shape of flame to use, he then disposes his furnace boundaries so as to avoid impingement and fouling, and to absorb the desired proportion of radiant heat.

It was pointed out, however, that these remarks apply to the larger water tube boilers and that there are still many of the older types, not only in use but still being made, in which means have to be devised to ensure that the requisite fuel is burnt in appropriate quantity within the confines of a fixed combustion space. This may demand considerable ingenuity to obtain the desired mixing. It is possible that some of the principles of mixing brought out in the conference may be of value here.

A specific problem of a fan about 2 ft. diameter working at 1,400 r.p.m in a temperature no less than 800°C. was mentioned. It was asked whether suggestions for redesigning the fan and/or the associated ducting could be made with a view to obtaining the same gas flow with lower fan speed, so as to reduce blade creep. One possible answer to this problem was to divide the total ducting among several fans.

Water Cooling Towers

A problem in aerodynamics was raised in connexion with hyperbolic water cooling towers for power stations. Tests had shown that a major portion of the air passes up the centre to the tower whereas uniform velocity over the cross-section was desired. The questioner thought it strange that, as the air entered the base from the perimeter in the turbulent flow, it did not tend to take the shortest path up near the side walls. There was considerable discussion as to whether this problem could suitably be studied by models. The principal difficulty envisaged in using models was proper simulation of the packing at the foot, and it was suggested that a section of a large-scale model might be studied. The meeting recommended, however, that it would be better to study a parallel-sided slice, or better still a small scale full model without attempting to simulate the packing exactly.

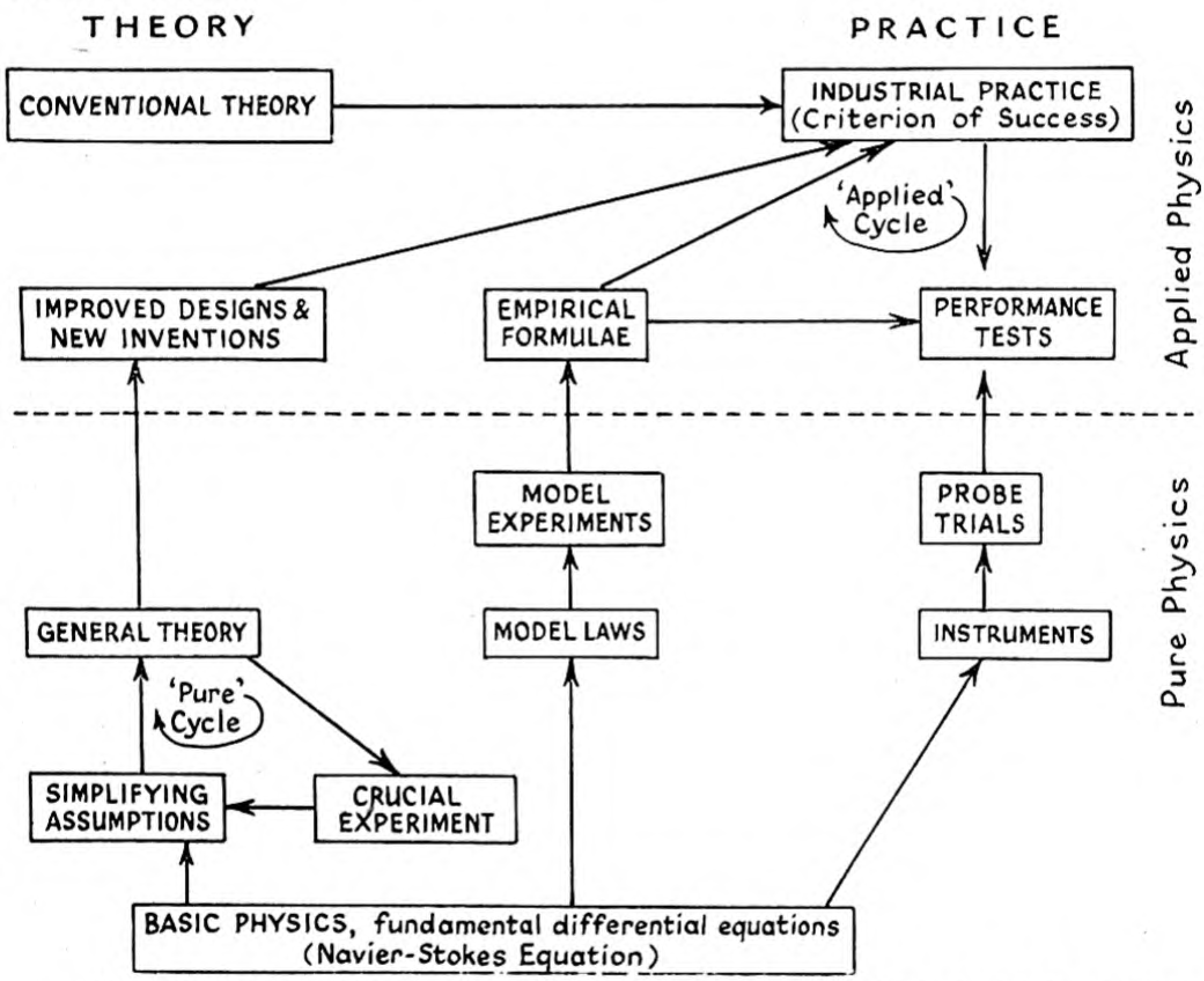
R. S. Silver.

CONCLUDING STATEMENT AND SUMMARY

by Mr. M. W. Thring

I think the main conclusion to be drawn from the conference is that the introduction of physics to the study of fluid flow in various systems, particularly inside medium sized chambers, is producing a " fluid flow physics " just as the introduction of physics to metallurgy has produced " metal physics ".

By medium sized chambers I mean those systems which are not so large in relation to the phenomenon that the effect of the walls can be neglected (as the effect of the walls of a wind tunnel are neglected when studying the airfoil) nor so small that the flow can be treated as essentially belonging to a single pattern, (as for example inside a tube where all the average velocities are parallel to the axis of the tube and one is concerned only with variations from point to point of the magnitude of the velocity and not of its direction).



The accompanying figure illustrates the way in which I think physics should be applied to industrial practice. Although admittedly no clear line can be drawn between them, in my view the upper half of the diagram roughly corresponds to engineering and the lower half to physics, while the left hand side of the diagram corresponds to the more theoretical aspects of the subject and the right hand side to the more practical or experimental aspects. In the top right hand corner of the diagram comes industrial practice. The subjects under this heading which have been discussed at the conference are aircraft, ships, furnaces, ejectors, fuel beds, the atomisation of fluids, cooling towers, and boilers. To each system there is attached

a criterion of success which in the long run comes down to the overall efficiency of the process in terms of labour and materials, and the first step in research connected with improving industrial practice must be the evaluation of this criterion of success. For example, one group at the conference discussed the relative importance of fuel economy, elimination of wear of refractories and increase of output as the criteria of success.

Simultaneously with the appearance on the scene of industrial practice in any given field one has also the appearance of what might be called "conventional theory", on which the first attempts at design are based; some of these themes may almost be called "old wives' tales." As examples of this, the theory that the volume of a furnace divided by the volume of gas flowing through it per second should give the quotient of 2 sec. had been mentioned and so had various forms of guesses at the way gas flows in a furnace and the theory that the visible flame should stop at the outgoing side of a furnace. While conventional theory must always play a part in deciding industrial practice, the aim should be to strengthen it by less intuitive processes.

There is normally a circulation between the three processes in the top right hand side of the diagram in which industrial practice is submitted to tests in which various designs are compared and their criteria of success measured. Performance tests in turn lead to empirical formulae under which heading some of the examples for formulae for pressure drop in fuel beds could be placed. Empirical formulae in turn lead to improvements in the design of the appliance. Successive steps around this closed "applied cycle" correspond to improvements in practice.

Turning now to physics, at the bottom of the diagram could be placed basic physics, i.e. the fundamental laws or differential equations which are well established as governing the processes going on in the given appliance. It is not quite appropriate to call this science "classical" physics since when the present diagram is applied to metallurgy and metal physics, the laws of wave mechanics are the laws in this fundamental section. In the present case of fluid flow the Navier-Stokes equation* is the complete expression of the basic physics concerned. The problem is then to represent the manner in which the Navier-Stokes equation of basic physics can be related to and can provide improvements in the design of industrial appliances involving fluid flow.

The diagram suggests that there are essentially three lines by which this relation takes place. The line on the right hand side of the diagram is fairly straightforward and the most empirical of the three. From the fundamental differential equations, physical instruments such as pitot tubes, the Schlieren process, orifices for metering, and the radon technique were designed, these being subjects all of which have been discussed at the conference, although the need for a probe suitable for measuring very low velocities is one of the unsolved problems thrown up by the conference. These instruments are then used in probe trials on the industrial appliance

* See for example Prandtl and Tietjens *Fundamentals of Hydro Aero-Mechanics* p. 259 equation (5).

in which the flow pattern occurring is evaluated in detail and these probe trials in turn lead to suggestions for performance tests with other variables.

The second line of approach from the basic physics is the development of model laws from the Navier-Stokes equation which gives rise directly to such criteria as the Reynolds and Froude criteria. These model laws can then be used to design model experiments wherein the flow processes corresponding to the complex geometry of the industrial system can be studied under conditions where they can be watched more closely, where the shape can be altered more readily and where the effects of other processes can be eliminated. In connexion with model experiments the name of Dr. P. O. Rosin should be especially mentioned as a pioneer in the use of model experiments for combustion systems.

A number of model experiments have been discussed at the conference including ship models and furnace models with cold air, cold water and small scale combustion and fuel bed models, but there has been rather a tendency to short circuit the model law step and carry out model experiments without, perhaps, enough detailed discussion of the validity of the laws on which the similarity was based.

The results of model experiments are expressed in the form $A = f(B,C,D)$ where A is the dimensionless criterion involving the dependent variable, and B,C,D are the dimensionless criteria involving the independent variables. These formulae then provide a direct evaluation of the empirical formulae of the cycle referred to above.

There has also been an example at the conference of the cross link from model experiments to a process on the third or left hand line, in the improved design of an open-hearth furnace, discussed by Dr. J. H. Chesters, which is now being tested on the full scale and which has arisen from laboratory scale models. There is, naturally, some reluctance to accept improvements, empirical formulae, and design based solely on model work which can only be overcome by the success of designs arising in such a way and this in turn must depend on the scientists basing their model work firmly on a careful analysis of the model laws and supplementing the improvements by the third line of contact, the theoretical one.

This line starts from basic physics but requires the process of hypothesis formation i.e. the introduction of simplifying assumptions, before it can proceed further. Certain examples of such simplifying assumptions have been produced during the conference as, for example, the ideas in the interaction of particles and fluids introduced by Mr. A. D. Denton and by Mr. P. G. W. Hawkesley, the suggestions for explaining the facts on fluid flow through orifices by Mr. H. E. Dall the boundary line theory discussed by Professor Young, general conceptions of recirculation and entrainment and the conception of eddy viscosity in jets. Nevertheless it does appear that it is in the introduction of fruitful new simplifying assumptions that the physics of the subject of medium sized chamber fluid flow can make the greatest advance in the near future ; particularly, we need an assumption whereby the general direction and velocity of flow at any point in a chamber in which recirculation, bouncing of jets off surfaces, and so on, are proceeding.

I consider the triangle on the left hand side as the " pure cycle ". From the simplifying assumptions proceed the purely mathematical development of a general theory. This in turn leads to certain conclusions which can be tested by specially designed experiments and these decide whether the choice of simplifying assumptions is a fruitful one in explaining a certain range of phenomena or not. These crucial experiments may of course be done in equipment entirely different both in size and shape from that involved in the industrial process. As for the " applied cycle " the " pure cycle " involves successive circulation just as applied scientists sometimes tend to regard their cyclic process as completely closed and to neglect the arrows going into it from outside, so pure scientists sometimes tend to regard their cyclic process as closed and to neglect the arrows leading from it. In fact, however, the main result of an improved general theory as far as the outside world is concerned, is the appearance of improved designs and new inventions as indicated on the diagram. General theory can also lead to improvement in the formulation of model laws by indicating which of all the significant criteria can be ignored and which can be carefully made equal in model and original.

In the case mentioned above of metal physics, the simplifying assumption which has proved particularly fruitful in establishing the physical part of the diagram is the dislocation hypothesis in which the dètailed wave mechanics of the nuclei and electrons are ignored and the metal is treated as a group of elastic spheres appropriately packed and able to yield plastically by the movement of dislocations along an atom plane. Another example of the applicability of the diagram is in the study of luminous radiation where the basic physics are the laws of electromagnetic radiation passing through a medium containing fine conducting particles and where the simplifying assumption which is needed relates to the number of such soot particles which will be in any given flame.

There had been considerable emphasis at the conference on the difficulty in introducing radically improved designs for large scale systems such as furnaces and boilers, but I feel that some way round this difficulty would undoubtedly be found when " fluid flow " physics is firmly established because thermodynamics shows clearly what is the ideal fluid flow system in such appliances and therefore as soon as " fluid flow " physics can show how to obtain these conditions the advantages of introducing them will be clear. The situation is, I believe somewhat analogous to the situation of the internal combustion engine half a generation ago when the theoretical advantages of eliminating reciprocating mechanisms was clear to everyone but the difficulties in the way of the internal combustion turbine appeared insuperable.

In conclusion I wish to stress that the conference indicates the need for a clear growth of the subject I have called " fluid flow physics " to increase and provide a firm basis for its application in practice, but that this growth requires the introduction of one or two master hypotheses before the theoretical aspects of the subject can be placed on a really firm foundation.

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